

# The Fractal Characteristics of the Landslides by Box-Counting and P-A Model

Zhiwang Wang<sup>1,2,3,\*</sup>, Fangfang Zhou<sup>1,2,3</sup> and Hao Cao<sup>1,2,3</sup>

<sup>1</sup>Changjiang River Scientific Research Institute, Wuhan, China.

<sup>2</sup>Research Center on Water Engineering Safety and Disaster Prevention of the Ministry of Water Resources, Wuhan, China.

<sup>3</sup>Research Center on National Dam Safety Engineering Technology, Wuhan, China.

\*Email: wzwhbwh@126.com

\*Corresponding author

**Abstract.** The landslide is a kind of complicated phenomenon with nonlinear inter-reaction. The traditional theories and methods are difficult to study the uncertainty characteristics of dynamic evolution of the landslides. This paper applies box-counting and P-A model to study the fractal characteristics of geometric shape and spatial distribution of the landslide hazards in the study area from Badong county to Zigui county in TGP reservoir region. The data obtained from the study area shows power-law distributions of geometric shape and spatial distribution of the landslides, and thus reveals some fractal or self-similarity properties. The fractal dimensions  $D_{AP}$  of the spatial distribution of landslides by P-A model shows that  $D_{AP}$  of the western landslides in the study area are smaller than those of the east, which shows that the geometry of the eastern landslide is more irregular and complicated than the western ones. The results show box-counting model and P-A model can be used to characterize the fractal characteristics of geometric shape and spatial distribution of the landslides.

## 1. Introduction

Landslide is a common geological disaster occurring regionally and frequently. It usually produces serious damages to factories, mines, cities and towns, villages, farmlands, transportation, communication, and even blocks rivers and destroys the water conservancy projects. It occurred regionally and frequently in the mountainous areas of China. Therefore, it is of importance and significance to study the landslide so as to improve the landslide prevention and control, and to guarantee the engineering safety and the environmental protection.

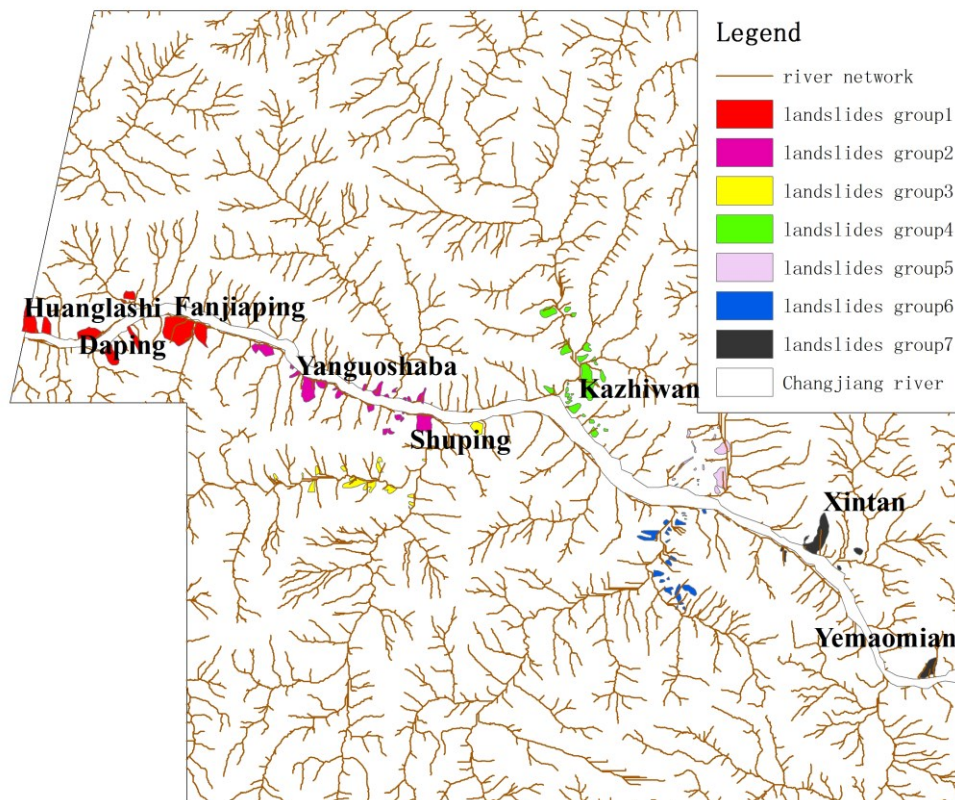
The landslide is a kind of complicated phenomenon with nonlinear inter-reaction. The traditional theories and methods are difficult to study the uncertainty characteristics of dynamic evolution of the landslide. In recent years, the fractal theory has become a new method to study such complicated phenomenon as earthquake (Turcotte, 1997) and landslide (Malamud et al., 2004). This paper uses the box-counting techniques to calculate the fractal dimensions of the geometric shape of landslides in the study area and uses the P-A model to characterize the fractal characteristics of spatial distribution of the landslides. The results show that these fractal models provide powerful tools for quantitative description of the geometric shape and spatial distribution of the landslide.

