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A Refined Model for Calculation of the Vortex Tube Thermal Characteristics

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Abstract. The article deals with the main types of vortex tubes, provides a brief description of the fundamental principles of the vortex interaction hypothesis. A physical process is represented reflecting the physical essence of the gas flow energetic separation process in the vortex tube due to the intensive turbulent heat exchange from the forced vortex to the free one. A method for refinement of the design characteristics for the cold and hot gas temperatures in a vortex tube through the employment of the gas-dynamic and thermodynamic corrections is proposed. A refined calculation method allows reaching close agreement between the cold gas temperature and the experimental values.

1. Introduction

The vortex effect of energetic gas separation discovered by J. Ranque in 1931, and then studied by R. Hilsch to a considerable detail, has been drawing close attention of scientists for 86 years. It should be noted that since before the Ranque's discovery, the works of Maxwell, Schmidt, K. Strakhovich had provided theoretical background for gas energetic separation in a swirling flow. Due to the studies carried out by the Russian scientists, optimal designs of the main types of vortex tubes were obtained. [1,2,3,4,5,6,7,8] They include: a separating tube, a cooled tube, a self-vacuuming tube, and a double-circuit vortex tube.

2. The essence of the hypothesis

A separating vortex tube, where the compressed gas is separated into two flows: a cold flow and a hot flow.

A cooled vortex tube cooling the gas flow through the walls using the external heat carrier to the lower temperature as compared to the temperature of the external heat carrier.

A self-vacuuming vortex tube allows transforming the vortex kinetic energy to the pressure energy using a rotating diffuser and thus sharply reducing the pressure at the energy separation chamber axis with substantially increasing a cooling effect in it.

A two-circuit vortex tube with the additional flow performs efficiently at $\mu>1$ thus ensuring high heat exchange intensity between the main and the additional flow.

In spite of the fact that the separating vortex tube (VT) has an extremely simple design, the processes occurring in it are so complex and paradoxical that it is only now that it is recognized that the vortex interaction hypothesis developed by professor A.P. Merkulov describes the physical essence of processes occurring in the energetic separation chamber to the fullest extent possible. [1, 2, 5]

The essence of this hypothesis is the following: after outflowing from the tangential nozzles, the gas flow forms a free vortex spreading to a certain radius and displacing along the tube axis to the throttle. A free vortex is resistant to the internal friction forces and is not destroyed by them. The vortex may become breaking down only at its radial boundaries due to the wall friction and the interaction with the near-axial elements. Furthermore, its swirling intensity decreases due to the decrease of the circumferential velocity when the vortex moves along the tube to the throttle, the static pressure radial gradient in the vortex flow decreases and the vortex moves closer to the axis. [5, 6, 7]

The decrease of radial gradient, in turn, results in the static pressure axial gradient, which forces the gas entering the near-axial region changing its axial motion initial direction to the opposite one, and moving to the nozzle section. While moving to the near-axial region, gas elements are becoming intensively turbulized. High turbulent viscosity forces the near-axial flow to swirl according to the solid body law. The reverse near-axial flow, with the advancement to the nozzle section, is swirled to form an increasingly intensive free vortex.

Apart from transferring the kinetic rotation energy from the free vortex to the forced vortex, an intensive turbulent heat exchange occurs between them with a high static pressure gradient being normal to the average flow velocity.

A radial displacement of the turbulent gas element is due to the radial turbulent pulsating velocity. When entering a higher or a lower pressure area, the element will shrink or expand adiabatically.

If after the displacement, the gas element temperature is different from the temperature of the surrounding elements, then heating or cooling of the elements will occur, i. e. turbulent elements pass refrigeration cycles by giving heat to the peripheral layers; the turbulence is the source of mechanical energy.

Turbulent heat transfer will cease (q=0) upon reaching the isentropic radial temperature distribution $\frac{dT}{dr} = \frac{k-1}{k} \cdot \frac{T}{p} \cdot \frac{dp}{dr}$.

This hypothesis makes it possible estimating the limiting possibilities of the vortex effect under the assumption that the energy exchange between the vortices has fully completed in the nozzle section.

