

Determination of the boundary conditions of the grinding load in ball mills

Rashid R Sharapov

Moscow State University of Civil Engineering (National Research University),
Yaroslavl highway 26, Moscow, 129337, Russia

Corresponding author: ptdm_zavkaf@mail.ru

Abstract. The prospects of application in ball mills for grinding cement clinker with inclined partitions are shown. It is noted that ball mills with inclined partitions are more effective. An algorithm is proposed for calculating the power consumed by a ball mill with inclined inter-chamber partitions in which an axial movement of the ball load takes place. The boundary conditions in which the ball load is located are determined. The equations of bounding the grinding load are determined. The behavior of a grinding load is considered in view of the characteristic cross sections. The coordinates of the centers of gravity of the grinding load with a definite step and the shape of the cross sections are determined. It is theoretically shown that grinding load in some parts of the ball mill not only consumes, but also helps to rotate the ball mill. Methods for calculating complex analytical expressions for determining the coordinates of the centers of gravity of the grinding load under the conditions of its longitudinal motion have developed. The carried out researches allow to approach from the general positions to research of behavior of a grinding load in the ball mills equipped with various in-mill devices.

1. Introduction

Cement production is the basic construction industry. The main aggregates for grinding clinker and additives are ball mills [1-3]. However, calculations have shown that the efficiency of ball mills does not exceed 2% [4]. To increase the efficiency of ball mills, they are equipped in-mill devices [5, 6]. All known theoretical studies are based on the solution of the analytic expression (1).

$$P = A_A \gamma_b V \sqrt{D} \{S_A\}, \quad (1)$$

where A_A is the constant S. E. Andreev; S_A – high-speed power factor.

Since in all these works conventional ball mills are considered in which the mill load factor does not change during one cycle, the power consumed by the mill drive remains unchanged. However, there are a number of works [5, 6], which show that the function $P(S)$ has a harmonic sinusoidal character.

In our proposed method for calculating the power consumption, we make an attempt, based on experimental data, to take into account the influence of the transverse form of loading along the length of the mill on the amount of power consumed by it, and also the change in the zone of active influence of the partition with a change in the angle of rotation of the mill's drum. In this case, the rotation of the mill drum is considered as a series of quasistatic states at a fixed angle of rotation of the drum ξ .

The algorithm for calculating the power consumed by a ball mill according to the proposed method is as follows:

1. The mill body is divided into sections with a constant cross-sectional shape of the load. 2. Within each section the loading is considered consisting of separate "parts" of constant thickness, normal to



the axis of rotation of the drum. 3. For a fixed position of the mill drum ($\xi_i = \text{const}$), the coordinates of the center of gravity of each of the "parts" along the length of the mill body are determined. 4. The moment created by the loading within each "part", relative to the axis of rotation of the mill's drum, is calculated. 5. The total load moment relative to the rotation axis for a fixed position of the mill drum ($\xi_i = \text{const}$) is calculated. 6. The power consumed by the ball load is determined at different positions of the mill drum.

2. Determination of mill drive power

The power of the drive mill, spent on moving the load, is determined by the formula, W :

$$P_{en} = \frac{P_0}{\eta_t \eta_{en}} \quad (2)$$

where P_0 – output shaft power, W ; η_t, η_{en} – the efficiency of the mechanical transmission and the engine, respectively.

$$P_0 = P_b + P_l, \quad (3)$$

where P_b – power used to move grinding media, Br ; P_l – power used to overcome losses in trunnion bearings.

To calculate the power required to move grinding bodies P_b , it is necessary to determine the boundary conditions in which the grinding load is located.

As can be seen from the calculation scheme (Fig. 1), the grinding load in the first mill chamber is limited by: the end cover of the mill, the side surface of the mill housing, the loading level A_1A_2 , the surface formed by the displaced volume A_2A_3 , the surface of the inclined partition.

