

# Numerical solution for stick-slip oscillator with geometric non-linearity

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**Abstract.** Linear spring mass framework controlled by moving belt friction have been subjected to various examinations. Dynamical attributes like amplitude and frequency of oscillations have been in a big way studied along by the whole of the different approach mechanisms for this model. Along by all of the dynamical characteristics, bifurcation structures also have been investigated. On the other hand, the corresponding self-excited SD oscillator has not instructed comparable attention. This complimentary presents the numerical investigation of the character of a self-excited SD oscillator resting on a belt moving with consistent speed and excited by dry friction. The moving belt friction is displayed as the Stirbeck friction (friction first decreases and then increase smoothly with interface speed) to figure the scientific model. It is demonstrated that the pure-slip oscillation phase influenced by system parameter  $\alpha$ . The influence of different system parameters on the dynamical characteristics was alongside considered.

Keywords: Self-excited SD oscillator, Stick-Slip, Pure-Slip and base frequency

## 1. Introduction

A lot of consideration has been paid to the self-excited vibration on account of friction in functional mechanical designing frameworks, i.e., brake, equipment tools, and others. The majority of the action of analysis in the literature are based on moving-belt in which mass is resting on the belt and supported by spring-damper to describe the friction-generated vibration in mechanical systems. The most of the study is done on the model which is described by a spring-mass-damper linear oscillator framework on the moving belt to move the dynamics which is directed by friction, such as periodic motion, chaotic behavior, and stick-slip. Thomsen and Fidlin [1] acquired analytical expression for stick-slip and pure-slip vibration amplitudes for the single degree of freedom linear system spring-mass on moving belt arrangement. Abdo and Abouelsoud [2] employed Liapunov second technique to approximate the amplitude of the velocity and displacement of the stick-slip movement of a single degree spring-mass-damper linear system on a driven belt. Thomsen [3] has furthermore demonstrated how friction-induced stick-slip motion are influenced by harmonic excitation and how high-frequency excitation can successfully expel the negative gradient effectively in the friction-velocity relationship, so it is vital to preventing self-excited oscillations. Further, Andreaus and Casini [4] have investigated the effect of belt velocity and friction modeling on the response of stick-slip oscillations in the



same model. Popp, Hinrichs and Oestreich [5] studied the bifurcations and chaotic behavior of a friction oscillator with simultaneous self and external excitation. They also studied the influence of different friction characteristics on the dynamic response of a friction oscillator. Hinrichs and Oestreich [6] have presented the dynamics of a non-smooth friction oscillator under self and external excitation and also the bifurcational behavior anticipated by numerical simulations and results compared with practical outcomes. Cheng et al. [7] have been concentrated the dynamic attributes of a mass on moving belt show, offered by external excitations. Furthermore, the parametric review was displayed in which the excitation emerges from the variable stiffness of a linear spring in the framework while the external excitation comprises of a harmonic force. Devarajan and Bipin [8] obtained analytical expression for stick-slip and pure slip vibration amplitudes for the classical duffing oscillator on-moving belt system. At present, there are numerous nonlinear friction models portrayed by geometric nonlinearities of versatile extensive distortion in designing, for example, the geometric nonlinear vibration instigates by contact between the brake plate and pad. Santhosh et al. [9] examined discontinuity initiated bifurcations seen in the geometric nonlinear smooth and discontinuous (SD) oscillator and in systems with friction are explored utilizing Filippov strategies. Li et al [10] proposed a modified archetypal self-excited SD oscillator with geometrical nonlinear friction oscillator moving on belt and demonstrated that the system characteristics of multiple stick regions, parabolic shape transition and friction give rise to asymmetry. Under perturbation, the framework initiated vibration of numerous stick-slip phenomena. Li and Cao [11] studied the multiple stick-slips chaotic motions of a self-excited SD oscillator which is driven by moving belt friction based upon the SD oscillator and classical moving belt. Li et al [12] investigated the local as well as global bifurcations on the basis of Stribeck friction function model of an geometrical nonlinear self-excited smooth and discontinuous (SD) oscillator. From the preceding cases, it is clear that extensive studies have been carried out on stick-slip and pure-slip oscillations of self-excited SD oscillator which is excited due to dry friction with several friction models and with or without harmonic excitation analytically as well as numerically. However, the amplitude response of stick-slip and pure-slip oscillation of self-excited SD oscillator moving on a belt excited by dry friction with constant velocity has not been studied. The main purpose of this paper is to study the influence of system parameter  $\alpha$  on the system vibration amplitude in stick-slip and pure-slip phases and the change in the frequency of the system and also studied the influence of friction difference on the stick-slip vibration amplitude of the system. This paper is arranged as follows. All the numerical models of related with this review are presented in segment 2. Section 3 provides numerical solution and discussion of the results. At long last, a few comments on this paper are closed in segment 4.

