

Development of A New Equations Describing Profile Losses in Axial Turbine Blade Row

O Baturin¹ and D Kolmakova²

¹ Associate professor, Samara National Research University, Samara, Russia

² Researcher, Samara National Research University, Samara, Russia

E-mail: oleg.v.baturin@gmail.com

Abstract. Based on the previous investigations, regression equation for calculating the profile losses was found by the least squares based on the analysis of the profile losses nature and using mathematical optimization techniques. This equation makes possible to define the profile loss of axial turbine more accurate than the existing models and equations. It allows the calculation of profile loss values in the axial turbine that differ from the actual value by $10\pm 61\%$ with a probability of 95%. The obtained equation is not only superior to other loss models in statistical criteria, but it also considers the increasing number of factors affecting the profile losses.

1. Introduction

Axial turbine is the most common device for production of mechanical work. The number of turbines operating in various industries amounts to tens of thousands. For this reason, the problem of improving the turbine efficiency is relevant and its solving promises the great economic effect.

Most of the researchers have focused on finding the ways to improve the structure of the flow in the turbine blade rows. A large part of these studies was conducted using the methods of computational fluid dynamics (CFD). The major disadvantage of CFD is that CFD is only confirmatory method. It is an expert system that allows drawing a conclusion about quality of the specific variant of design.

Design of turbomachinery channels and formation of geometry of the blades is still carried based on 1D and 2D computations. CFD calculations only allow identifying and correcting design errors, as well as considering the features of the flow, which are not accounted in 1D and 2D calculations.

The better the design calculation was made, the smaller iterations with computationally expensive CFD models will be required, and the sooner the best variant will be found. For these reasons, the improvement of methods of 1D and 2D design of turbines is a promising task.

The most important aspect affecting the accuracy of prediction of turbine characteristics in 1D and 2D calculations is a reliable prediction of the energy losses in its flow part. Currently, more than ten complex models are known, allowing the calculation of the losses in axial turbines, as well as dozens of equations that allow calculating the individual loss components [1-5].

In the research described in [6], authors compared existing loss models. Report [7] of the Central Institute of Aviation Motors (CIAM) (Russian) [8] was used for there purpose. This report contains the results of experimental determination of the profile losses in more than 170 nonswirling cascades of axial turbines with constant height section. The study showed that although it is possible to establish the best loss model, these models can give significant errors during the calculation. For this reason, the



decision was made to create a new equation for calculating the profile losses based on the available experimental data. This was the goal of current research described in the paper.

2. Selection of the variables that affect losses. Justification of the selection of the equation forms

The first task towards this goal was to choose the variables affecting the value of the loss and type of the equation. For this purpose, the analysis of scientific literature was carried out [9, 10, 11, 12], which showed that the energy losses on the surface of the streamlined blades meant by the profile losses excluding the effect of the end surfaces (the hub and shroud endwalls). It is considered that the profile losses consist of four components: friction, edge, wave and losses of flow separation.

In the analysis of the components of profile losses the following fact was highlighted. All components of the profile losses except for the wave losses vary within the entire range of flow velocities. Moreover, friction and edge losses will decrease and asymptotically tend to a certain value as velocity increases. There are no wave losses at low flow rates and they occurred only at $\lambda_{w2s} = 0.75 \dots 0.85$. With the velocity growth, this type of loss increases (Figure 1). The opposite effect of these two factors determines the form of the dependences $\xi = f(\lambda_{w2s})$.

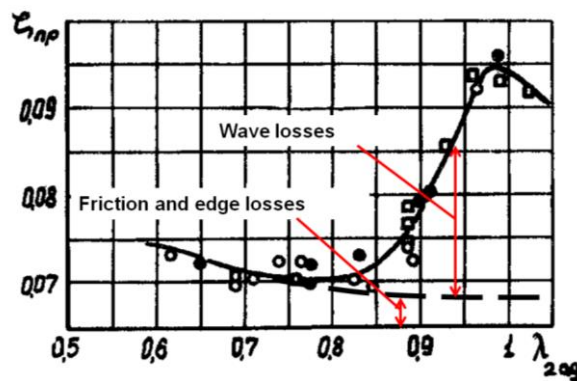


Figure 1. For an explanation of the ratio between friction, edge and wave losses.

Based on the analysis, the geometrical and operational parameters of the profile have been chosen that have a significant impact on the value of profile losses. According to the authors, these parameters must be considered when deriving the equation for the calculation of profile losses. Since derivable equation should be universal, most of the selected parameters were normalized. Table 1 provides a list of selected variables.