By using the radial velocity distribution laws for free and forced vortices, the conditions for gas mechanical equilibrium, the isentropic radial distribution of the static temperature of the forced vortex, the mechanical and thermal conjugacy of the vortices, the expressions for distributing the parameters of the nozzle section radius may be obtained, and in case of the defined geometry the mean-integral temperature of the cold and hot flows of the vortex tube as a function of the main parameters of the vortex tube may be determined. For the relative temperature of the cold and hot flows of the vortex tube, these dependencies are as follows: [3, 4 ,8]

$$\theta_x = \theta\left(\pi, \mu, \overline{F}_c, \overline{d}_Д\right); \quad \theta_Г = \theta'\left(\pi, \mu, \overline{F}_c, \overline{d}_Д\right). \tag{1}$$

They provide a qualitative correspondence with the experimental characteristics of the vortex tube as well as describe the reversing phenomenon in the vortex tube (θ_x>1 at $\mu \rightarrow 0$) and are indicative of the possibility of cooling at μ=1 in the cooled VT or at μ>1 in the double-circuit VT or at μ=0 in self-vacuuming VT. Furthermore, the design and experimental values of temperature are considerably different (Figure. 1).

To refine the design characteristics of the VT, the losses of the total gas pressure when outflowings through the nozzle inlet (gas dynamic calculation), as well as the heat recovery in counter-flow motion of the cooled near-axial and heated peripheral gas flows in the energy separation chamber (thermodynamic analysis) shall be considered. The vortex interaction hypothesis about the existence of two vortices - a free vortex with the potential flow $V_\tau \cdot r = const$ and a forced vortex $\omega = \frac{V_\tau}{r} = const$ - is assumed as the basis of the gas dynamic calculation of the VT.

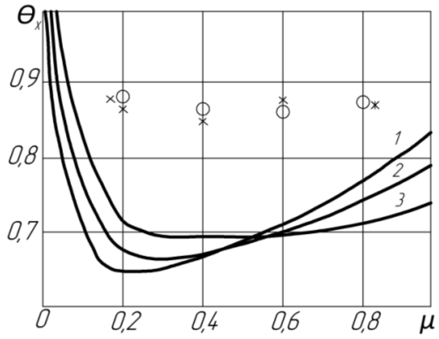

Figure 1. The limiting design characteristics of the VT:
The experimental data
K=1.4; π=5; $\overline{F}_c$ =0.1; x – Merkulova; * - Gulyayeva;
O – Martynova; 1-$\overline{r}_д$ =0, 4; 2-$\overline{r}_д$ =0.45; 3·$\overline{r}_д$ =0.5.

The momentum equation for a non-viscous gas may be integrated

$$\frac{dp}{dr} = \rho \cdot \frac{V_\tau^{\,2}}{r} \tag{2}$$

with obtaining the radial gas parameter variation laws, p, T, ρ.

The limiting condition for solving this equation with respect to p is determined based on the solution of the flow-rate equation for the diaphragm section, which lies in the VT nozzle section.

$$\mu G_1 = \int_0^{\overline{r}_д} \rho \cdot V_z \cdot 6,28 \cdot r \cdot dr. \tag{3}$$

Here μ is the fraction of the cooled gas flow,

$\overline{r}_д$ - the diaphragm opening radius,

V_z is the axial speed in the diaphragm opening,

G_1 is the gas mass flow through the VT.

The axial speed, in turn, is determined based on the expression,

$$V_z = \pm\sqrt{\frac{2k}{k-1} R T_1 \left(\frac{p}{p_1}\right)^{\frac{k-1}{k}} \Big/ \left[1 - \left(\frac{p_x}{p}\right)^{\frac{k-1}{k}}\right]}, \tag{4}$$

where p_1 is the pressure at the energy separation chamber periphery.

The gas flow direction and the sign of the expression V_z are determined depending on the ratio of the gas pressure p and the pressure p_x of the medium into which the cooled gas flows out.

The value p_1 is obtained under the condition of radial adiabatic distribution of the static gas parameters within the expression as follows:

$$p_1 = \frac{\mu G_1 R T_1}{6.28 \cdot \left[-\int_0^{r_a} \left(\frac{p}{p_1}\right)^{\frac{1}{k}} V_z r dr + \int_{r_a}^{r_д} \left(\frac{p_x}{p_1}\right)^{\frac{1}{k}} V_z r dr \right]}. \tag{5}$$

