

Optimizing Parameters of Axial Pressure-Compounded Ultra-Low Power Impulse Turbines at Preliminary Design

D S Kalabukhov¹, V M Radko¹ and V A Grigoriev¹

¹Theory of Aircraft Engines Department, Samara University, Samara, 443086, Russia

E-mail: dskalabukhov@ssau.ru

Abstract: Ultra-low power turbine drives are used as energy sources in auxiliary power systems, energy units, terrestrial, marine, air and space transport within the confines of shaft power $N_{\text{td}} = 0.01 \dots 10$ kW. In this paper we propose a new approach to the development of surrogate models for evaluating the integrated efficiency of multistage ultra-low power impulse turbine with pressure stages. This method is based on the use of existing mathematical models of ultra-low power turbine stage efficiency and mass. It has been used in a method for selecting the rational parameters of two-stage axial ultra-low power turbine. The article describes the basic features of an algorithm for two-stage turbine parameters optimization and for efficiency criteria evaluating. Pledged mathematical models are intended for use at the preliminary design of turbine drive. The optimization method was tested at preliminary design of an air starter turbine. Validation was carried out by comparing the results of optimization calculations and numerical gas-dynamic simulation in the Ansys CFX package. The results indicate a sufficient accuracy of used surrogate models for axial two-stage turbine parameters selection

1. Introduction

Ultra-low power turbines (ULPT) also known as "microturbines" and they are widely used in the industry, energy and transport sets [1]. Their shaft power usually is up to 10 kW. ULPT have extremely low gas consumption up to 0.1 kg/s, miniature dimensions (average diameter of turbine wheel is typically less than 100 mm and the height of nozzle diaphragm blade is about 5 mm) and a supercritical pressure drop ratio in turbine $\pi_t = 2 \dots 100$. Rotation speed n depends on the type of power consumer and can vary from a few thousand to $(300 \dots 500) \cdot 10^3$ rpm [1, 2]. A turbine wheel is driven by pressurized working fluid (it is usually an air) that stored in a receiver. There are number of features of this type of turbines in comparison with full-size turbines. These include: a thick boundary layer on blades surfaces, large leak through gaps, significant secondary wave losses and other kind of specific losses that cause low energy efficiency of ULPT. Typically, such turbines are made impulse type to reduce leakage through radial gaps.

Ultra-low power turbines usually work on a multi-mode operation and have a variety of scheme designs. Impulse axial and radial centripetal turbines are the most widely distributed among this class of turbomachinery. Studies [3] have shown that in the most cases, an axial small-sized turbine is more effective over a wide range of operating conditions than other types of turbines. Single-stage axial ULPT are most widely used due to the requirements of technological simplicity and severe restrictions on the layout of turbine unit. However, high values $\pi_t \geq 4$ [1] cause increased working loading of turbine stage, which determines the intensive reduction of power efficiency coefficient η_t . In addition, single-stage ULPT are typically operate on the left branch of $\eta_t = f(y_t)$ characteristic (Figure. 1), where



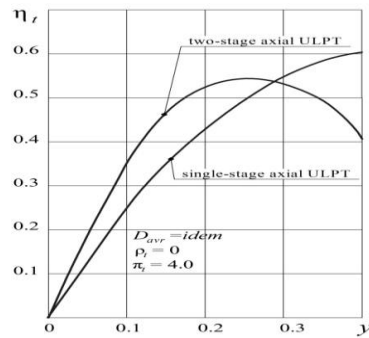


Figure 1. Typical power efficiency characteristics of impulse axial ultra-low power turbines

$$y_t = \frac{u}{c_s} = \frac{\pi D_{avr} n}{60 \left[2 \frac{k}{k-1} R T_0^* \left(1 - \pi_t^{\frac{1-k}{k}} \right) \right]^{1/2}}. \quad (1)$$