Table 1. Variables affecting the value of profile losses.

| Designation | Nomination | Loss type on which it affects |
|-----------------------------|--|-------------------------------|
| $\frac{\Delta\beta}{100}$ | Relative value of flow deflection angle in turbine cascade | Friction losses |
| $\bar{X}_c = \frac{X_c}{b}$ | Relative position of maximum thickness | Friction losses |
| $\frac{C_m}{b}$ | Relative maximum blade thickness | Friction losses |
| λ_{w2s} | Isentropic specific velocity at the cascade outlet | Friction losses, wave losses |
| $\frac{t}{b}$ | Relative spacing | Friction losses, edge losses |
| $\frac{i}{\beta_1}$ | Relative blade angle | Separation losses |
| Re | Reynolds number | Friction losses |
| β_2 | Flow angle at the cascade outlet | Edge losses |

| | | |
|----------|--------------------------|-------------|
| δ | Bending angle of profile | Edge losses |
| r_2 | Trailing edge radius | Edge losses |

Based on the analysis of the form of the equations describing the profile losses in the existing models of losses [1-5,9,13], the expected form of the equation has been selected:

$$\xi_{prof} = (K_i \cdot K_{re} \cdot \xi_{fr} + \xi_{eg}) + \xi_{\lambda} \quad (1)$$

where $\xi_{fr} = A_2 \left(\frac{\Delta\beta}{100} \right)^2 + A_3 \bar{X}_c + A_4 \left(\frac{c_m}{b} \right) + A_8 \lambda_{w2s} + A_7 \left(\frac{t}{b} \right) + A_9$ - a term, considering the profile losses of friction in the boundary layer in the reference conditions;

$K_i = 1 + A_1 \left(\frac{i}{\beta_1} \right)^2$ - adjustment coefficient considering the losses of flow separation;

$K_{re} = A_6 \left(\frac{2 \cdot 10^5}{Re} \right)^{0.25}$ - adjustment coefficient considering the effect of Reynolds number;

$\xi_{eg} = A_5 \frac{r_2}{t \sin(\beta_2 + \delta)}$ - a term considering the edge losses.

$\xi_{\lambda} = A_{10} \sin(A_{14} \lambda_{w2s} + A_{11}) + A_{12} \lambda_{w2s} + A_{13}$ - a term considering the wave losses.

$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}$ - coefficients of equations.

The desire to ensure that all terms of the equation are of the same degree, as well as preliminary study of equations of another type by algorithm described above, influenced the choice of the form of equation.

Thus, the problem of the derivation of a new equation of profile losses reduced to finding the numerical values of the coefficients.

3. Search algorithm of equation coefficients

The problem of finding the coefficients of the equation is a search of function extremum. Indeed, such a combination of coefficients must be found that provides a minimum difference between the results of calculation and experimental data. For this reason, the optimization program IOSO was used to find the values of coefficients [14-16].

The core of IOSO software is optimization method based on the construction of response surface, which is being refined and evolves at each iteration. Each iteration of IOSO contains two steps. At the first step, the response function in the form of multi-level graph is constructed based on the earlier accesses to the models with different combinations of varied parameters. The next step is finding of the extremum of a given function. This approach allows users to constantly adjust the response surface in the optimization process. As a result, an unusually small number of starting points are required to its construction and the first results obtaining [15, 17, 18].

IOSO algorithm has a low sensitivity to the topology of the objective functions and allows to accomplish the tasks for smooth, unimodal, multiextremal and non-differentiable functions.

The unknown coefficients of the equation A1-A14 have been selected as variable parameters while setting the optimization task. Their values varied in the range of ± 150 .

An average integral deviation of calculated values from experimental data in the whole considered range was specified as the optimization criterion:

$$\delta \xi_{\Sigma} = \frac{\sqrt{\sum_{i=1}^n (\delta \xi_i)^2}}{n} \quad (2)$$

where $\delta \xi_i$ - relative deviation from the expected experimental value found for the specific values of conventional specific velocity and the specific cascade.

n - total number of calculated values of the relative deviation for various combinations of conventional specific velocity and definite cascade.

Apparently, the smaller this value is, the better equation describes an experimental data array. Thus, the task was set to IOSO program in a process optimization to find such a combination of factors in which $\delta \xi_{\Sigma}$ would be minimal.