## 2. Problem Explanation and Equations of Motion

The mathematical model consists of differential equations with discontinuous right hand side. Assuming a small but finite difference in static and kinematic friction. This system comprises with a mass  $M$ , which is supported by a moving belt and associated with a settled support through an inclined linear spring of stiffness coefficient  $K$ , and a viscous damping coefficient  $C$ , as shown in Figure 1. The mass vibrates due to dry friction  $F_f$ , which is modeled as Stribeck friction on the contacting surface. The belt moves with a steady speed  $V_b$  and is supposed to be non-deformable. Here, the mass ought to move with the driven belt in the horizontal direction without losing the contact. The state of the mass on the moving belt at some specific time  $t$  is taken as  $X$ .

From the figure 1 we can write:

$$\sin \gamma = \frac{X}{\sqrt{X^2 + H^2}} \quad \cos \gamma = \frac{H}{\sqrt{X^2 + H^2}}$$

Spring force =  $K\delta_0 + K\left(\sqrt{X^2 + H^2} - L\right) = F'$ ,  $K\delta_0$  is very small term so it can be neglected.

Horizontal component of the system:

$$M\ddot{X} = F_f - F' \sin \gamma - C\dot{X} \quad (1)$$

Where Friction force:  $F_f = \mu(v_r)F_n$ , and  $\mu(v_r)$  is the coefficient of friction between the mass and the belt.

And vertical component:

$$F_n = Mg - F' \cos \gamma = Mg - KH \left( 1 - \frac{L}{\sqrt{X^2 + H^2}} \right) \quad (2)$$

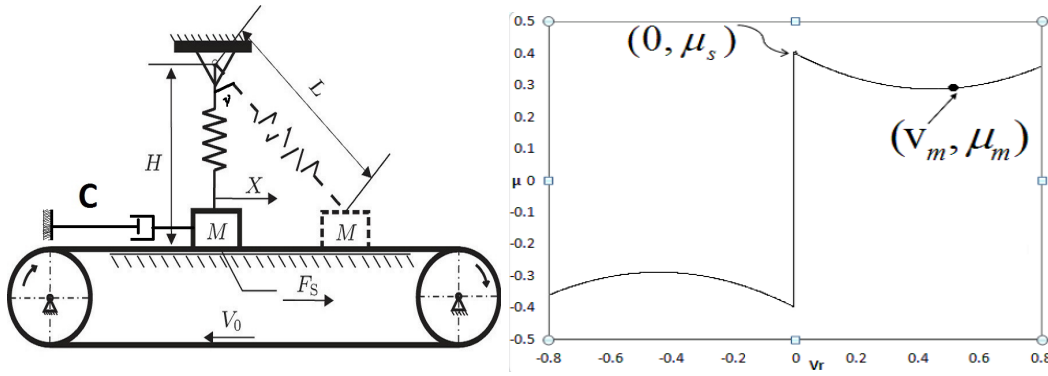


Figure 1: a).The dynamical framework as self-excited SD oscillator where a mass-spring-damper on a moving belt b).Stribeck friction model

The equation of motion for the above model can be obtain from Eqn.(1), by substituting the value of  $F_f$ ,  $F' \sin \gamma$  and  $F_n$

$$M\ddot{X} + C\dot{X} + KX \left( 1 - \frac{L}{\sqrt{X^2 + H^2}} \right) = -\mu(v_r) \left( Mg - KH \left( 1 - \frac{L}{\sqrt{X^2 + H^2}} \right) \right) \quad (3)$$

The dimensionless equation of motion for the given framework can be gotten by expecting  $x = \frac{X}{L}$   $2\zeta = \frac{C}{\omega_0 M}$   $\omega_0 = \sqrt{\frac{K}{M}}$   $\alpha = \frac{H}{L}$   $v_0 = \frac{V_0}{L\omega_0}$   $\tau = \frac{t}{t_0}$   $t_0 = \frac{1}{\omega_0}$   $g_1 = \frac{g}{L\omega_0^2}$

