

Dynamic performance of a suspended reinforced concrete footbridge under pedestrian movements

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Abstract. In the paper the dynamic analysis of a suspended reinforced concrete footbridge over a national road located in South Poland was carried out. Firstly, modes and values of natural frequencies of vibration of the structure were calculated. The results of the numerical modal investigation shown that the natural frequencies of the structure coincided with the frequency of human beings during motion steps (walking fast or running). Hence, to consider the comfort standards, the dynamic response of the footbridge to a runner dynamic motion should be calculated. Secondly, the dynamic response of the footbridge was calculated taking into consideration two models of dynamic forces produced by a single running pedestrian: a 'sine' and 'half-sine' model. It occurred that the values of accelerations and displacements obtained for the 'half-sine' model of dynamic forces were greater than those obtained for the 'sine' model up to 20%. The 'sine' model is appropriate only for walking users of the walkways, because the nature of their motion has continuous characteristic. In the case of running users of walkways this theory is unfitting, since the forces produced by a running pedestrian has a discontinuous nature. In this scenario of calculations, a 'half-sine' model seemed to be more effective. Finally, the comfort conditions for the footbridge were evaluated. The analysis proved that the vertical comfort criteria were not exceeded for a single user of footbridge running or walking fast.

1. Introduction

Many pedestrian bridges erected nowadays have modal properties (natural frequencies) that are close to the critical frequencies of dynamic excitations produced by pedestrians. Potentially, users of this structures walking or running on a footbridge may cause a resonance phenomenon. There are several models of forces produced by a single human being running or walking along a footbridge [1, 2, 3].

In this work the dynamic investigation of a suspended footbridge that carry people over the S7 national roadway in Gaj, South Poland was performed. The central purpose of the research was to compare the dynamic behaviour of the structure obtained for two different models of dynamic forces generated by a single running user of the footbridge. On the basis of the achieved effects of the study, the vibration comfort standards were also assessed.

2. Vibration comfort criteria

The leading aim of dynamic study on footbridges is the evaluation of their vibration comfort standards. There are some standards which are dedicated to the assessment of the influence of oscillations on human beings who are the users of walkways, such as British Standards [4] or ISO [5]. The rules for



design practice are also delivered by Eurocodes [6]. The critical allowed value of vertical acceleration, given by Eurocode (EN 1990:2002/A1:2005), is 0.7 m/s^2 . The rules are appropriate for pedestrian bridges with natural frequencies less than 5 Hz. Other useful rules for the evaluation of the vibrational comfort of pedestrian bridges are proposed by SÉTRA [7]. The acceleration ranges in horizontal and vertical course with comfort stages assigned, provided by this document, are summarized in Table 1. The classification in the frequency domain of the human being motion in the Tab. 2 is presented.

Table 1. Comfort of use levels in the acceleration domain (SÉTRA 2006) [7].

<i>Comfort level</i>	<i>Ranges of comfort [m/s^2]</i>	
	<i>Vertical</i>	<i>Horizontal</i>
Maximum comfort	0.0-0.5	0.00-0.15
Mean comfort	0.5-1.0	0.15-0.30
Minimum comfort	1.0-2.5	0.30-0.80
Uncomfortable	> 2.5	> 0.8

Table 2. The classification in the frequency domain of the human being motion [1, 2].

<i>Type of movement</i>	<i>Frequency ranges [Hz]</i>			
	<i>Total range</i>	<i>Slow</i>	<i>Normal</i>	<i>Fast</i>
Walking	1.40-2.40	1.40-1.70	1.70-2.20	2.20-2.40
Running	1.90-3.30	1.90-2.20	2.20-2.70	2.70-3.30