## 2. Study Area and Data Sets

The study area is located in Hubei Province from Zigui to Badong counties. There are most of the strata from the Pre-Sinian system to the Quaternary system except for the lower Devonian system, the upper Carboniferous and part of the Cretaceous and tertiary system in the area. There are many



landslides (as shown in Figure 1), for example, Xintan landslide, Huangtupo landslide and Huanglashi landslide, which are usually located in the section with “hard” rock and “soft” rock jointly and along the bank of the river. The landslides often caused serious damage to the area before. For instance, the Huangtupo landslide forced the new town of Badong county to be relocated to Xirangpo, about 6 km to the west of Huangtupo. Another recent landslide, on 3rd March 2002, moved more than 20 million cubic metres of debris down slope, threatening the new town of Wushan County (Wu et al., 2001).



**Figure 1.** The distribution plan of known landslides in the study area.

The main data for this study is the 1:50,000 scales digitized geological map, from which the geometric shape and spatial distribution of the landslides were obtained.

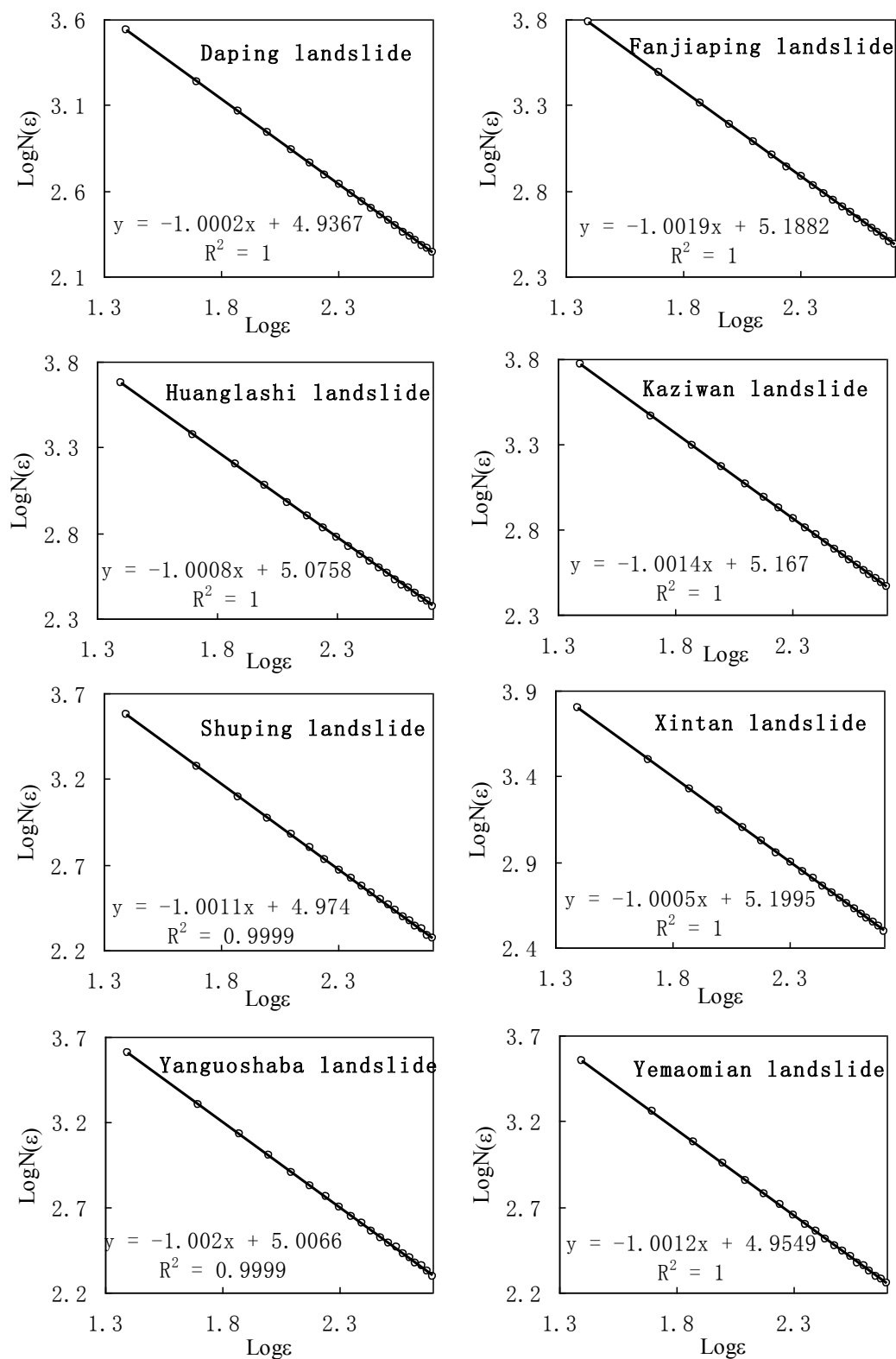
### 3. The Fractal Dimensions of the Geometric Shape of Landslides by Box-Counting Model

There are many methods of measuring and calculating the fractal dimension. For a certain area of statistical measurement, the box-counting method is simple, objective and practical and thus it is now widely used. The box-counting method is a convenient and objective fractal dimension method (Falconer., 1990). If the study area is divided into