The equation of motion is, in non-dimensional form:

$$\ddot{x} + 2\beta\dot{x} + x \left( 1 - \frac{1}{\sqrt{x^2 + \alpha^2}} \right) = -\mu(v_r) \left( g_1 - \alpha \left( 1 - \frac{1}{\sqrt{x^2 + \alpha^2}} \right) \right) \quad (4)$$

And according to Stribeck friction model, which shows the friction-velocity relationship for the above model with boundary lubrication, friction function  $\mu(v_r)$ :

$$\mu(v_r) = \mu_s \operatorname{sgn}(\dot{x} - v_b) - k_1(\dot{x} - v_b) + k_3(\dot{x} - v_b)^3 \quad (5)$$

With the help of Eqn.(5) one can transform the equation of motion into

$$\ddot{x} + 2\beta\dot{x} + x \left( 1 - \frac{1}{\sqrt{x^2 + \alpha^2}} \right) + \left( \mu_s \operatorname{sgn}(v_r) - k_1(v_r) + k_3(v_r)^3 \right) \left( g_1 - \alpha \left( 1 - \frac{1}{\sqrt{x^2 + \alpha^2}} \right) \right) = 0 \quad (6)$$

where  $v_r = \dot{x} - v_b$  is non-dimensional relative velocity amongst mass and belt, and

$$k_1 = \frac{3}{2} \frac{(\mu_s - \mu_m)}{v_m} \quad k_3 = \frac{1}{2} \frac{(\mu_s - \mu_m)}{v_m^3}$$

Note: where the dot indicates derivative w.r.t. time  $\tau$

This equation of motion can be divided into two different phase on the basis of relative velocity, the slip phase and the stick phase. Amid the slip phase, the mass and belt have diverse speeds ( $\dot{x} \neq v_b$ ), so equation for the slip mode is given by:

$$\ddot{x} + 2\beta\dot{x} + x \left(1 - \frac{1}{\sqrt{x^2 + \alpha^2}}\right) + \left(\mu_s \text{sgn}(v_r) - k_1(v_r) + k_3(v_r)^3\right) \left(g_1 - \alpha \left(1 - \frac{1}{\sqrt{x^2 + \alpha^2}}\right)\right) = 0 \quad (\text{slip}) \quad (7)$$

Throughout the stick phase, the mass and belt have equal velocity and so there is no acceleration in the system ( $\dot{x} = v_b$ ). Eqn.(6) can thus be written as,

$$\ddot{x} = 0 \quad 2\beta\dot{x} + x \left(1 - \frac{1}{\sqrt{x^2 + \alpha^2}}\right) \leq \mu_s \quad (\text{stick}) \quad (8)$$

The above equations are differential equation and Eqn.(7) is having discontinuity. The solution of the above differential equation should be continuously differentiable and smooth in its given domain. Here, spring is linear and the system is strongly non-linear due to the geometrical configuration and a variable friction force, which is not only a function of the velocity  $\dot{x}$ , but also of the position  $x$ .

By using the Taylor series expansion for  $\frac{1}{\sqrt{x^2 + \alpha^2}}$ , rewriting the equation:

$$\frac{1}{\sqrt{x^2 + \alpha^2}} = \frac{1}{\alpha} - \frac{x^2}{2\alpha^3}. \quad \text{Neglecting higher order terms,}$$

$$\ddot{x} + 2\beta\dot{x} + x - \frac{x}{\alpha} + \frac{x^3}{2\alpha^3} - \frac{x^2}{2\alpha^2}(\mu_r) + ((\mu_r)(g_1 - \alpha + 1)) = 0 \quad \text{for } (\dot{x} \neq v_b) \quad (\text{slip}) \quad (9)$$

$$\ddot{x} = 0 \quad 2\beta v_b + x - \frac{x}{\alpha} + \frac{x^3}{2\alpha^3} \leq \mu_s \quad \text{for } (\dot{x} = v_b) \quad (\text{stick}) \quad (10)$$

where  $\mu_s$  is coefficient of static friction and  $v_m$  is the velocity at which coefficient of kinetic friction  $\mu_m$  is minimum, where  $\mu_m \leq \mu_s$  and  $k_1, k_3 > 0$ . From the Figure 1(b), it is clear that  $\mu \leq \mu_s$ , when the mass and the belt moves with same velocity ( $\dot{x} = v_b, v_r = 0$ ), the phase known as stick phase. When mass slides on the belt, friction force first decreases with the velocity and then increases, this phase is known as slip phase ( $\dot{x} \neq v_b, v_r \neq 0$ ).