3. Models of dynamic forces produced by a runner

Vibrant loading normally produced by people on walkways results from walking or running. The dynamic behavior of a footbridge under pedestrians movements is usually obtained by the time history analysis in which the dynamic loading has to be modelled. This type of forces are periodic as well as change in time and space [8]. There are number of mathematical models of forces caused by a single user walking or running. In this study two different models are introduced for the dynamic analysis. Firstly, the force generated by a single walker or runner was used as a sum of static and dynamic parts by the equation [3]:

$$F(t) = G \left[1 + \sum_{i=1}^n A_i \sin(2i\pi f_s t - \varphi_i) \right] \quad i = 1, 2, \dots, n \quad (1)$$

where: G – weight of the human body; f_s – major frequency of the motion; φ_i and A_i – the phase angel of the i -th harmonic and amplitude, respectively. The Fourier constants in formula (1) for different forms of people motion are presented in [3].

The ‘sine’ model provided by formula (1) is adequate in the case of forces produced by walking people because each step overlaps with the previous one thus making the function of loading force continuous. However, in the situation of running this postulation seems to be unfitting, since the loading caused by a running user of footbridge has a discontinuous nature. In this situation, a ‘half-sine’ mathematical model [1] appears to be more adequate. Forces produced by a runner are calculated from the formula:

$$F_z(t) = \begin{cases} k_z G \sin\left(\pi \frac{t}{t_c}\right) & \text{for } t \leq t_c \\ 0 & \text{for } t_c < t < t_u \end{cases} \quad (2)$$

where: k_z – impact factor ($k_z = F_{max}/G$); F_{max} – maximum load produced by runner; G – weight of the human body; t_c – period of contact; t_u – period of load produced by runner; f_u – frequency of running.

4. Geometry and material data of the footbridge

The span of the analysed structure is 50.0m, the whole width is equal to 4.50m and the dimension (height) of the both pylons are equal to 15.5m and 17.5m (Fig. 1). The structural system of the investigated footbridge consists of two reinforced concrete girders (85.00 x 40.00cm). The girders are linked by ten crossbars (thickness 0.16cm, diameter 2.44cm, material: steel) along the span and two crossbars at both ends (90.00 x 50.00cm, material: concrete). The thickness of the footbridge deck is equal to 22.00cm. The deck is supported by twenty trusses (diameter 0.56cm) from steel material pylons.

Elastomeric bearings (height 20.00cm, cross-section 30.00 x 30.00cm) support the footbridge deck. The values of material parameters are summarized in Table 3.



Figure 1. The footbridge located in Gaj (South Poland) along the S7 roadway

Table 3. Parameters of materials of the structure of the footbridge.

<i>Material</i>	<i>Elasticity modulus [GPa]</i>	<i>Poisson's ratio [-]</i>	<i>Mass density [kg/m³]</i>
Concrete	32	0.2	2500
Structural steel	210	0.3	7800
Steel trusses	200	0.3	7800

5. The numerical model and the dynamic characteristics of the structure

The finite element model (FEM) of the investigated footbridge with control points of the walkway selected for dynamic study is presented in Fig. 2. The model was assembling in the ABAQUS software program [9].

In the FEM of the footbridge beam elements were used for crossbars and girders. Deck was solved with shell elements. Solid finite elements were applied for the foundations. Fixed boundary conditions were applied in all cases. This numerical approach reflect the high rigidity of subsoils. The steel hangers were solved as truss finite elements with the 'no compression' material option [9] in order to assurance that compressive effects would not be produced during dynamic time history analysis (THA).