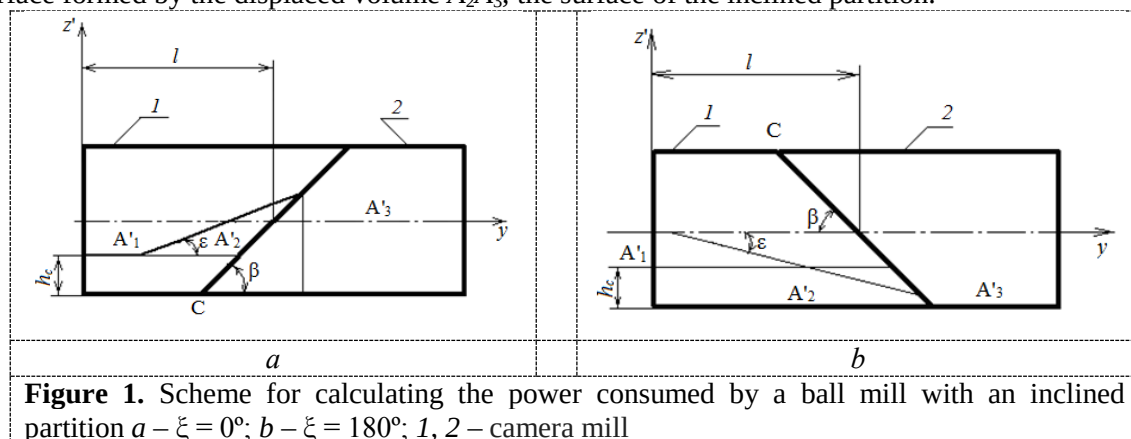


Figure 1. Scheme for calculating the power consumed by a ball mill with an inclined partition $a - \xi = 0^\circ$; $b - \xi = 180^\circ$; 1, 2 – camera mill

3. The definition of the equations of planes, limiting the load

To determine the equation of the surface of a newly formed volume, we consider Fig. 2.

To simplify the calculations, we introduce a mobile coordinate system $x'0z'$ rigidly connected with the angle of rotation of the mill's drum ξ .

As is known, the equation of the plane has the form:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) + D = 0. \quad (4)$$

To determine the coefficients A, B and C , we use the scheme (see Figure 2).

$$A = N \cos \varepsilon \cdot \sin \xi = \cos \varepsilon \cdot \sin \xi, \quad x_0 = 0;$$

$$B = -N \sin \varepsilon \cdot \cos \xi = -\sin \varepsilon \cdot \cos \xi, \quad y_0 = l - l_{pj}; \quad (5)$$

$$C = N \cos \varepsilon \cdot \cos \xi = \cos \varepsilon \cdot \cos \xi, \quad z_0 = -(R - h_c),$$

where N is the unit normal vector ($N = 1$); ε is the angle of the natural slope of the grinding load, deg; l – length of the first chamber of the mill along the axis, m ; l_{pj} – the size of the zone of active influence of the partition at an arbitrary angle of rotation of the drum (see Figure 1), m ; R is the radius of the mill drum, m ; h_c is the level of the grinding load, m .

It is seen that the normal vector N for the coordinates z' and x' is constant regardless of the rotation angle of the drum ξ , and therefore we obtain the following equation of the plane of the newly formed volume:

$$-[y-(l-l_{pj})]\sin \varepsilon \cdot \cos \xi + [z'+(R-h_c)]\cos \varepsilon = 0. \quad (6)$$

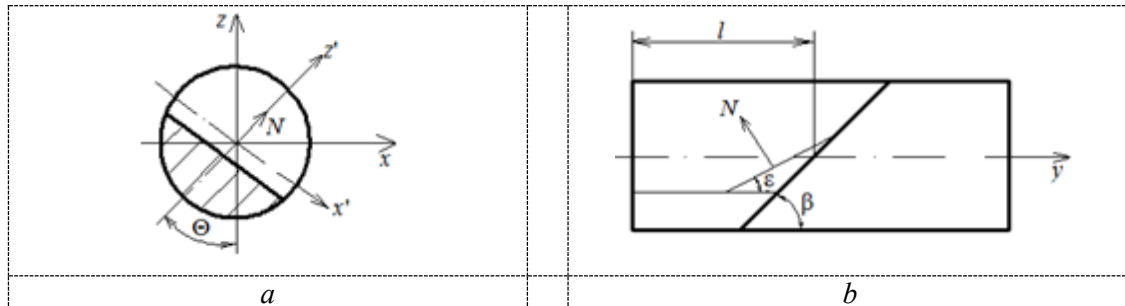


Figure 2. The scheme to determine the coefficients of the equation of the surface of the newly formed volume: *a* - the cross section of the drum; *b* - longitudinal section of the drum

In the formula (6) the parameter l_{pj} the author [6] recommends defining as:

$$l_{pj} = l_{pa} - \frac{1}{2}(l_{pa} - l_{pi}) \cdot (1 - \cos \xi), \quad (7)$$

where l_{pa} is the maximum radius of the zone of active influence of the partition, for $\xi = 0^\circ$ (see Fig. 1, *a*); l_{pi} - the minimum radius of the zone of active influence of the partition, at $\xi = 180^\circ$ (see Fig. 1, *b*).