An operation at low values $y_t \leq 0.4$ in full-admission turbines leads to high losses of output flow velocity. It is known from theory and practice of impulse turbines [4] that the blade-jet speed ratio in multistage pressure-compounded turbines increases by $\sqrt{z}$ times ($y_{tp} = u\sqrt{z}/c_s$, where z – number of stages). During operation on the left branch ($y_{tp} < y_{tp\ opt}$), this leads to an increased efficiency with respect to a single-stage turbine. This is explained by using an output velocity in the next stage. Also the wave losses reduce in pressure stages due to reducing pressure ratio π_{st} in each stage (provided that $\pi_t = idem$). Furthermore, the higher multistage turbine efficiency is provided by the heat recovery effect [4]. With the proviso that $y_{tp} \gg y_{t\ opt}$ a reducing effect of output velocity losses is generally offset by the significantly increase in secondary losses, gaps losses, the friction on turbine wheels, the uneven flow around turbine wheel blades. The optimum ratio $(u/c_s)_{opt}$ shifts towards lower values with increase of z number. These values, that typical for ULPT, are usually range between 0.1...0.4 [5].

A typical scheme of two-stage pressure-compounded impulse turbine is shown in Figure. 2.

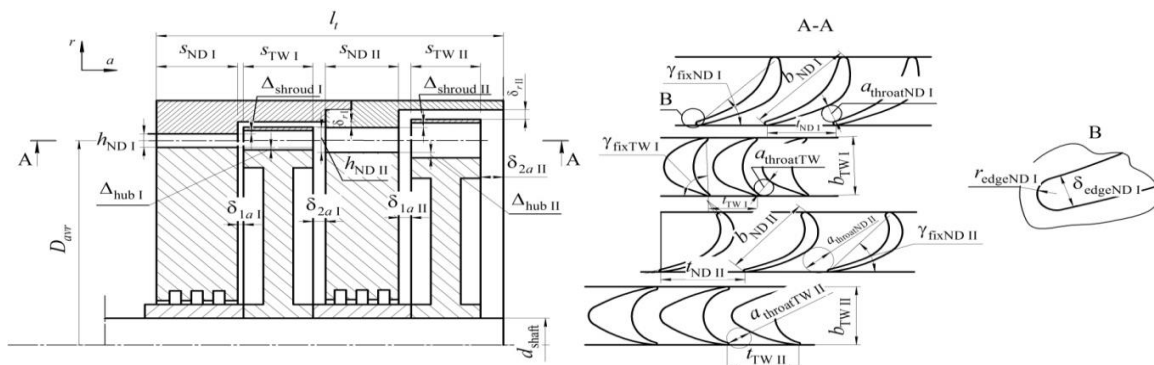


Figure 2. Meridional section of a two-stage pressure-compounded impulse ultra-low power turbine

Thus, the prospect of wider use of two-stage pressure-compounded axial ULPT is justified by the possibility of obtaining the greater efficiency than in single-stage turbines due to the same initial conditions (see Figure. 1). Therefore, the actual problem is the development of rational choice of two-stage pressure-compounded ULPT parameter values. Authors of this paper have already developed a method for single-stage ULPT parameters selecting which allows meet the complex of contradictory requirements. It is shown to be effective at the preliminary design [6]. This work follows, and extends our previous one. We propose a method for selecting the rational parameter values of impulse two-stage pressure-compounded ULPT.

2. Particulars of surrogate models in the method of selecting parameters of two-stage axial pressure-compounded ULPT

The developed method of selecting parameters of a two-stage axial ULPT is based on the principles

that was set out in [6] for a single-stage ULPT. The method is implemented in the mathematical package Matlab Optimization Toolbox. Subroutine for the optimization calculation is based on the well-known algorithm of one-dimensional gas-dynamic calculation of a turbine stage [7, 8]. It is based on empirical relations and the amendments on influence of a small size to turbine efficiency.

For example, the regression empirical relation of power efficiency from the above-mentioned operating and geometric parameters has been used in this algorithm. It was obtained by the author of paper [9].

The main feature of the proposed approach to determining the efficiency of a two-stage axial pressure-compounded ULPT is a formation of surrogate models of criteria for evaluating the two-stage turbine efficiency on the basis of mathematical models for a single-stage turbine. The main relationships and dependencies for creating such models are described below.

Gas-dynamic loading of a multi-stage turbine with pressure stages is characterized by the blade-jet speed ratio $y_t = \sqrt{\Sigma u^2} / c_s$, but in the special case of rotation speeds equality $y_t = u \sqrt{z} / c_s$ [4]. We propose to derive the blade-jet speed ratio of whole turbine through ratios of it stages y_{stI} and y_{stII} . To do this, it is necessary to perform the following transformations.