From Eqn.(9), the mass has a static equilibrium at  $x = \bar{x}$  and  $\ddot{x} = 0, \dot{x} = 0$

$$\bar{x} - \frac{\bar{x}}{\alpha} + \frac{\bar{x}^2}{2\alpha^2} \left(\mu_s - k_1 v_b + k_3 v_b^3\right) + \frac{\bar{x}^3}{2\alpha^3} - (\mu_s - k_1 v_b + k_3 v_b^3)(g_1 - \alpha + 1) = 0 \quad (11)$$

One can find  $\bar{x}$  by solving above equation numerically for particular cases, and it is taken that  $\bar{x} = B$  (Expression for  $B$  is given in the Appendix). To study about the motion near this equilibrium, origin should be shifted by defining,

$$u(t) = x(t) - \bar{x}, \quad \dot{x}(t) = \dot{u}(t) \quad \ddot{x}(t) = \ddot{u}(t) \quad (12)$$

By substituting the above value in the Eqn.(8) and (9), equation can be transformed into

$$\ddot{u} + u + h(u, \dot{u}, \text{const}) = 0 \quad \text{for } (\dot{u} \neq v_b) \quad (\text{slip}) \quad (13)$$

Where

$$h(u, \dot{u}, \text{const}) = B + 2\beta\dot{u} - \frac{u + B}{\alpha} - \frac{(\mu_s \text{sgn}(v_r) - k_1(v_r) + k_3(v_r)^3)(u + B)^2}{2\alpha^2} + \frac{(u + B)^3}{2\alpha^3} + (\mu_s \text{sgn}(v_r) - k_1(v_r) + k_3(v_r)^3)(g_1 - \alpha + 1) \quad (14)$$

$$v_r = \dot{u} - v_b$$

$$\ddot{u} = 0 \quad u + 2\beta v_b + B - \frac{u + B}{\alpha} + \frac{(u + B)^3}{2\alpha^3} - \mu_s \leq 0 \quad \text{for } (\dot{u} = v_b) \quad (\text{stick}) \quad (15)$$

The equilibrium of the system at  $u = \dot{u} = 0$  for the Eqn.(13) compatible to the steady state sliding, where it is assumed that mass is at rest and belt slides along a smooth surface while maintaining continuous contact with constant velocity  $v_b$ . For Eqn.(13) and (15), two different types of periodic solutions are possible. First one is pure-slip oscillation, where  $\dot{u}(t) < v_b$ ; which means that the velocity of the mass is always lesser than the speed of the belt  $v_b$ , and second one is stick-slip oscillation, where  $\dot{u}(t) \leq v_b$ ; it means that the velocity of the mass can be equal(stick) to or lesser than the belt velocity(slip). Here viscous damping and dry friction will take care of the energy stored by the spring, until a stationary state is achieved.