a number of squares with the length of  $\varepsilon$  in the selected area, the number of lattices of the geometry is determined to be  $N$ , then change  $\varepsilon$  to find the corresponding  $N(\varepsilon)$ , and so on, and finally find the fractal dimension  $D$  by the following formula.

$$D = -\lim_{\varepsilon \rightarrow 0} \frac{\log N(\varepsilon)}{\log \varepsilon} \quad (1)$$

The box-counting method was used to calculate the fractal dimension of the boundary trajectory of the typical landslides such as Huanglashi landslide, Fanjiaping landslide, Daping landslide, Kaziwan landslide, Shuping landslide, Yanguoshaba landslide, Xintan landslide and Yemaomian landslide (Figure 1), the calculation results are shown in Figure 2. It can be seen from the  $\log N(\varepsilon) \sim \log(\varepsilon)$  plot

that there is a clear linear relationship between them and the line fitting degree is high, which shows that the boundary trajectories of these landslides have obvious fractal characteristics.



**Figure 2.** The fractal dimensions plot of the geometric shape of landslides in the study area

#### 4. The Fractal Dimensions of the Spatial Distribution of Landslides by P-A Model

At present, there are many fractal models that describe the irregularities of the geometric objects and mode shapes. Among them, the P-A model is a very effective method to describe the irregular geometric features of the objects with similar geometric characteristics and has been widely used (Cheng, 2004; Zhang et al., 2001). This paper uses the P-A model to quantitatively describe the irregularity of regional landslide distribution.