The kinetic energy of entire turbine is the sum of the each stage kinetic energy

$$\frac{c_{1s}^2}{2(1-\rho_t)} = \frac{c_{1sI}^2}{2(1-\rho_{stI})} + \frac{c_{1sII}^2}{2(1-\rho_{stII})}; \quad c_s^2 = c_{sI}^2 + c_{sII}^2, \quad (2)$$

where ρ_{stI} , ρ_{stII} and ρ_t – degree of the first turbine stage, the second stage reaction and the average level of turbine reaction, respectively.

Then

$$y_t = \frac{(\Sigma u^2)^{1/2}}{c_s} = \left(\frac{u_I^2}{c_{sI}^2 + c_{sII}^2} + \frac{u_{II}^2}{c_{sI}^2 + c_{sII}^2} \right)^{1/2} = \left(\frac{u_I^2 / c_{sI}^2}{1 + (c_{sII}^2 / c_{sI}^2)} + \frac{u_{II}^2 / c_{sII}^2}{(c_{sI}^2 / c_{sII}^2) + 1} \right)^{1/2} = \left(\frac{y_{stI}^2}{1 + \frac{c_{sII}^2}{c_{sI}^2}} + \frac{y_{stII}^2}{1 + \frac{c_{sI}^2}{c_{sII}^2}} \right)^{1/2} \quad (3)$$

Calculation of the blade-jet speed ratio by formula (3) turned out to be more accurate results than by traditional method through reactivity of each stage instead of generalized, with some error, reaction ρ_t .

The gas turbine work is the sum of individual stages gas works

$$L_t = L_{stI} + L_{stII} \text{ or } \eta_t L_{ts} = \eta_{stI} L_{stIs} + \eta_{stII} L_{stIIs} \quad (4)$$

Then, the coefficient of power efficiency

$$\eta_t = \frac{\eta_{stI} L_{stIs} + \eta_{stII} L_{stIIs}}{L_t} \quad (5)$$

where $L_{stIs} = \frac{k}{k-1} R T_{0I}^* \left(1 - \pi_{stI}^{\frac{1-k}{k}} \right)$ and $L_{stIIs} = \frac{k}{k-1} R T_{0II}^* \left(1 - \pi_{stII}^{\frac{1-k}{k}} \right)$ – isentropic gas flow work in the first

and second stages respectively;

T_{0I}^* and T_{0II}^* – total temperature at the inlet of the first and second stage respectively;

$\pi_{stI} = p_{0I}^* / p_{2I}$ and $\pi_{stII} = p_{0II}^* / p_{2II}$ – pressure drop ratio in the first and second stages respectively; multiplication $\pi_{stI} \pi_{stII}$ determines the pressure ratio π_t of whole turbine;

η_{stI} and η_{stII} – coefficients of total-static efficiency for each stage that can be calculated by the formula [9]:

$$\begin{aligned} \eta_{stI} = & 0.146 + 1.672 y_{stI} - 3.165 y_{stI}^2 - 4.018 (h_{ND} / D_{avr})_I + 65.827 (h_{ND} / D_{avr})_I^2 + 0.0205 \alpha_{1efI} - \\ & - 0.0011 \alpha_{1efI}^2 - 0.11 (b/t)_{TWI}^2 - 0.002 (s/h)_{NDI}^2 + 0.091 y_{stI} \pi_{stI} + 6.022 y_{stI} (h_{ND} / D_{avr})_I + \\ & + 4.345 (h_{ND} / D_{avr})_I (b/t)_{TWI} - 0.068 \pi_{stI} \bar{\delta}_{edgeNDI} + 0.12 \bar{\delta}_{edgeNDI} (b/t)_{TWI} - \end{aligned}$$

$$- 0.006y_{sti}(b/t)_{TWi} + 0.195y_{sti}\bar{D}_{avr i} + 0.045y_{sti}(s/h)_{NDi} - 0.0019\pi_{sti}(s/h)_{NDi} + 1.106(h_{ND}/D_{avr})_i(s/h)_{NDi} + 0.033(b/t)_{TWi}(s/h)_{NDi} \quad (6)$$