### 3. Numerical Simulations and discussion of the results

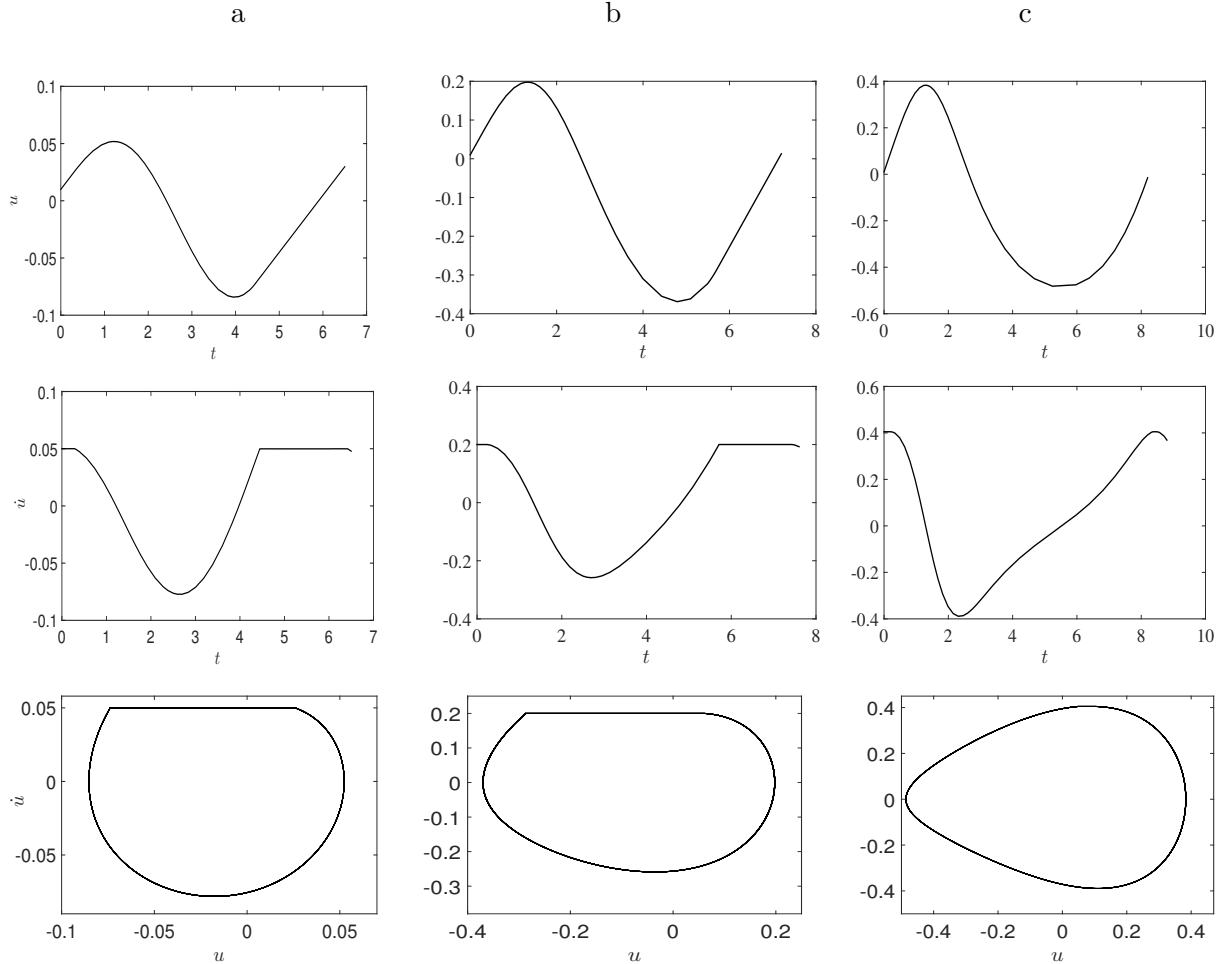


Figure 2: One period of stick-slip displacements (first), velocity (second), and phase plane plots (third row) for three different values of belt velocity  $v_b$ . The value of parameters were used for the plots:  $\beta = 0.05$ ,  $\mu_s = 0.20$ ,  $\mu_m = 0.05$ ,  $\alpha = 0.8$  and (a)  $v_b = 0.05$ , (b)  $v_b = 0.20$ , (c)  $v_b = 0.405$

Figure 2 describes the time history and phase plot figures for single cycle of stationary stick-slip oscillations. These curves represent numerical simulation results of Eqn.(13) and (15).

Each figure is plotted for one particular value of excitation speed  $v_b$  with the parameter values mentioned in figure 2. In figure 2(a) the belt velocity is small and corresponding to that, oscillation amplitudes are reduced and stick phase takes a substantial part. From figure 2(b), the belt velocity is mid-range velocity which yields stable oscillations, whose magnitude is higher than that of figure 1(a); also stick phase is observed to be reduced. It is clear that when belt velocity increases, stick region reduces and finally becomes pure slip at  $v_b = v_{b0}$ . It means that there is no stick (horizontal line shows stick) region at a particular velocity, known as critical belt velocity.

