Shear rate analysis of water dynamic in the continuous stirred tank

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Abstract. Analysis of mixture in a continuous stirred tank reactor (CSTR) is an important part in some process of biogas production. This paper is a preliminary study of fluid dynamic phenomenon in a continuous stirred tank numerically. The tank is designed in the form of cylindrical tank equipped with a stirrer. In this study, it is considered that the tank is filled with water. Stirring is done with a stirring speed of 10rpm, 15rpm, 20rpm, and 25rpm. Mathematical modeling of stirred tank is derived. The model is calculated by using the finite element method that are calculated using CFD software. The result shows that the shear rate is high on the front end portion of the stirrer. The maximum shear rate tend to a stable behaviour after the stirring time of 2 second. The relation between the speed and the maximum shear rate is in the form of linear equation.

1. Introduction

Continuous stirred-tank reactors (CSTR) are widely applied in the chemical, food, and pharmaceutical engineering fields. Zhang *et. al.* [1] did 3-dimensional Computational Fluid Dynamics (CFD) simulations that carried out to portray the flow characters in the CSTR of the mini-plant using speed of 400rpm. Studies about shear rate has been performed by some researchers. Kumar [2] analysed the effect of geometric parameters on the energy dissipation and shear rate. Pérez *et. al.* [3] had studied the relationship the rotational speed and the average shear rate in a stirred vessel by using a rigorous theoretical analysis. Safrit *et. al.* [4] noted that the effects of monomer content, molecular weight, temperature, and shear rate had to be investigated in the modeling of the reaction mass viscosity.

Johnsen [5] looked at particles in different rheology to develop a methodology for measuring settling velocities in static systems. In the study the variation of shear rates it has been used to get the comparison of experimental and theoretical velocities for Power-law values. Vlaev has simulated the shear rate distribution in the flow fields of two basic impellers comprising flat and fluid-foil blades and compared with measurement data obtained by electro-diffusion. Pérez *et. al.* [3] also showed that for both Newtonian and non-Newtonian power law fluids agitated in stirred vessels, the average shear rate γ in the fluid is a function of the rotational speed N of the impeller, i.e. $\gamma = \text{constant} \cdot N$ for the case of laminar flow, and $\gamma = \text{constant} \cdot N^{3/2}$ for the case of turbulent flow

This paper will show the distribution of shear rate in the slice of domain at the level of baffles. The distributions are considered in the speed variation.

2. Shear rate problem

Shear rate is an important parameter in a surface aerator, but it is not easy to characterize. Knowledge of the shear rate is essential for the design and operation of surface aerators. The specific energy dissipation rate in a stirred tank well known to depend on the shear rate γ and the shear stress τ, as follows:

$$\frac{P}{V} = \tau\gamma \tag{1}$$

where P is the power input and V is the volume of fluid in the tank. Furthermore, for Newtonian fluids, the viscosity μ is the ratio of shear stress and shear rate, i.e.

$$\mu = \frac{\tau}{\gamma} \tag{2}$$

Therefore, the Equation (1) can be written as follows:

$$\frac{P}{V} = \mu\gamma^2 \tag{3}$$

or

$$\gamma = \sqrt{\frac{1}{\mu}\frac{P}{V}} \tag{4}$$

The Equation (4) applies to laminar, turbulent and transitional flow. For agitation under laminar flow, the power number (N_p) and the agitator Reynolds number (R_r) are related as follows:

$$N_p = \frac{C}{R_r} \tag{5}$$

where the constant C depends on the geometry of the tank and impeller.