The value of multiple coefficient of determination is equal to 0.967, which indicates a high correlation between effective and independent variables. But we should note that we obtained Eq. (6) on the basis of tests results of single-stage turbines. It is fully justified for the first stage because it does not take into account the effect of non-uniform parameters fields in axial gap on the efficiency of subsequent stages. However, it is quite acceptable to optimize the parameters using Eq. (5) and (6) at the stage of preliminary design. It was shown by the following results of verification of the surrogate model for an efficiency evaluation.

Surrogate models of the turbine size and weight were formed similarly by summing the length and mass of its stages.

Solution of the optimal design task of any technical system always presupposes the availability of different kinds of conditions and constraints. Implementation of these conditions indicates the compliance of output data to the requirements of technical task. The parametric constraints such as $X_{\min} \leq X \leq X_{\max}$ and functional limitations, which relate the parameters of vector X , are imposed in the object existence region.

The parametric conditions and constraints of the optimization problem were selected from the variation ranges of the independent variables x_j , which are given in [10].

There is a basic functional condition to provide a given mass flow rate G_{tu} through a turbine unit or to provide a given shaft power N_{tu} . Mass flow rate and power are defined through optimized parameters and some initial data:

$$G_{tu} = \frac{2.5 \cdot 10^{-3} \cdot m p_{in}^* \sigma_{in} \sigma_{throat} \cdot (h_{ND}/D_{avr})_1 \bar{D}_{avr 1}^2 (\sin \alpha_{1ef 1}) q(\lambda_{throatND 1})}{(1 + \delta_{edgeND 1}) \sqrt{T_{in}^*}} \quad (7)$$

$$N_{tu} = G_{tu} L_{tus p} \eta_{tu} \quad (8)$$

where $m = 0.0404 \left(\frac{\text{kg} \cdot \text{K}}{\text{J}} \right)^{-0.5}$ - a physical constant that depends on the properties of the working fluid;

σ_{in} - total pressure loss in the area from the entrance of the turbine to the turbine;

σ_{throat} - total pressure loss in the area from the nozzle diaphragm inlet to throat; approximately defined as $0.5(1 + \sigma_{in})$ [5];

$p_{in}^* = p_0^*/\sigma_{in}$ and $T_{in}^* = T_{01}^*$ - stagnation pressure and temperature at the inlet of turbine;

$q(\lambda_{throatND 1})$ - relative flux density in the critical section (throat) of the nozzle unit of the first turbine stage;

$\eta_{tu} = \eta_{in}\eta_t\eta_{out}$ - coefficient of power efficiency, a multiplication of coefficients of the input merit η_{in} , the output merit η_{out} and the coefficient of turbine power efficiency η_t [5].

Calculated values G_{tu} and N_{tu} in the multimode turbine unit are not known in advance, but their possible values must be within the range that defined by the load diagram. Thus, one of the constraints given by inequality

$$G_{tui \min} \leq G_{tu \text{ design}} \leq G_{tui \max} \quad \text{or} \quad (9)$$

$$N_{tui \min} \leq N_{tu \text{ design}} \leq N_{tui \max} \quad (10)$$

In the case of single-mode turbine unit these conditions should meet the equations

$$G_{tu \text{ design}} - G_{\text{set}} = 0 \quad \text{and} \quad (11)$$

$$N_{tu \text{ design}} - N_{\text{set}} = 0, \quad (12)$$

where G_{set} and N_{set} - set values of the mass flow rate and of a turbine unit power that are specified in the technical task.

In addition, it is necessary to set equality $G_{st I} = G_{st II}$ for matching the flow paths of turbine stages at the mass balance condition.

Cyclogram of the designed turbine drive limits the operating ranges of rotation speed n . In this

case, assumptions of surrogate mathematical models for evaluation criteria require a consideration of restrictions on the magnitude $y_{sti} \leq 0.4$ in all operating modes.