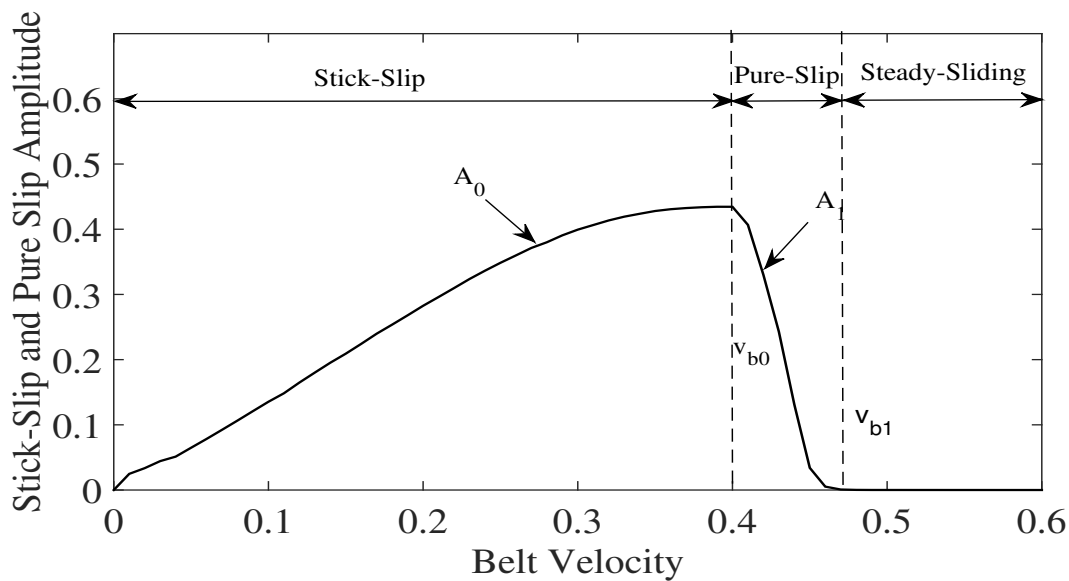


Figure 3: Plot between stick-slip and pure-slip amplitude vs. belt velocity parameters used:  $\beta = 0.05$ ,  $\mu_s = 0.20$ ,  $\mu_m = 0.05$ ,  $\alpha = 0.8$ .

Figure 3 is the variation of stick-slip and pure-slip displacement amplitudes with belt velocity for particular values of typical parameters. Here  $A_0$  shows the amplitude during stick-slip oscillation for  $v_b < v_{b0}$  and  $A_1$  is the amplitude for pure slip oscillation. When the belt velocity increases from zero, the stick-slip oscillation occurs with increasing amplitude until  $v_b = v_{b0}$  beyond which pure-slip oscillation takes over. This pure-slip region occurs for very contracted range of belt velocity until  $v_{b1}$ , above which oscillation stops and steady slip attains stable type of motion. Different types of graph are plotted below with belt velocity by changing the other parameters. In figure 4(a) graph is plotted between amplitude vs belt velocity for different value of geometrical parameter  $\alpha$ , which shows amplitude increase as the value of  $\alpha$  increases. But the pure-slip region is same in both cases. In figure 4(b), graph is plotted between amplitude vs belt velocity for different values of  $\alpha$ , which shows amplitude increase when the value of  $\alpha$  is increased. Figure 4(b) which shows frequency is reduces as the value of  $\alpha$  increased. But the pure-slip region is same in both cases. Figure 5. shows the significance of friction difference in stick-slip and pure-slip oscillation amplitude as a function of belt velocity. If the difference between friction coefficient is small, the largest amplitude oscillation occurs at the critical belt velocity  $v_{b0}$ , provided the non-linearity is small. For the largest friction difference, the occurrence of largest amplitude shifts to a velocity which is less than  $v_{b0}$ . If the friction difference is small, pure slip region is large and as the friction difference increases the pure slip region gets smaller. For the smaller value of friction difference, the relationship between

stick-slip amplitudes and the excitation speed is almost linear for smaller value of geometrical parameter  $\alpha$ .