3. Finite element analysis for fluid dynamic in stirred tank

To analyze the shear rate with dealt to the fluid dynamic in the tank, it is considered the equation of Navier-Stokes is assumed to give the dynamic as follows.

$$\rho\frac{\partial u}{\partial t} + \rho(u \cdot \nabla)u = \nabla \cdot \left[\rho i + \mu(\nabla u + (\nabla u)^T) - \frac{2}{3}\mu(\nabla \cdot u)i\right] + F \tag{6}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \tag{7}$$

The wall is assumed in the model as in the Equation (8), and the rotating wall equation as in Equations (9) and (10). The rotating domain is using the Equation (11). In the model it is also assumed the symmetry for upper of domain as in the Equations (12) - (14).

$$u = 0 \tag{8}$$

$$u = v_{wall} \tag{9}$$

$$\left(\frac{\partial x}{\partial t}\right)\Big|_x = v_{wall} \tag{10}$$

$$dx = dx(r_{bp}, \omega, t) \tag{11}$$

$$u \cdot n = 0 \tag{12}$$

$$K - (K \cdot n)n = 0 \tag{13}$$

$$K = [\mu(\nabla u + (\nabla u)^T)]n \tag{14}$$

The geometric model is imported from the existing from the COMSOL document [7]. By using Finite Element Method, the mesh is generated using tetrahedron as depicted in Figure 1 . A careful selection of the mesh type and number is considered before the simulation. These have a significant influence for calculation accuracy and efficiency. In 90% of the overall volume of the CSTR hexahedron mesh type is used in the meshing scheme for its higher calculation accuracy in 3-dimension multi-phase simulation.
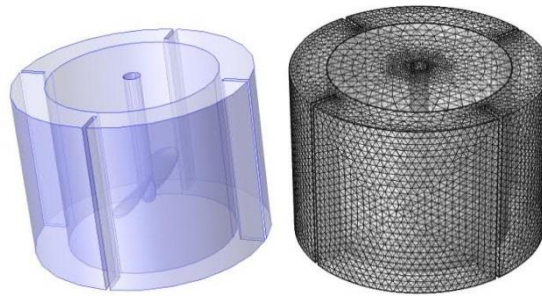

Figure 1. Model and mesh of the domain

4. Result and discussion

This section will show the distribution of shear rate in the slice of domain an the level of baffles. The distributions are considered in the speed of 10 rpm, 15 rpm, 20 rpm, and 25 rpm time by time. Figure 2 shows the distribution of shear rates in the case of speed 25 rpm.
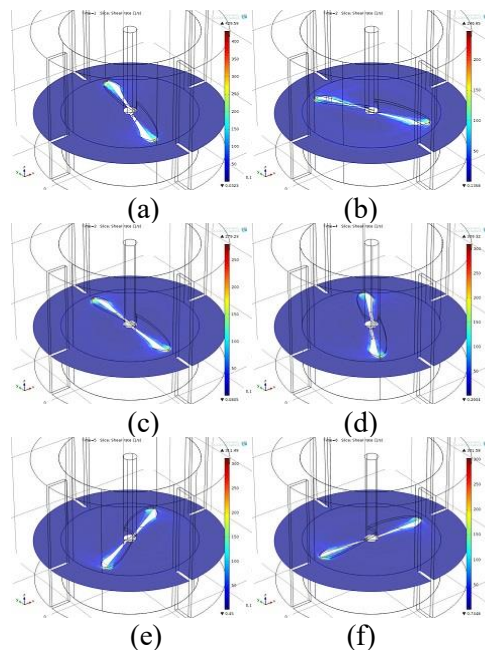

(a)   (b)

(c)   (d)

(e)   (f)

Figure 2. Shear rate distribution at the slice in the level of blade for
(a) 1s, (b) 2s, (c) 3s, (d) 4s, (e) 5s, and (f) 6s in the case of speed 25 rpm.