IOSO formed some combination of varied parameters in the performance of its internal algorithms. They were loaded into a file of Microsoft Excel, which calculates the value of the relative deviation from the expected experimental values $\delta\xi_i$ for different isentropic specific velocities λ_{w2s} and for all 170 cascades. The value of λ_{w2s} for each cascade was varied in the range 0.5-1.4 with a step of 0.05. In total, more than 27.5 thousand values of $\delta\xi_i$ were calculated for a varied combination of variables.

Ultimately, the Excel file calculated the average integral deviation of calculated values from the experimental data in the whole range of $\delta\xi_\Sigma$, as well as the mean values of the whole range of mathematical expectation $\mu_{\Delta\xi}$ and standard deviation $\sigma_{\Delta\xi}$.

Considering that the wave component of the profile losses is not appears at all values of velocity, optimization was carried out in two stages. At the first stage, the coefficients (A1, A2, A3, A4, A5, A6, A7, A8, A9), which consider the friction losses, edge and flow separation losses, were found based on the values of deviations for λ_{w2s} in the range of 0.5...0.75. Then, the coefficients of the term considering the wave losses (A10, A11, A12, A13, A14) were calculated at fixed values of found coefficients, based on the values of the deviations in the whole range of $\lambda_{w2s} = 0.5 \dots 1.4$.

4. Final equation and its comparison with other loss models

As a result of optimization, the minimum value of $\delta\xi_\Sigma$, which is equal to 33, was reached. At the same time, mathematical expectation $\mu_{\Delta\xi}$ and standard deviation $\sigma_{\Delta\xi}$ were -10 and 31 respectively. The coefficients of the equation that give such result are shown in Table 2.

Table 2. Equation coefficients

| Coefficient | Value | Coefficient | Value |
|-------------|----------|-------------|-----------|
| A1 | 5,52661 | A8 | -37,16360 |
| A2 | 1,16463 | A9 | -53,81240 |
| A3 | 99,99734 | A10 | 19,82116 |
| A4 | 69,11952 | A11 | -53,727 |
| A5 | 0,68967 | A12 | 1,650231 |
| A6 | 0,00036 | A13 | 16,11946 |
| A7 | 99,96786 | A14 | -9,93227 |

Thus, the derived equation to calculate the profile losses has the following form:

$$\xi_{prof} = (K_i \cdot K_{re} \cdot \xi_{fr} + \xi_{eg}) + \xi_\lambda \quad (3)$$

where $\xi_{fr} = 1,16463 \left(\frac{\Delta\beta}{100}\right)^2 + 99,99734 \cdot \overline{X}_c + 69,11952 \cdot \left(\frac{C_m}{b}\right) - 37,16360\lambda_{w2s} + 99,96786 \left(\frac{t}{b}\right) - 53,81240$ – a term, considering the profile losses of friction in the boundary layer in the reference conditions;

$K_i = 1 + 5,52661 \cdot \left(\frac{i}{\beta_1}\right)^2$ – adjustment coefficient considering the separation losses;

$K_{re} = 0,00036 \cdot \left(\frac{2 \cdot 10^5}{Re}\right)^{0.25}$ – adjustment coefficient considering the effect of Reynolds number;

$\xi_{eg} = 0,68967 \frac{r_2}{t \cdot \sin(\beta_2 + \delta)}$ – a term considering the edge losses.

$\xi_\lambda = 19,82116 \cdot \sin(-9,93227 \cdot \lambda_{w2s} - 53,727) + 1,650231 \cdot \lambda_{w2s} + 16,11946$ – a term considering the wave losses.

It was interesting to compare the statistical parameters of the new equation with similar data for other models of profile losses obtained earlier. The obtained equation shows better statistical data, compared with all the loss models discussed above. The derived equation shows the value of the mathematical expectation like CIAM model (both models underestimate the magnitude of profile losses by about 10%), but has better standard deviation by 25% (rel.).

These conclusions are illustrated by Figure 2, where the dependences of the profile loss coefficient ξ from the isentropic specific velocity at the cascade outlet λ_{w2s} are constructed as an example for the cascades No. 42, 55, 119, 135.

Figure 3 shows how the most probable value of the deviation of calculated data from experimental $\Delta\xi$ change depending on the values of isentropic specific velocity λ_{w2s} for all considered loss models.

Obtained maximum and minimum possible values of deviations $\Delta\xi$ with a probability of 95% indicated in the same figure. Data for CIAM loss model, the best of previously considered, are shown for comparison in Figures 2 and 3.