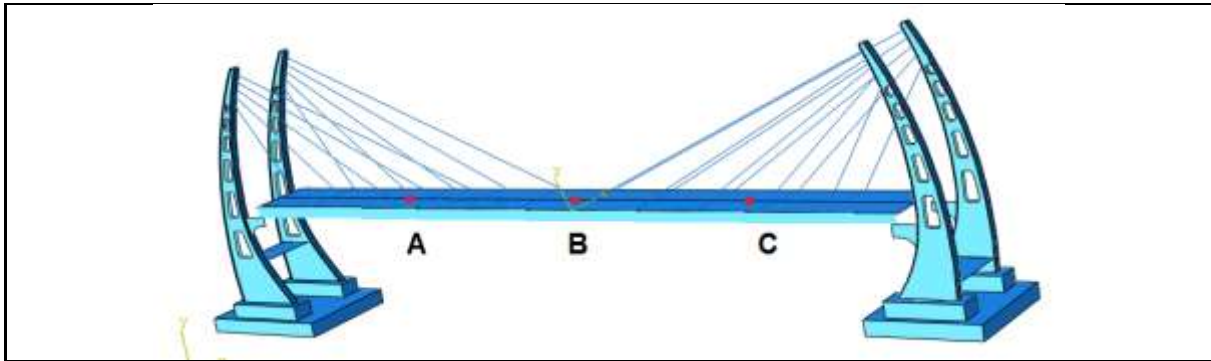


Figure 2. FEM of the investigated footbridge with the selected control points

The evaluation of modal characteristics of pedestrian bridges has been extensively studied [1, 2, 10, 11]. On the basis of the other authors results, it was recognised that the basic natural frequencies of these structures are located generally in the range of 1.4-3.4 Hz. The typical frequency of vertical oscillations produced by running or walking user is also 1.4-3.4 Hz (see Table 2). The evaluated lowest natural frequency of vibrations in vertical direction of the studied footbridge is equal to 1.92 Hz. This value is adequate for the walking as well as running user of the walkways. Taken into consideration the maximum values for the forces generated by walkers and runners, the dynamic evaluation for runner was conducted in the next stage of the study.

6. Two models of forces generated by a runner

In the second step, the response of the footbridge to a single running user were evaluated. The THA was prepared with ABAQUS. The calculations were carried out, during which, the twenty lowest modes were considered with a damping ratio equal to 2% for each mode. Two mathematical models of forces generated by a runner were introduced and the results obtained for both variants were compared. Firstly, the function of forces produced by a single running user, provided by equation (1), was discussed [3]. For the analysis purposes, the values were assumed as follow: pedestrian weight – $G = 800 \text{ N}$; frequency of running – $f_u = 1.92 \text{ Hz}$; period of running – $t_u = 0.52 \text{ s}$; period of contact – $t_c = 0.26 \text{ s}$. In the Fig. 3 the function of the force produced by a runner is presented.

In the next stage of the investigation, the ‘half-sine’ mathematical model of loading generated by a single running user, provided by equation (2), was discussed [1]. For this model, the values were assumed as follow: pedestrian weight – $G = 800 \text{ N}$; impact factor – $k_z = 3$ (according to [2]); frequency of running – $f_u = 1.92 \text{ Hz}$; period of running – $(t_u = 1/f_u) t_u = 0.52 \text{ sec}$; period of contact with the deck – $t_c = 0.26 \text{ s}$. The curve of the function of forces produced by a runner is presented in Fig. 4.