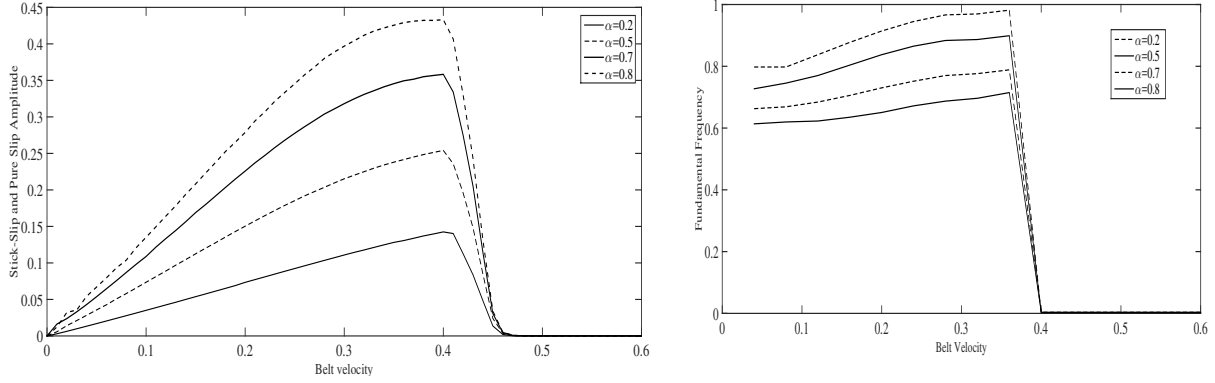


Figure 4: (a) : Plot between Stick-slip amplitude Vs Belt velocity for different values of  $\alpha$ , (b) Plot between fundamental frequency Vs Belt velocity for different values of  $\alpha$ .

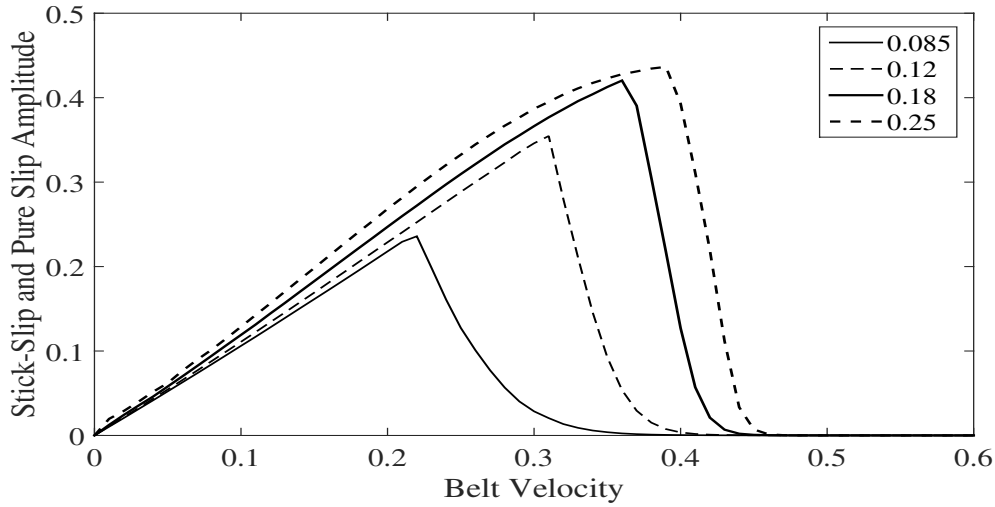


Figure 5: Plot between stick-slip and pure-slip amplitude vs Belt velocity for different values of  $\frac{\mu_s - \mu_m}{v_m} = (0.085, 0.12, 0.18, 0.25)$  and other parameters are  $\beta = 0.05$ ,  $\mu_s = 0.20$ ,  $v_m = 0.5$ .

#### 4. Conclusion

This work attempts to study the amplitude of pure-slip and stick-slip oscillation of a self excited SD oscillator placed on a belt moving with constant velocity under the presence of dry friction. The mechanical model and mathematical formulation is done. Figure 3 shows pure slip oscillations occur at higher speeds compare to stick-slip oscillation  $v_b = [v_{b0}; v_{b1}]$  for a narrow range, beyond which the model shows steady sliding with no oscillation. It is also clear that pure slip is effected be the geometrical parameter  $\alpha$ . Influence of system parameters along with friction difference on vibration characteristics, such as amplitude and base frequency, is also studied.



## Appendix

$$B = \frac{1}{3} \left( \frac{\alpha^2 (-6\alpha + (-k_1 v_b + k_3 v_b^3 + \mu_s)^2 + 6)}{\sqrt[3]{a+b}} + \sqrt[3]{a+b} - \alpha\mu_s + \alpha k_1 v_b - \alpha k_3 v_b^3 \right)$$

$$A = \alpha^3 D (D^2 - 27g_1 - 18) - 18\alpha^4 D$$

$$B = \sqrt{\alpha^6 \left( (6(\alpha - 1) - D^2)^3 + D^2 (18(\alpha - 1) + D^2 - 27g_1)^2 \right)}$$

$$D = -k_1 v_b + k_3 v_b^3 + \mu_s$$

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