Table 1 Maximum shear rates

Time (s)	Maximum shear rate (s^{-1})			
	25rpm	20rpm	15rpm	10rpm
0.25	23.904	19.163	14.397	9.5939
0.5	122.39	100.12	77.292	53.175
0.75	274.18	222.04	169.46	115.95
1	429.59	344.45	261.23	178.13
1.25	517.25	414.87	313.25	211.74
1.5	478.69	391.35	295.41	200.93
1.75	334.91	299.39	221.64	149.56
2	246.65	212.67	171.33	111.67
2.25	285.25	212.58	169.98	115.04
2.5	303.6	218.47	169.33	119.14
2.75	317.95	239.04	165.26	117.72
3	279.23	242.73	165.85	118.76
3.25	286.27	240.35	173.03	114.38
3.5	292.41	234.9	172.6	116.81
3.75	300.67	214.9	176.74	113.3
4	309.32	218.58	181.09	113.55
4,25	289.08	220.03	176.61	108.77
4.5	285.13	235.17	170.08	111.59
4.75	301.77	239.91	170.38	114.67
5	311.49	227.6	155.69	117.19
5.25	301.12	211.79	157.82	118.67
5.5	287.79	226.8	159.75	118.12
5.75	284.12	221.33	161.17	118.26
6	301.58	243.29	168.8	116.65

The distributions are depicted at the several time started by 0.25 second and followed by 0.5 second in sequence until 6 second. From the Figure 2, it can be seen that the highest shear rates are happen near the end of the baffle.

According to the maximum shear rate depend on the time (Table 1), it can be seen that the highest share rate in all type of speed are happen in the time of 1.25 second, i.e. 517.25 s^{-1}, 414.87 s^{-1}, 313.25 s^{-1}, and 211.74 s^{-1} for the speeds of 10rpm, 15rpm, 20rpm and 25rpm, respectively. The shear rate comes to a stable condition after the time 2 second.

Figure 3 shows the graphic of the maximum shear rates vs time. From the figure it can be seen that there is a relationship between the speeds and the maximum shear rates. It can also be seen that the shear rates tend to stable condition after the time of 2 second.
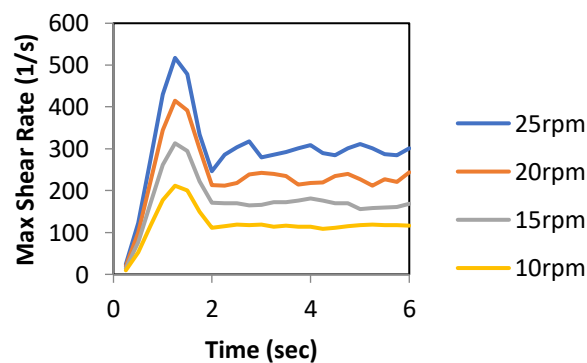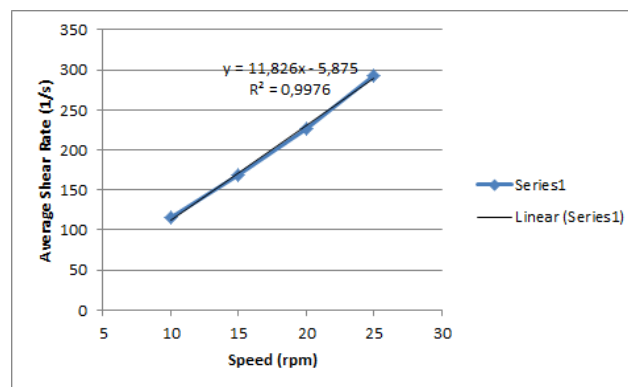

Figure 3. Maximum shear rate vs time

Table 2 The average maximum shear rate after the time of 2 second

Speed (rpm)	Maximum shear rate (s^{-1})
10	115.546
15	168.559
20	227.067
25	293.143

Table 2 shows the average maximum shear rates after the time of 2 second. From the table it can be found that the average shear rates have a relationship depend on the speed. According to the average shear rates. Figure 4 shows that the relationship form a linear relation as follows.

$$y = 11.826x + 5.875 \qquad (15)$$

with $R^2 = 0.9976$. It can be concluded that the relation is linear.

Figure 4. Graph of average shear rate and speed

5. Conclusion

The computational reasoning presented here leads to equations for correlating the average shear rate in continuous stirred tank operated with various types speed in laminar flow regimes. In Newtonian fluids, the average shear rate in laminar flow is confirmed to depend on the impeller rotational speed N.

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