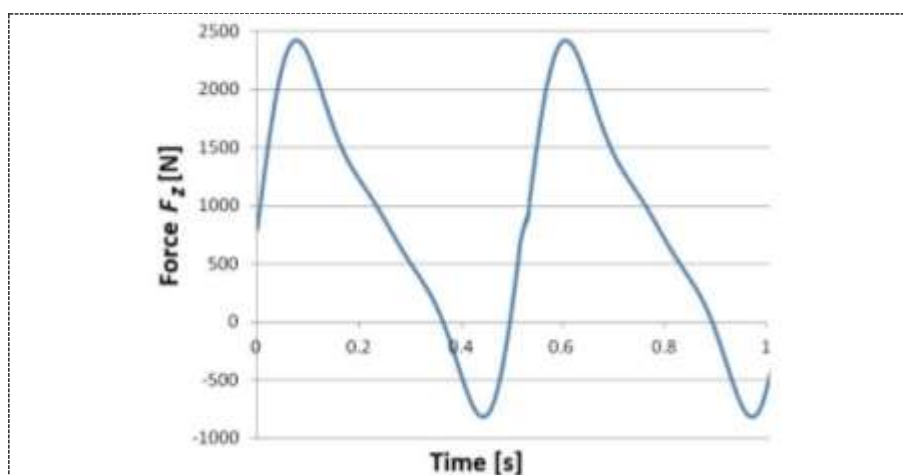
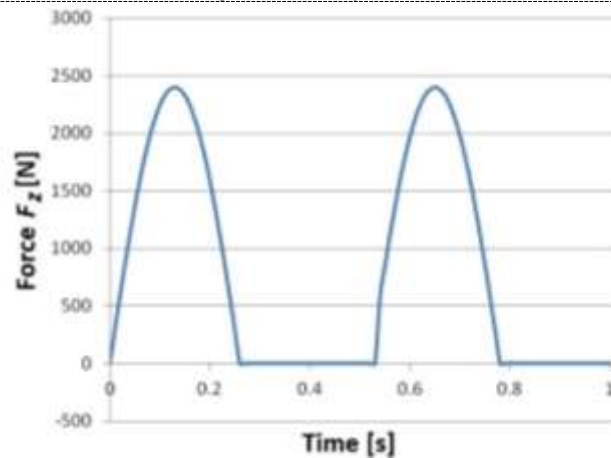
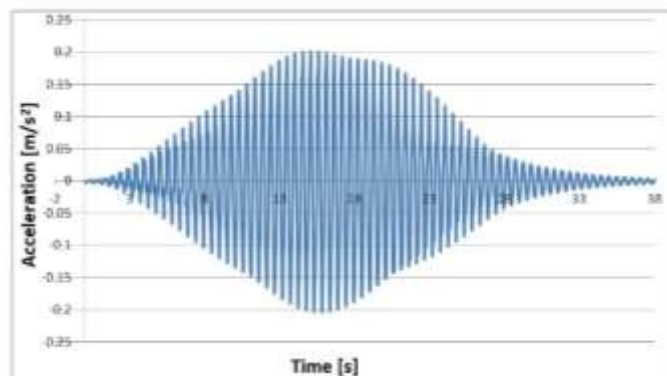
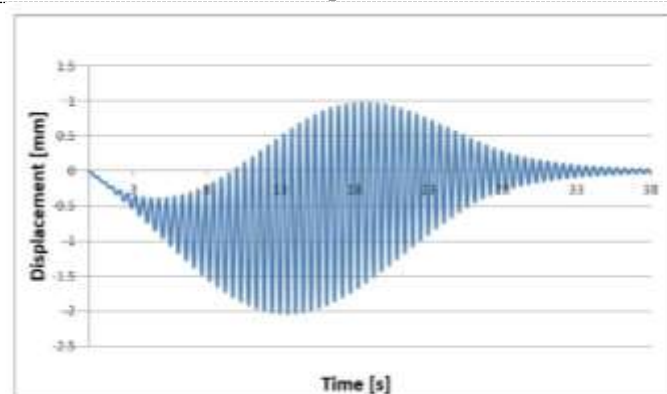


Figure 3. Periodic forces F_z generated by a runner – ‘sine’ model [3]**Figure 4.** Periodic forces F_z generated by a runner – ‘half-sine’ model [1]

7. Comparison of the results obtained for the different models of dynamic forces

The time histories of accelerations and displacements obtained at point B (midspan), for the first ‘sine’ model of dynamic forces [3] (see Fig. 3) are presented in Fig. 5 and 6. The accelerations and displacements obtained for the second ‘half-sine’ model [1] (see Fig. 4) are presented in Fig. 7 and 8.