Apparently from Figure 2, the curves obtained with the new equation describe well the experimental data for most of cascades. Often, calculated dependence of $\xi = f(\lambda_{w2s})$ using the new equation is closer to the experimental points than the curve obtained by CIAM equations. It should be noted that dependences $\xi = f(\lambda_{w2s})$ have similar character.

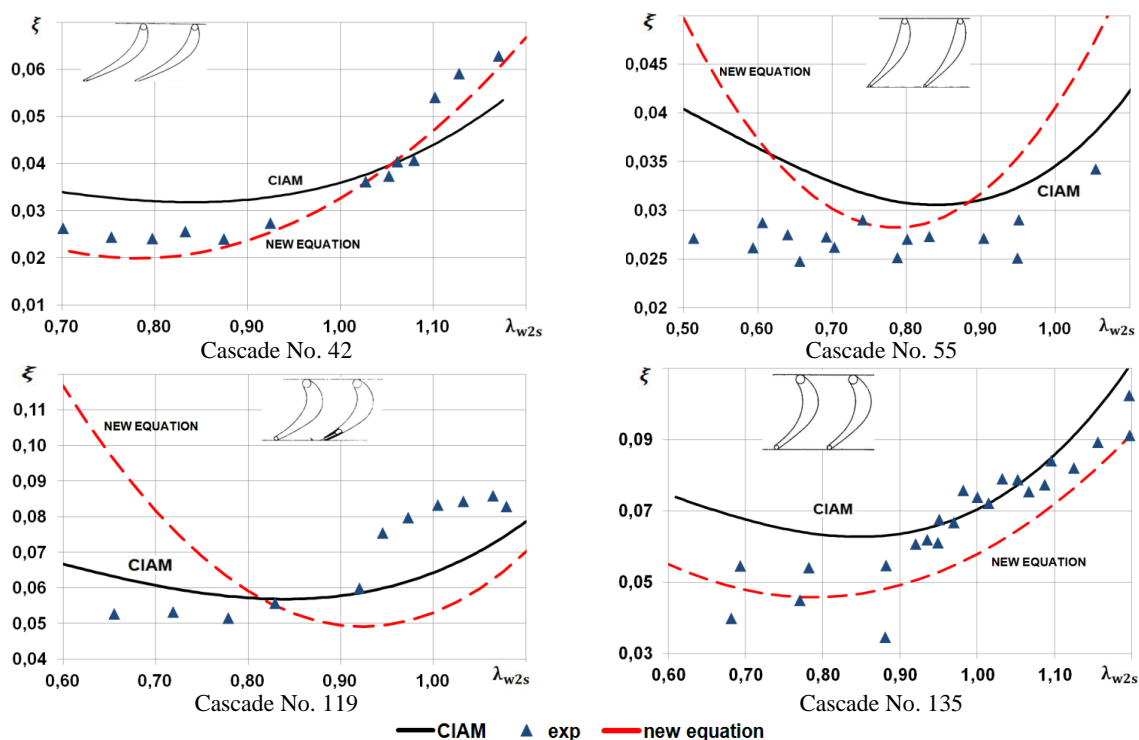


Figure 2. Comparison of the calculation results of profile losses obtained by a new loss model with experimental data and calculations with CIAM model.

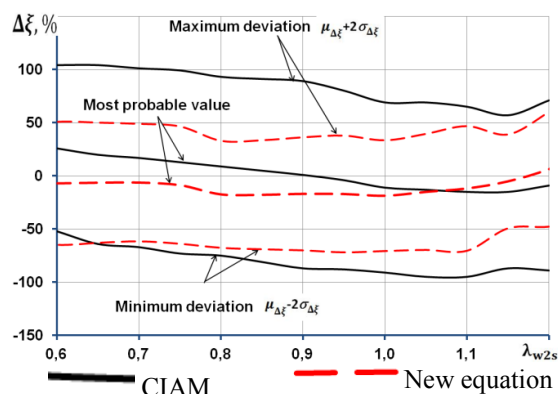


Figure 3. Comparison of changes in the most probable value of the estimated deviation of calculated profile loss values from real value and boundaries of scattering with the probability of 95%, depending on λ_{w2s} for derived equation and CIAM model.

Information provided in Figure 3 confirms the conclusions of the qualitative results in the evaluation of profile losses by the new equation and CIAM model. However, mathematical

expectation of the new equation varies slight and is close to zero, indicating the proximity of the calculated and experimental data. New equation provides a smaller dispersion of the experimental and calculated data.