The P-A general model (Cheng, 1995) has the following expression:

$$P(> A) \propto A^{\frac{1}{2}D_{AP}} \quad (2)$$

Where P is the perimeter, A is the area,  $\propto$  stands for "proportional to" and  $D_{AP}$  is the exponent of the power-law relationship.

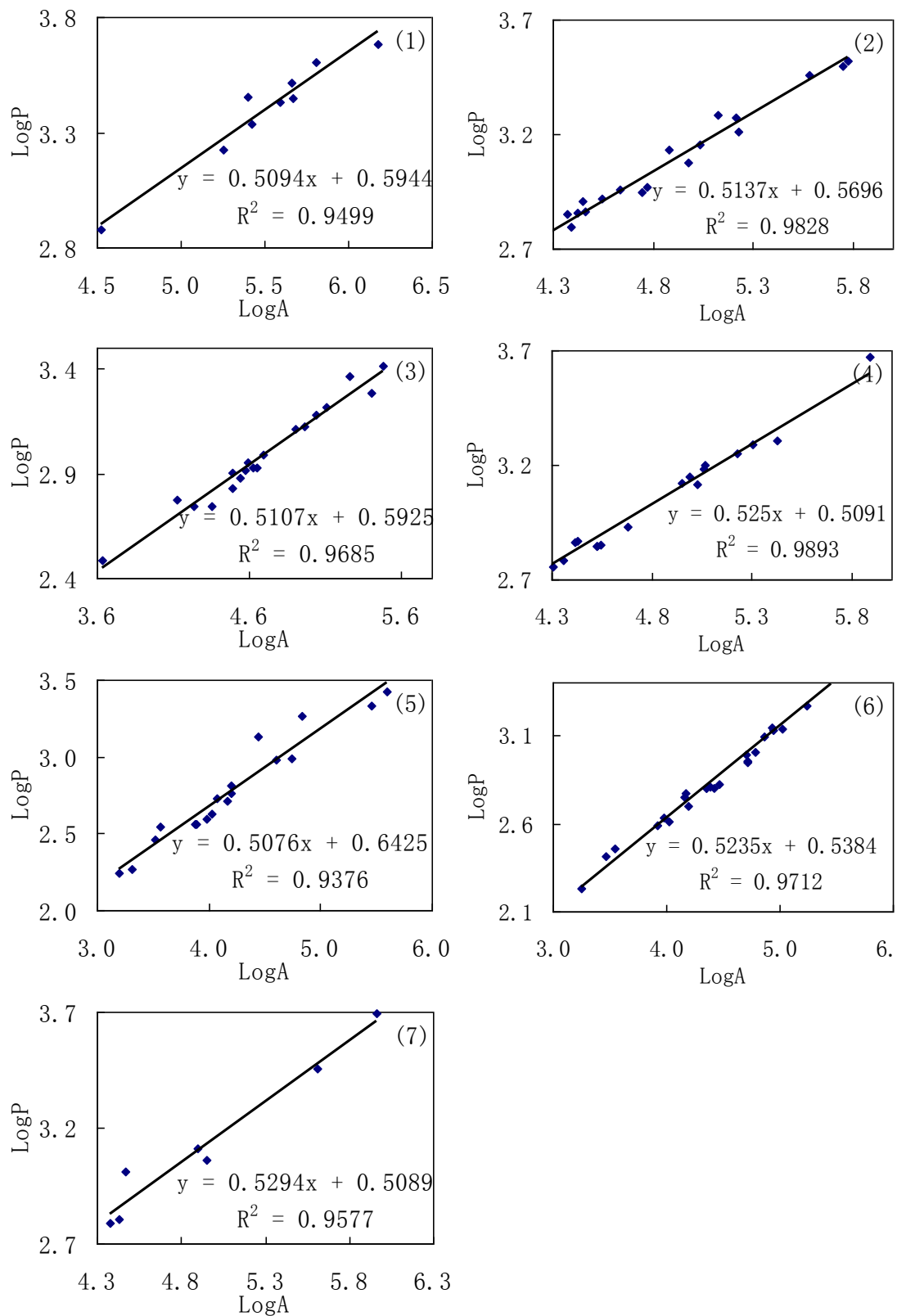
In general,  $D_{AP}$  ranges between 1 and 2. If  $D_{AP}=1$ , then  $P=A^{0.5}$ , and it implies regularly shaped sets, i.e. the squares or circles. The higher the value of  $D_{AP}$ , the greater the irregular shape of the grains. If  $D_{AP}=2$ , then  $P=A$ , and it implies extremely irregular sets so that the perimeter changes at the same rate as area. So,  $D_{AP}$  is an indicator that shows the irregularity of the shaped set (Cheng, 1995).