Substituting equation (7) into equation (6) we finally obtain the equation of the plane of the newly formed volume:

$$-\left[y - \left(l_{pa} - \frac{1}{2}(l_{pa} - l_{pi})(1 - \cos \xi)\right)\right] \sin \varepsilon \cdot \cos \xi + [z' + (R - h_c)] \cos \varepsilon = 0. \quad (8)$$

Similarly, we obtain the equation of the inclined partition, depending on the angle of rotation of the drum. To determine the coefficients *A*, *B* and *C*, we use the calculation scheme (Fig. 3):

$$A = N \sin \xi \cos \beta;$$

$$B = -N \cos(90 - \beta) = N \sin \beta; \quad (9)$$

$$C = N \cos \xi \cos \beta.$$

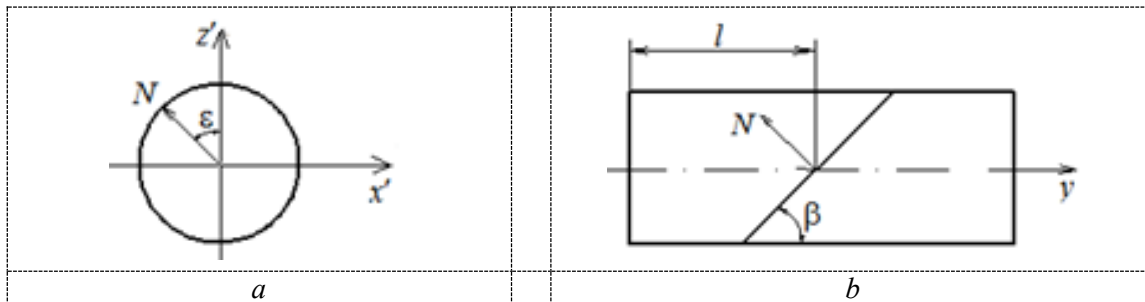


Figure 3. The scheme to determine the coefficients of the equation of the plane of the inclined partition: *a* - the cross section of the mill drum; *b* - longitudinal section of the mill drum

Putting the origin of coordinates in the center of the inclined partition, we get that x_0 and z_0 are equal l . Thus, the equation of the partition becomes:

$$-x' \sin \xi \cos \beta - (y - l) \sin \beta + z' \cos \xi \cos \beta = 0. \quad (10)$$

Ultimately, the load consuming power in the first chamber is limited to planes:

$$\begin{aligned} &-\left[y - \left(l_{pa} - \frac{1}{2}(l_{pa} - l_{pi})(1 - \cos \xi)\right)\right] \sin \varepsilon \cdot \cos \xi + [z' + (R - h_c)] \cos \varepsilon = 0; \\ &-x' \sin \xi \cdot \cos \beta - (y - l) \cdot \sin \beta + z' \cos \xi \cdot \cos \beta = 0; \\ &x'^2 + z'^2 = R^2; \\ &z' = -(R - h_c); \\ &y = 0. \end{aligned} \quad (11)$$

In Fig. 4 we can see that in the first chamber of the mill there are three areas with the characteristic behavior of the load:

I area. On this area, the level of loading of grinding bodies does not change and the length of the section is determined from the expression: $0 \leq y_1 \leq l_1 - l_{pj}$

Here y_1 is the axis coordinate of the area I.

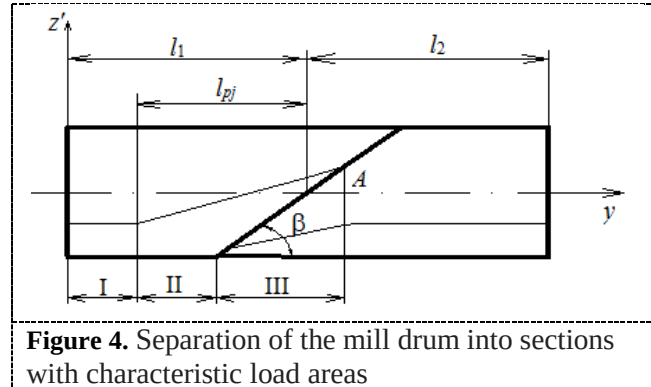


Figure 4. Separation of the mill drum into sections with characteristic load areas

II area. It is characterized by a change in the load by the plane of the newly formed volume from above and is determined from expression:

$$l_1 - l_{pj} \leq y_2 \leq l_1 - R \operatorname{ctg} \beta \quad \text{under } 0 < \xi < 90^\circ \text{ or } 270^\circ < \xi < 360^\circ;$$

where y_2 – is the axis coordinate of the area II.