Here r_a is the radius at which the axial component of the velocity V_z changes its sign. The limiting condition in the equation for the gas temperature at the energy separation chamber radius, $T=T_1$ is obtained from the energy equation for the gas flow in a free vortex near the chamber wall:

$$T_1 = \frac{T_1^*}{1 + \frac{k-1}{2}(M_1^2 - M_2^2)} \quad . \tag{6}$$

The tangential velocity M_1 at the periphery of the energy separation chamber is expressed through the ratio of pressures

$$M_1 = \sqrt{\frac{2}{k-1}\left[\left(\frac{p_k^*}{p}\right)^{\frac{k-1}{k}} - 1\right] - M_z^2} \; . \tag{7}$$

The relative axial velocity M_z is found from the equation of flow conservation through the free vortex region assuming the constancy of the radial axial velocity

$$M_z = \frac{G_1}{\sqrt{kRT_1} \cdot 6.28 \cdot \int_{r_2}^{r_1} \rho r dr} \, . \tag{8}$$

Full gas pressure on the walls of the energy separation chamber pk* is found considering the pressure losses in a tangential nozzle

$$p_k^* = p_1^* \left(\frac{\alpha_c}{\varphi_c}\right)^{\frac{k-1}{k}} , \tag{9}$$

Where the speed factor φc may be found by the flow factor αc based on the expression

$$\frac{\alpha_c}{\varphi_c} = \frac{1 - \frac{k-1}{k+1}\lambda_c^2}{1 - \frac{k-1}{k+1}\varphi_c^2 \lambda_c^2} \, . \tag{10}$$

The latter expression comprises the relative speed of the gas in the outgoing section of the tangential nozzle inlet. It is connected with the tangential component of the gas velocity component at the periphery of the energy separation chamber. The form of the expression λ_c on M_1 is obtained based on the momentum-of-momentum equation

$$\lambda_c = \frac{M_1}{\sqrt{\frac{2}{k+1} \cdot \frac{k-1}{k+1}(M_1^2 + M_z^2)}} \cdot \frac{r_1}{r_c} \, . \tag{11}$$

The gas flow G1 is determined considering the flow factor αc using the relative velocity λc:

$$G_1 = \alpha_c \frac{p_1^* F_c \sqrt{k} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \bar{q}(\lambda_c)}{\sqrt{RT_1^*}} \, . \tag{12}$$

where the gas dynamic function of the flow of the medium velocity in the nozzle

$$\bar{q}(\lambda_c) = \frac{\int_0^h q(\lambda_c) b \, dh}{F_c} \, . \tag{13}$$

The last undetermined value necessary for closing the set of equations is the vortex separation radius r2, which is assumed to be determined based on the variation principle of the flow maximum entropy:

$$G_1 R \cdot \ln \frac{p_1^*}{p_k^*} = \max \; . \tag{14}$$

The system includes an empirical value, i.e. the flow factor of the tangential nozzle input, for whose calculation the following experimental dependence was obtained:

$$\alpha_c = 1 - \frac{1.63}{\mathrm{Re}^{0.25}}(1.03 - \lambda_c^4). \tag{15}$$

The represented set of equations is solved by the iteration method and allows determining the gas flow through the VT, the radial distribution of gas parameters in the nozzle section, and the averaged total gas temperature in the diaphragm section (the so-called theoretical limiting temperature).

The design temperature of the cooled gas is determined by the expression

$$\tilde{T}_{XT} = \frac{\int_0^{r_Д} \rho V_z \left(T + \frac{V_\tau^2}{2C_p} \right) r dr}{\int_0^{r_Д} \rho V_z r dr}. \tag{16}$$

To determine the pressure of the heated gas, the semiempirical formula is proposed

$$p_Г = \frac{1}{2}\left[(p_{oc} + p_2) + \mu (p_1 - p_{oc}) \right], \tag{17}$$

assuming the linear behaviour of the dependence of the fraction of the cold gas flow on the pressure of the heated gas flow and connecting the pressure values at the characteristic radii (r=0, r=r2, r=r1) of the nozzle section.

The thermodynamic analysis of the VT performance consists in determining the temperature of the cooled and heated gas by finding and considering the conditions resulting in the adiabatic distribution of the static gas parameters the along the nozzle section radius. With these conditions, a relation between the temperature $\tilde{T}_{XT}$ and the cooled gas temperature T_X in the idealized process approximating the heat transfer process in the VT, may be found.

The idealization of the heat transfer process in VT is based on the assumption that this process occurs according to the regenerative pattern and proceeds in the following way.