To determine area-perimeter exponent  $D_{AP}$ , the data set of P and A plotted as log-log scale will show linear relationship between the log A and log P, which can be fitted with straight line using the least square method. The slope of linear regression can be taken as the estimation of  $1/2D_{AP}$

$$\log P = C + \frac{1}{2} D_{AP} \log A \quad (3)$$

The landslides in the study area are divided into 7 different divisions according to geographical distribution (Figure 1). Using the P-A model, the calculated landslides perimeter and area data of different divisions are respectively plotted on the log-log map (Figure 3). Through the least square method, the slope of P-A (perimeter-area) of the landslide can be calculated and the P-A fractal dimensions can be calculated.

As can be seen from Figure 3, there is a clear linear relationship between the logarithm of the area and the logarithm of the circumference, and the correlation coefficient  $R^2$  is greater than 0.9, which indicates that there is a good linear correlation. The analysis of the geometry of the seven zonal landslides shows that the average of  $D_{AP}$  is about 1.04 and the variation range is between 1.02 and 1.06. From the perspective of regional distribution,  $D_{AP}$  of the western landslides in the study area are smaller than those of the east, which shows that the geometry of the eastern landslide is more irregular and complicated than the western ones. The results indicate that the fractal theory provides an effective measure to further study the morphological characteristics and formation conditions of landslides.



**Figure 3.** The LogA-LogP plot of the spatial distribution of landslides in the study area.

## 5. Concluding Remarks

Box-counting model and P-A model can be used to characterize the fractal characteristics of geometric shape and spatial distribution of the landslides. The data obtained from the study area show power-law distributions of geometric shape and spatial distribution of the landslides, and thus reveal some fractal or self-similarity properties. These fractal models provide powerful tools for quantitative description of the geometric shape and spatial distribution of the landslide.

## 6. Acknowledgments

This research was financially supported by the National Natural Science Foundation of China (No.51379023), the Public Welfare Research Project sponsored by Ministry of Water Resources of China (201501033-3) and Science and Technology Special Fund Project of Guizhou Provincial Water Resources Department (KT201604).

## 7. References

- [1] Malamud B.D., Turcotte D.L., Guzzetti F., Reichenbach P., 2004. Landslide inventories and their statistical properties. *Earth Surface Processes Landforms* 29 687
- [2] Turcotte D.L., 1997. *Fractals and Chaos in Geology and Geophysics*, second ed. Cambridge University Press, Cambridge, 412pp.
- [3] Wu S, Shi L, Wang R, Tan C, Hu D, Mei Y and Xu R 2001 *Engineering Geology* 59 51
- [4] Mandelbrot B B 1974 *J. Fluid Mechanics*, 62 331
- [5] Falconer K, 1990 *Fractal Geometry*. Wiley, New York, pp 288
- [6] Zhang Z., Mao H., Cheng Q 2001 *Math. Geol.*,33 217
- [7] Cheng Q 2004 *Math. Geol.*,36 345
- [8] Cheng Q 2005 *Math. Geol.*, 38 915