Great importance in the design of ultra-low power turbine drive has a design and technological constraints. The main constructive restriction is the tolerance to nozzle diaphragm blades height:

$$0.5 \text{ mm} \leq h_{ND} \leq 5 \text{ mm}, \quad (13)$$

where $h_{ND I} = 50 (h_{ND} / D_{avr})_I \bar{D}_M$ - the blades height of the first stage of ND;

$h_{ND II} = 1.8 h_{TW I}$ - the blades height of the second stage ND;

The heights of turbine wheel blades of the first and second stages are defined by formula $h_{TW i} = 1.8 h_{ND i}$.

The blade height increasing of each subsequent by 1.8 times is caused by the fact that the optimum ratios of hub and shroud overhang $\Delta_{hub i}$ and $\Delta_{shroud i}$ (Figure. 1) to $h_{ND i}$ are about 0.3 and 0.5 respectively.

Considering that

$$h_{TW i} = h_{ND i} [1 + (\Delta_{hub i} / h_{ND i}) + (\Delta_{shroud i} / h_{ND i})] = 1.8 h_{ND i} \quad (14)$$

In addition to gas-dynamic, structural-technological and dimensional constraints, the strength limitations are usually taken into account while turbines optimizing. However, at the preliminary stage of design these factors could not be defined by the following reasons:

- strength factors do not play a determining role in the parameters selection for ultra-low power turbines that operate at low temperatures of the working fluid T_{in}^* during the limited operating time [11];
- parametric restrictions are imposed so as to minimize the probability of selecting the geometry of turbine that is not providing its static strength in wide ranges of mode parameters.

Strength limitations in the mathematical optimization model of ULPT do not included for reasons noted above.

3. Approbation of surrogate models while selecting rational parameters values of two-stage axial pressure-compounded turbine of an air starter

The largest application was received by turbine type air starters (turbine starters), which are interchangeable with electric starters. Typically, such turbine starters have two velocity stages and less often - pressure stages [12].

3.1 Initial data for designing.

The article deals with the design concept of a turbine driven air starter for diesel engine with the initial data corresponding to the standard atmospheric conditions: corrected rated power of turbine drive $N_{td corr} = 7.5 \text{ kW}$; corrected rotation speed on the rated mode $n_{corr} \leq 20,000 \text{ rpm}$. Inlet pressure p_{in}^* was kept constant and determined by the pressure drop ratio π , which ranged from 2 to 6 while optimizing process, and by the outlete pressure p_{out}^* . In the calculations we took the values $p_{out}^* = 99 \text{ kPa}$ and $T_{out}^* = 233 \text{ K}$.

Cyclogram of turbine drive operating was not set because starter was designed for a conditional diesel engine whose parameters are not known. Turbine optimization was performed only under the nominal mode since a starter was calculated for a conventional diesel engine, the cyclogram of which is not known. Starter is supposed to run for 20 seconds at nominal mode [13].

3.2. Selection of criteria for efficiency evaluation.

According to the systematic approach for the turbine unit designing, the turbine parameters should be optimized on the evaluation criteria of efficiency for the entire turbine unit. Energy efficiency is characterized by specific mass flow rate through the turbine unit $G_{tu sp} = G / N_{tu}$ whereas mass efficiency is characterized by turbine unit (air starter in our case) specific mass $M_{tu sp} = M_{tu} / N_{tu}$. However, for the convenience of comparing the values of criteria based on the optimization results, it is desirable that they would have the same dimensionality. Therefore, instead of the specific air consumption, a

criterion of the required specific mass of compressed air $M_{air\ sp} = M_{air}/N_{tu} = G\tau/N_{tu}$ was used for an air starter operation during operating time $\tau = 10$ s. Then both criteria are measured in kg/W.

3.3 Selection of varying ranges of optimization parameters.

The ranges of varying operational and geometric parameters (which are included in Eq. (6)) were chosen on the recommendations [14]. The range corresponding the values of $D_{avr} = 80 \dots 100$ mm was chosen due to the analysis of the modern industrial air starters dimensions.

3.4 Features of optimization techniques.

Optimization was performed in MATLAB Optimization Toolbox package in the order that was described in detail in the paper [6].