5. Conclusion

Based on the analysis of the profile losses nature and using mathematical optimization techniques, a new equation was proposed, which allows defining the profile loss of axial turbine more accurate than the investigated models. It allows the calculation of profile loss values in the axial turbine that differ from the actual value by $10\pm 61\%$ with a probability of 95%. The proposed new equation allows considering more geometric and operational factors affecting the value of losses.

This work is the first step to finding the best loss model in axial turbine. It considers only profile losses. In the future, it is planned to analyze other types of losses occurring in the blade passage of axial turbine in a similar way. As a result, it is planned to reliably determine which model is the best, or to develop a new one, which is better in accuracy than existing.

Acknowledgments

The work was financially supported by the Ministry of education and science of Russia in the framework of the implementation of the Program of increasing the competitiveness of SSAU among the world's leading scientific and educational centers for 2013-2020 years.

References

- [1] Wei N 2000 Significance of Loss Models in Aerothermodynamics Simulation for Axial Turbines *Doctoral Thesis KTH ISBN 91-7170-540-6*
- [2] Dahlquist A N 2008 Investigation of Losses Prediction Methods in 1D for Axial Gas Turbines Thesis for the Degree of Master of Science Division of Thermal Power Engineering Lund Institute of Technology (Lund University, Sweden)
- [3] Horlock J H 1965 Axial Flow Turbines (Butterworths, UK)
- [4] Ainley D G, Mathieson G C R 1955 An Examination of the Flow and Pressure Losses in Blade Rows of Axial-Flow Turbines Reports & memoranda; no.2891 (British ARC)
- [5] Dunham J, Came P M 1970 Improvements to the Ainley/Mathieson Method of Turbine Performance Prediction ASME Journal of Engineering for Power 92(3) pp 252-256
- [6] Baturin O, Kolmakova D, Gorshkov A and Popov G 2015 Selection of models to assess the profile losses in blade rows using the methods of mathematical statistics Proc. of ASME 2015 Gas Turbine India Conference, GTINDIA 2015 Paper No. GTINDIA2015-1245
- [7] Venediktov V D, Granovsky A V 1990 Atlas eksperimental'nykh kharakteristik ploskikh reshetok okhlazhdaemykh gazovykh turbin (The Atlas of Experimental Performances of Cooled Gas Turbine Blade Cascades) p 393
- [8] Central Institute of Aviation Motors, Accessed November 01, 2015. <http://www.ciam.ru/>.
- [9] Abiants V Kh 1979 Teoriya gazovykh turbin reaktivnykh dvigatelei (The theory of the gas turbine jet engines) (Mashinostroenie, Moscow) p 246
- [10] Belousov A N, Musatkin N F and Rad'ko V M 2003 Teoriya i raschet aviatsionnykh lopatochnykh mashin (Theory and calculation of aircraft turbomachinery), (Samara State Aerospace University, Samara) p 344
- [11] Denton J D 1993 The 1993 IGTI Scholar Lecture: Loss Mechanisms in Turbomachines J. of Turbomachinery 115(4) pp 621-656
- [12] Lewis R I 1996 Turbomachinery performance analysis, Elsevier Science & Technology Books Pub, p 329
- [13] Kacker S C, Okapuu U 1982 A Mean Line Prediction Method for Axial Flow Turbine Efficiency ASME J. of Engineering for Power 104(2) pp 111-119
- [14] Sigma Technology, Accessed October 10, 2014 <http://www.iosotech.com>

- [15] Egorov I N, Kretinin G V, Leshchenko I A, Kuptzov S. V 2002 IOSO Optimisation Toolkit - Novel Software to Create Better Design 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimisation, (Atlanta, Georgia)
- [16] Dennis B H, Egorov I N, Sobieczky H, Dulikravich G S, Yoshimura S 2003 Parallel Thermoelasticity Optimization of 3-D Serpentine Cooling Passages in Turbine Blades American Society of Mechanical Engineers, International Gas Turbine Institute, Turbo Expo (Publication) IGTI GT2003-38051
- [17] Komarov O V, Sedunin V A, Blinov V L 2014 Application of Optimisation Techniques for New High-Turning Axial Compressor Profile Topology Design Proc. of ASME Turbo Expo Paper No. GT2014-25379.
- [18] Rybakov V N, Kuz'Michev V S, Tkachenko A Y and Krupenich I N 2016 Thermodynamic multi-criteria optimization of the unified engine core for the line of turbofan engines Proc. of ASME Turbo Expo Paper No. GT2016-57854.