The gas, entering the VT, moves along the energy separation chamber periphery to the VT throttle and receives some quantity of heat q from the near-axial gas layers, thus heating to the temperature $T_Г$. The gas is separated in two parts near the throttle: one its fraction (1-μ) goes out through the throttle as a heated flow with the temperature $T_Г$, and its second fraction μ, while moving to the diaphragm, gives the heat q along its way to the periphery gas layers with cooling to the temperature T_X. Furthermore, the pressure of near-axial gas layers is decreased from $p_Г$ near the throttle to p_X downstream the diaphragm. This is the first thermodynamic process in the VT. Without considering the heat transfer mechanism, the temperatures $T_Г$ and T_X may be calculated by using the thermodynamic analysis method.

Let's consider that the adiabatic distribution of the parameters in the nozzle section is achieved when such a quantity of heat is diverted from the peripheral layers, which is supplied to them in the actual process in the BT from the near-axis layers.

Let's consider the other thermodynamic process. The gas with flow rate G_1, while moving to the throttle, acquires heat q from the near-axial layers and heats up to the temperature of the heated gas $T_Г$ in the real energy separation process. A quantity of heat q is isobarically removed outside from this gas flow; furthermore the flow temperature is reduced from $T_Г$ to some temperature T_z. A part of the gas with a flow of μG_1 forms a near-axial zone and, while moving counter-flow to the peripheral flow, gives it heat q due to the vortex effect.

The temperature of this portion of gas is decreased from T_z to the theoretic temperature T_{XT}. Furthermore, the gas pressure is decreased from the heated gas pressure $p_Г$ to the cooled gas pressure p_X. This is the second thermodynamic process considered.

Heat removed per unit time to the outside of the peripheral flow considering the isobaricity of the process is determined

$$q = -G_1 C_p (T_Г - T_z) \,. \tag{18}$$

On the other hand, the heat transferred from the near-axial gas layers to the peripheral layer is equal to

$$q' = G_x \int_{S_z}^{S_{XT}} T\,dS \tag{19}$$

or is approximated by the expression

$$q' = G_x \frac{T_z + T_{XT}}{2} R \cdot \ln\left[\frac{p_Г}{p_x} \left(\frac{T_{XT}}{T_z} \right)^{\frac{k}{k-1}} \right]. \tag{20}$$

The temperature Tz may be found by the iteration method from the balance of enthalpies for the second thermodynamic process:

$$G_1 C_p T_1 - G_Г C_p T_z - G_x C_p T_x + q' = 0 \,. \tag{21}$$

The equation q and q' is used to determine the temperature of the hot flow TГ in the second thermodynamic process, which is equal to the temperature of the hot gas flow in the first thermodynamic process

$$T_Г = T_z + \frac{k-1}{2k} \mu \left(T_z + T_{XT} \right) \ln\left[\frac{p_Г}{p_x} \left(\frac{T_{XT}}{T_z} \right)^{\frac{k}{k-1}} \right]. \tag{22}$$

The cooled gas temperature TX of the first thermodynamic process will be determined from the balance equation of its enthalpies.

$$G_1 C_p T_1 - G_Г C_p T_z - G_x C_p T_x = 0 \,. \tag{23}$$

The proposed calculation method allows reaching a close agreement between the value of the cooled gas temperature and the experimental values (Figure. 2).

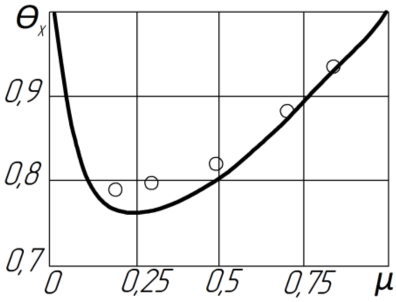

Figure 2. A refined temperature characteristics of the SVT:
- design; o – experiment at π=5

The above thermodynamic analysis may be applied to more complex cases of realizing the vortex effect, for example, a cooled VT, a self-vacuuming VT, and a two-circuit VT.

3. Conclusion

As a result of studying the VT performance, a method for refining the calculation of characteristics for the SVT and other types of the VT with the use of gas-dynamics, thermodynamics and thermal corrections was developed.

Consideration of empirical corrections allows performing mathematical modelling of the VT processes in aircraft vortex cooling systems. According to the proposed methods, the techniques and programs for calculating the actual characteristics of vortex tubes for aircraft cooling systems were developed. [4, 7, 8-10]

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