We used following methods to the solutions searching: the goal attainment method as solver, two objectives ($M_{air\ sp}$ and $M_{tu\ sp}$), two vectors of goals values ($M_{air\ sp}^*$ and $M_{tu\ sp}^*$) and weights ($\Delta M_{air\ sp}$, $\Delta M_{tu\ sp}$), arrays with 16 inputs (divided into 8 parameters for each stage) lower bounds, upper bounds and start point and a special subroutine of nonlinear constraints that has been briefly described above.

3.5 The results of optimization.

At the first step of optimization, the optimal parameters have been found on criteria $M_{air\ sp}$ and $M_{tu\ sp}$ separately, i.e. single-criterion optimization was conducted. Next, the task of two-criterion optimization was set. Two-criterion optimization results are given in Table 1.

Table 1. Results of two-criterion optimization

No.of stage	Optimized parameter	Parameter value	Criterion value	
			$M_{air\ sp}$ kg/kW	$M_{tu\ sp}$ kg/kW
I	y_{st}	0.321	0.407	2.567
	π_{st}	2.039		
	h_{ND}/D_{avr}	0.014		
	α_{1ef}	14.69		
	$\bar{\delta}_{edgeND}$	0.178		
	$(b/t)_{TW}$	1.116		
	$(s/h)_{ND}$	7.59		
	$\bar{D}_{avr}$	2		
II	y_{st}	0.277	0.407	2.567
	π_{st}	2.986		
	h_{ND}/D_{avr}	0.046		
	α_{1ef}	7.633		
	$\bar{\delta}_{edgeND}$	0.209		
	$(b/t)_{TW}$	1.471		
	$(s/h)_{ND}$	3.783		
	$\bar{D}_{avr}$	2		

Criteria deviation are calculated by the formula

$$\Delta y_{rp} = \frac{y_{rp} - y_{rp}^{\min}}{y_{rp}^{\min}} \cdot 100\% \quad (15)$$

when $y_{rp}^{\min}$ - criterion value determined by results of single-criterion optimization.

Criteria deviation amounted to $\Delta M_{air\ sp} = 1.96\%$ and $\Delta M_{tu\ sp} = 2.39\%$. The values of the deviations are small and fit within the range of error in determining the values of these criteria, which makes it advisable to use two-criterion optimization in this case.

According to the Equations (3-5), the design blade-jet speed ratio $y_t = 0.298$ and pressure drop ratio $\pi_t = \pi_{t\ st\ I} \pi_{t\ st\ II} = 6$. Design value of corrected rotation speed $n_{corr} = 19,900$ rpm.

4. Results of surrogate models validation

Validation of surrogate models has been conducted by comparing the optimization results and computational experiment data. Comparison was implemented by integral gas-dynamic parameters: coefficient of power efficiency η_t and corrected torque $M_{torq\ corr}$.

Computational experiments were performed using the commercial CFD software package Ansys CFX. The computational fluid flow domain corresponds to optimized turbine geometry. The turbine computational domain is shown in Figure. 3. We used the turbulence model SST, which is more suitable for the flow computations in small-sized turbines [15]. The flow parameters at the interface of the stationary and rotating elements transmitted through a cylindrical interface surface «frozen rotor», i.e. without averaging parameters in the circumferential direction. The flow models in ULPT were previously verified by field experiments [15].

Validation was conducted in two steps. At the first step we performed a comparison of CFD simulation data with the results of parameters selection for the design mode according to the developed method (Table 2).

Table 2. Comparison of calculation results and computational experiment

	$\eta_{st\ I}$	$\eta_{st\ II}$	η_t	$M_{torq\ corr}$, N m	$N_{t\ corr}$, W
Optimization results	0.568	0.647	0.607	3.6	7500
Computational experiments data	0.603	0.579	0.59	3.54	7396
Relative error, %	-5.8	11.74	2.88	1.69	1.41