III area. It is characterized by a change in the load by the plane of the newly formed volume from above and the plane of the inclined partition from below and is determined from the expression:

$$l_1 - R \operatorname{ctg} \beta \leq y_3 \leq y_A \quad \text{under } 0 < \xi < 90^\circ \text{ or } 270^\circ < \xi < 360^\circ,$$

$$y_A \leq y_3 \leq l_1 + R \operatorname{ctg} \beta \quad \text{under } 90^\circ < \xi < 270^\circ,$$

where y_3 – is the axis coordinate of the area III; y_A - the maximum value at the intersection of the plane of the newly formed volume, the inclined partition and the drum body. This value is obtained from the solution of a system of three equations:

$$\begin{cases} - \left[y - \left(l_{pa} - \frac{1}{2} (l_{pa} - l_{pj}) (1 - \cos \xi) \right) \right] \sin \varepsilon \cdot \cos \xi + [z' + (R - h_c)] \cos \varepsilon = 0 \\ - x' \cdot \sin \xi \cdot \cos \beta - (y - l) \sin \beta + z' \cdot \cos \xi \cdot \cos \beta = 0 \\ x'^2 + z'^2 = R^2. \end{cases}$$

Thus, in each of the three sections where the grinding load consumes power, the equations of the load restricting planes are determined at an arbitrary angle of rotation of the mill drum ξ .

4. Determining the coordinates of the center of gravity of the sections

The coordinates of the center of gravity of a homogeneous planar figure, representing the cross-section of the grinding load along characteristic sections (see Figure 4), are determined by the formulas :

$$x_c = \frac{\int dz \int x dx}{\int dz \int dx}; \quad z_c = \frac{\int z dz \int dx}{\int dz \int dx}; \quad x_c = \frac{I_z}{S}; \quad z_c = \frac{I_x}{S}. \quad (12)$$

We introduce a moving coordinate system rigidly connected with the angle of rotation of the drum of the mill ξ . In this case, the coordinate $x_c' = 0$ (since in the I and II areas the loading is symmetrical about the Oz' axis), and the coordinate z_c' is determined from the expression:

$$z_c = \frac{\int_a^b \int_{\varphi_1(z)}^{\varphi_2(z)} z dx}{\int_a^b \int_{\varphi_1(z)}^{\varphi_2(z)} dx} \cdot \quad (13)$$

In this case, the limits of integration in expression (13) are equal to:

$$\varphi_1(z') = \sqrt{R^2 - z'^2}; \quad \varphi_2(z') = -\sqrt{R^2 - z'^2}; \quad a = -R; \quad b = -(R - h_c).$$

As is known, the coordinate z'_c for this section is determined by the formula

$$z'_c = -\frac{2}{3} R \sin^3 \frac{\Omega}{2} / \pi \varphi, \quad (14)$$

where Ω – central angle of the load, rad.

According it can be seen that

$$\Omega = 2 \arccos \frac{R - h_c}{R}. \quad (15)$$

Let us now consider the second section, which is characterized by the fact that the level of loading along z' varies along the length of the drum y and with the rotation angle ξ .

According to the equation (8), the change in the loading position z' will occur according to the following law:

$$z' = D = \left[y - \left(l_{pa} - \frac{1}{2} (l_{pa} - l_{pj}) (1 - \cos \xi) \right) \right] \operatorname{tg} \varepsilon \cdot \cos \xi - (R - h_c), \quad (16)$$

where h_c – the load level in the I area.

The central loading angle Ω' will then be equal:

$$\Omega' = 2 \arccos \frac{z'}{R}, \quad (17)$$

where z' is determined from equation (16).

The third section is characterized by the fact that the contour of the load section is limited by the plane of the newly formed volume and the plane of the inclined partition.

From the equation of the plane of the partition we have:

$$x' = -\frac{(y-l)\sin\beta - z'\cos\xi \cos\beta}{\sin\xi \cdot \cos\beta} = -\frac{(y-l)\operatorname{tg}\beta}{\sin\xi} + z'\operatorname{ctg}\xi. \quad (18)$$

Depending on the position of the lower intersection point relative to the loading level and the Oz' axis, the integration limits and the form of the formulas for determining the coordinates of the load center of gravity will be different.