**Figure 5.** Vertical accelerations occurring at point B for the ‘sine’ model of dynamic forces adopted from [3]**Figure 6.** Vertical accelerations occurring at point B for the ‘sine’ model of dynamic forces adopted from [3]

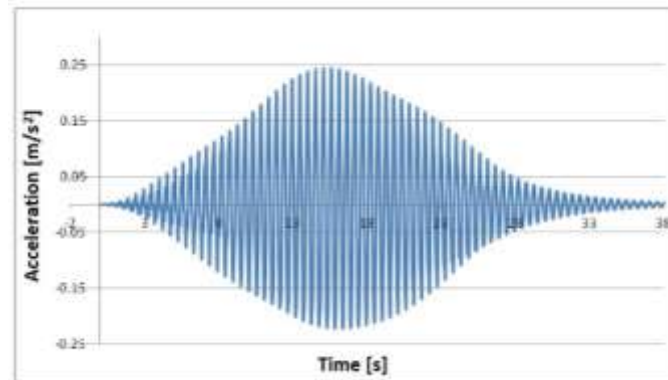


Figure 7. Vertical accelerations occurring at point B for the ‘half-sine’ model of dynamic forces adopted from [1]

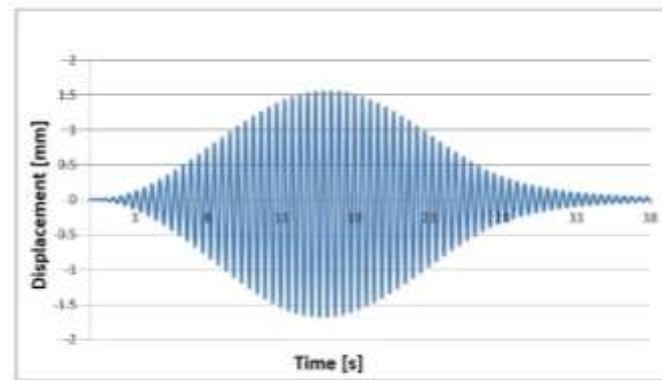


Figure 8. Vertical accelerations occurring at point B for the ‘half-sine’ model of dynamic forces adopted from [1]

The maximum values of accelerations and displacements that occurred at points A, B and C during the pedestrian passage along the footbridge, for both models of dynamic forces are summarised in Table 4.

Table 4. Comparison of maximum accelerations and displacements at points A, B and C obtained for two models of dynamic forces.

<i>Point</i>	<i>Maximum accelerations [m/s²]</i>		<i>Maximum displacements [mm]</i>	
	<i>‘Sine’ model of forces [3]</i>	<i>‘Half-sine’ model of forces [1]</i>	<i>‘Sine’ model of forces [3]</i>	<i>‘Half-sine’ model of forces [1]</i>
A	0.18	0.23	1.98	1.58
B	0.20	0.24	2.03	1.67
C	0.18	0.17	1.42	1.23

It can be observed from Table 4 that for both models of forces generated by a runner, the requirements of vibration comfort standards recommended by Eurocode for vibrations in vertical direction, are fulfilled. The level of oscillation does not exceed 0.7 m/s^2 . Taken into consideration, the recommendation taken from of SÉTRA document (see Table 1), the comfort of use the investigated footbridge is on the maximum level.

8. Conclusions

Firstly, modal properties, i.e. modes and natural frequencies of the suspended footbridge were calculated. Secondly, the dynamic response of the footbridge was assessed taking into consideration two models of dynamic forces produced by a single running user of the structure. Finally, the comfort of footbridge use was checked. The following concluding remarks can be expressed:

- The results of the modal analysis shown that the first, vertical natural frequencies of the structure coincided with the frequency of pedestrian generated loading (walking or running) which could result in the resonance phenomenon.
- The values of accelerations and displacements obtained for the second half-sine model of dynamic forces [1] were greater than those obtained for the first model [3] up to 20%. The mathematical model recommended by [3] is adequate only in the case of walking, because each step overlaps with the previous one thus making the force continuous. In the example of running this postulation is inappropriate. The loading generated by runner has a discontinuous nature. For the loading produced by a runner the 'half-sine' model is more complete. The dynamic analysis of a footbridge, performed with first 'sine' model of dynamic force, may underestimate the dynamic response of the pedestrian bridge.
- The vertical comfort criteria are not exceeded for a single pedestrian walking fast or running.

9. References

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