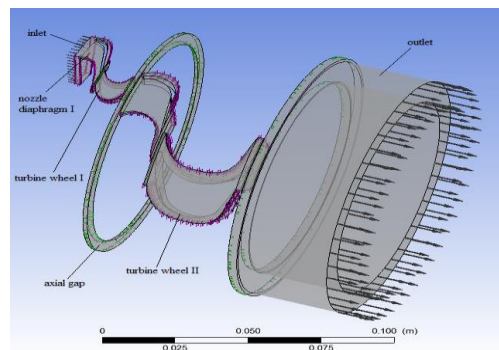


Figure 3. Fluid flow domain for CFD simulation of the optimized turbine geometry

As can be seen from Table 2, the computational experimental data practically coincide with the results of calculation, since the relative error fit within the experimental errors of the efficiency measurement [15]. However, the errors in determining the efficiency of individual stages turned out to be quite large. The reasons for this could be explained as follows. The higher experimental values η_{st} are caused by the influence of positive degree of reaction $\rho_{st\ I}$, which was not considered in the equation (6). At the same time, a decrease of $\eta_{st\ II}$ in comparison with calculation data is probably due to the additional axial gap losses by the uneven flow field. These losses are not taken into account in the surrogate optimization model. It is interesting that eventually these discrepancies cancel each other out so that the relative error of power efficiency determining by Eq. (6) has a very low value.

At the second step, we compared turbine characteristics that had been obtained on the optimization results by developed surrogate model with turbine characteristics by CFD simulation. Validation was carried out in the following sequence:

- computational experiments were carried out on the experimental design *in which the combinations of parameters* $\pi t = 2; 3; 4; 5; 6$ and $n_{corr} = 13,000; 19,500; 26,000$ rpm were varied;
- the characteristics $\eta_t = f(\pi t)$ and $M_{torq\ corr} = f(\pi t)$ were described according to the results of these experiments;
- the torque $M_{torq\ corr}$ and power efficiency η_t values were calculated in the Matlab subroutine with the use of surrogate models by the developed method (based on the calculated parameters $y_{st\ i}$, $\pi_{st\ i}$ and on the already known geometric ratios of turbine stages);

- the results were compared with each other (Figure. 4a and Figure. 4b).

Efficiency values that are calculated in MATLAB subroutine by the surrogate model have a very good agreement with the computational experiment data throughout the considered rotation speed range at the design value of turbine pressure drop ratio $\pi_t = 6$ (Figure. 4a).

However, within a deviation of π_t values from the design value, the divergences in most cases are significant and the characteristics curves are not congruent.

5. Comparison with results of an axial single-stage ULPT optimization

The method for selecting rational parameters of single-stage ULPT was first described in paper [6]. In this work we supplemented this method with surrogate models for two-stage axial pressure-compounded ULPT optimization. Now, let us compare the optimization results that were obtained in [6] for single-stage axial turbine and results we have gotten at present work for two-stage axial turbine, provided at the nominal mode with operating period $\tau = 20$ sec (Table 3).

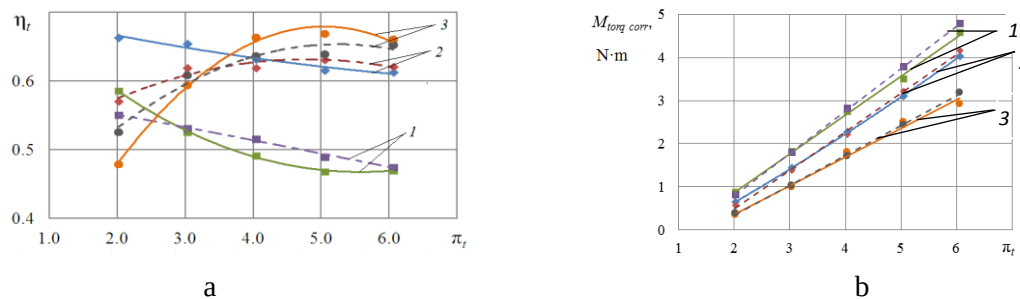


Figure 4. Calculated (—) and CFD-Simulated (---) efficiency (a) and torque (b) characteristics of the air starter ULPT (1 - $n_{corr} = 13,000$ rpm; 2 - 19,500 rpm; 3 - 26,000 rpm)