Solving equation (10) together with the equation of the lateral surface of the drum, we obtain that the lower point is equal to:

$$\begin{aligned} z = F = & \operatorname{tg}\beta \cdot y \cdot \cos \xi - \operatorname{tg}\beta \cdot L \cdot \cos \xi - \\ & - \sqrt{\operatorname{tg}\beta^2 \cdot y^2 \cdot \cos^2 \xi - 2 \cdot \operatorname{tg}\beta^2 \cdot y \cdot \cos \xi^2 \cdot L + \operatorname{tg}\beta^2 \cdot L^2 \cdot \cos^2 \xi -} \\ & - \operatorname{tg}\beta^2 \cdot y^2 + 2 \operatorname{tg}\beta^2 \cdot y \cdot L - \operatorname{tg}\beta^2 \cdot L^2 + \sin^2 \xi \cdot R^2}. \end{aligned} \quad (19)$$

The lower coordinate x' from equation (10) will be:

$$x' = \frac{(y-l)\sin\beta - z'\cos\xi \cos\beta}{\sin\xi \cdot \cos\beta} = \frac{(y-l)\operatorname{tg}\beta}{\sin\xi} - F \operatorname{ctg}\xi. \quad (20)$$

The coordinates of the center of gravity of the cross-sections x'_c and z'_c are determined by the formula (12).

If the lower intersection point of the trace of the plane of the inclined partition with the surface of the drum is greater than the coordinate z' of the intersection of the trace of the plane of the newly formed volume:

$$F \geq D = \left[y - \left(l_{pa} - \frac{1}{2} (l_{pa} - l_{pj}) (1 - \cos \xi) \right) \right] \operatorname{tg} \varepsilon \cdot \cos \xi - (R - h_c), \quad (21)$$

then for all ξ $x' = 0$, and z' is computed by the formulas (14) and (17).

If, in this area, the inclined partition is in contact with the charge,

$$F \leq D = \left[y - \left(l_{pa} - \frac{1}{2}(l_{pa} - l_{pj})(1 - \cos \xi) \right) \right] \operatorname{tg} \varepsilon \cdot \cos \xi - (R - h_c),$$

then the coordinates of the center of gravity of the load section are determined by the formulas (12) for each loading position and the inclined partition.

Taking into account the fact that the constantly changing load section in area *III* does not make it possible to derive a general formula for finding the centers of gravity that takes into account the initial parameters, then the area and moments of inertia are determined by calculating the integrals to determine the coordinates of the centers of gravity of the load:

$$\begin{aligned} I_z &= \int_b^a z' dz' \int_{\varphi_2(z')}^{\varphi_1(z')} dx' - \int_d^c dx' \int_{\varphi_4(x')}^{\varphi_3(x')} z' dz'; \\ I_x &= \int_b^a x' dx' \int_{\varphi_2(z')}^{\varphi_1(z')} dz' - \int_d^c dx' \int_{\varphi_4(x')}^{\varphi_3(x')} x' dz'; \\ S &= \int_b^a dz' \int_{\varphi_2(z')}^{\varphi_1(z')} dx' - \int_d^c dx' \int_{\varphi_4(x')}^{\varphi_3(x')} dz', \end{aligned} \quad (22)$$

where a, b, c и d – limits of figures along the axis z' ; $\varphi_1(z), \varphi_2(z), \varphi_3(z)$ and $\varphi_4(z)$ – equations of surfaces that bound these figures.

the limits of integration in the expressions (22) will be equal to:

$$a = \left[y - \left(l_{pa} - \frac{1}{2}(l_{pa} - l_{pj})(1 - \cos \xi) \right) \right] \operatorname{tg} \varepsilon \cdot \cos \xi - (R - h_c);$$

$$b = -R;$$

$$d = \operatorname{tg} \beta \cdot y \cdot \cos \xi - \operatorname{tg} \beta \cdot L \cdot \cos \xi -$$

$$- \sqrt{\operatorname{tg} \beta^2 \cdot y^2 \cdot \cos^2 \xi - 2 \operatorname{tg} \beta^2 \cdot y \cdot \cos \xi^2 \cdot L + \operatorname{tg} \beta^2 \cdot L^2 \cdot \cos^2 \xi -}$$

$$- \operatorname{tg} \beta^2 \cdot y^2 + 2 \operatorname{tg} \beta^2 \cdot y \cdot L - \operatorname{tg} \beta^2 \cdot L^2 + \sin^2 \xi \cdot R^2};$$

$$c = \left[y - \left(l_{pa} - \frac{1}{2}(l_{pa} - l_{pj})(1 - \cos \xi) \right) \right] \operatorname{tg} \varepsilon \cdot \cos \xi - (R - h_c);$$

$$\varphi_1(z') = \sqrt{R^2 - z'^2};$$

$$\varphi_2(z') = -\sqrt{R^2 - z'^2};$$

$$\varphi_3(x') = \frac{(y-l)\operatorname{tg}\beta}{\cos\xi} + x'\operatorname{tg}\xi; \quad \varphi_4(x') = -\sqrt{R^2 - x'^2}.$$

5. References

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