Table 3. Single-stage axial ULPT vs. two-stage axial ULPT

Initial data	Single-stage ULPT	Two-stage ULPT
Inlet pressure p_0^* , kPa	594	592
Shaft power $N_{t\ corr}$, W	7500	7500
Torque $M_{torg\ corr}$, N m	3.6	3.6
Rotating speed n , rpm	19,900	19,900
Output data	Single-stage ULPT	Two-stage ULPT
Mass flow air consumption G , kg/s	0.147	0.119
Power efficiency η_t	0.455	0.607
Mass of air starter M_{air} , kg	4.91	16.28
Specific mass of a fluid (air) $M_{air\ sp}$, kg/W	0.754	0.407
Specific mass of an air starter $M_{tu\ sp}$, kg/W	0.312	2.567
Overall specific mass $M_{\Sigma\ sp}$, kg/W	1.066	2.974

As it can be seen from Table 3, we obtained different output results with identical initial data values. Energy efficiency was increased with the use of two pressure stages instead single one. But in this way the mass efficiency of turbine unit is several times worse. Thus, the overall efficiency of the two-stage ULPT will be the higher the longer the operating time on design mode. For short-term operating turbine units like air starters we would recommend to use the single-stage turbines.

6. Conclusions

Analysis of the obtained characteristics allows making some conclusions. The developed surrogate models for evaluating the energy efficiency criteria provide a fairly good agreement with the results of CFD simulation at the proviso of the optimization at a given operational mode when total inlet pressure is kept constant ($p_0^* = const$). This corresponds to the fact that the average degree of reaction

of entire turbine p_t is equal to or close to zero.

If pressure drop ratio values π_i are differ from the calculated by more than 1.5 times (based on the estimates of errors from Figure. 4), we are not recommend this surrogate models for estimating the energy efficiency of a two-stage ULPT to avoid unreliable results. This means that the correction of surrogate model of turbine efficiency in view of degree of reaction influence is mandatory for an adequate selection the parameters of multimode turbine drive with $p_0^* = var$ during the operation. This will require the formulation and implementation of additional natural and computational experiments for further research.

So, the application of these surrogate models in the proposed method of parameter selection is most advisable for turbine units with maintaining a constant pressure at the pressurized air receiver like air starters turbines.

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Nomenclature

a_{throat}	blade passage throat, mm;	Subscripts	
b	blade chord, mm;	a	axial direction
c	absolute flow velocity, m/s;	avr	average parameter
d, D	diameter, mm;	c	velocity
G	mass flow rate of an air , kg/s;	$corr$	corrected to the standard atmospheric conditions
h	blade height, mm;	ef	effective
k	adiabatic index ($k = 1.4$ for air);	i	No. of operational mode;
l	length, mm;		No. of turbine stage
L	specific isentropic heat drop, J/kg;	j	No. of optimization parameter
M	weight, kg;		pressure; No. of a set
M	isentropic Mach number;	p	initial uncertain data
n	rotation speed, rpm;	q	No. of an optimized solution variant
N	shaft power, W;		radial direction; No. of efficiency criterion
p	flow pressure, Pa;	r	isentropic parameter
$q(\lambda)$	relative flux density	s	specified parameter
R	universal gas constant (for air it equals 287 J / kg K);	sp	turbine stage
s	blade width, mm;	st	turbine
t	blade pitch, mm;	t	turbine drive
T	flow temperature, K;	td	torque
u	blade linear velocity, m/s;	$torq$	turbine unit
y	blade-jet speed ratio;	tu	design parameter value;
z	number of stages;	design	inlet
Greek		in	outlet
α	angle of the absolute flow (nozzle angle), °	out	predefined (set) value of a parameter
β	angle of the relative flow (angle of a rotor blade), °	set	nozzle diaphragm
δ	gap, mm;	ND	turbine wheel
δ_{edge}	blade edge thickness, mm;	TW	before a nozzle diaphragm
Δ	blade overhang, m; difference or change of parameter, m;	0	before a turbine wheel
		1	

ρ	degree of reaction;	2	behind a turbine wheel
σ	total pressure loss;	I	first turbine stage
τ	operational time, s;	II	second turbine stage
(b/t)	chord-pitch ratio;	Superscripts	
(s/h)	relative extension of a blade (width-height ratio);	*	stagnation gas-dynamic parameter (total parameter); desired value (in optimization task)
λ	reduced velocity;		
η	coefficient of total-to-static power efficiency		
π_t	turbine pressure drop ratio		

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