Biped Robot Gait Planning Based on 3D Linear Inverted Pendulum Model

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Abstract. In order to optimize the biped robot's gait, the biped robot's walking motion is simplify to the 3D linear inverted pendulum motion mode. The Center of Mass (CoM) locus is determined from the relationship between CoM and the Zero Moment Point (ZMP) locus. The ZMP locus is planned in advance. Then, the forward gait and lateral gait are simplified as connecting rod structure. Swing leg trajectory using B-spline interpolation. And the stability of the walking process is discussed in conjunction with the ZMP equation. Finally the system simulation is carried out under the given conditions to verify the validity of the proposed planning method.

1. Foreword

Biped robot is a multivariable, nonlinear, strongly coupled and variable structure complex dynamics system. And it is a comprehensive application of new technologies in many fields such as computer technology, electronics, mechanics and artificial intelligence. Gait generation is a relatively basic and key technique in biped robot research. We must study practical and effective gait planning methods to achieve and improve the robot's walking performance. People want to use fewer variables to get the gait walk that is as real humans among many techniques. Based on this idea, the three-dimensional linear inverted pendulum model is applied to the generation of robot gait model [1, 2].

Inverted pendulum control system is a complex, unstable, nonlinear system. It is an ideal experimental platform for control theory teaching and various control experimenting. Among many research methods of robots, the inverted pendulum system is used to simulate biped robot walk in recent years due to the great similarities between inverted pendulum system and biped robot walking, and it has great significance in the research of biped robot gait planning and stability control. Become a research direction of biped robot [3].

This paper systematically analyzes the robot's motion system, establishes a genealogical data structure and determines the robot's motion transmission path. Then, the robot with single-leg support period is simplified to 3D inverted pendulum movement, and the movement trajectory of CoM is obtained according to the relationship between ZMP and CoM and the initial conditions. Using link constraints to find the desired angle of each joint. Combined with ZMP theory [4, 5], the system simulation was carried out to verify the effectiveness of the proposed method.

2. Biped Robot System Analysis and Modeling

The object of this paper is a biped robot with 21 degrees of freedom. A miniature servomotor is used as the driving element to simulate a joint. Among them, the hip of the leg has 3 mutually perpendicular degrees of freedom, the knee has 1 degree of freedom, ankle joints has 2 mutually perpendicular degrees of freedom, the upper extremity shoulder has 2 mutually perpendicular degrees of freedom and the elbow has 1 degree of freedom, the head has 2 degrees of freedom, the waist has 1 degree of freedom. The system of biped robot is first analyzed because of its complexity. According to the number of joints, the robot is divided into 21 masses. Each mass contains a joint. The torso has no joint. When the model newly joins a joint, it can be added to the corresponding position. When programming, you can use a unified function to handle programming issues for each mass. Each mass has two branches, the bottom left is the parent-child relationship, and the bottom right is the brother-sister relationship. Then these masses are connected in a certain order to form the robot's motion database, which is convenient for recording and updating the value of each individual. At the same time, it shows the power transmission route. For example, when the robot supports the left foot swinging, The 13th individual's movement affects the 12th individual's movement, that is the route is:13-12-11-10-9-8-1-14-15-16-17-18-19, you can also understand the robot motion database as a genealogical tree Structure shown in Figure 1. The genealogical tree structure is converted into a data list shown in Table 1, starting from the root of the tree marked as 1, giving each link a number (ID), and showing the connection between the links in Table 1.

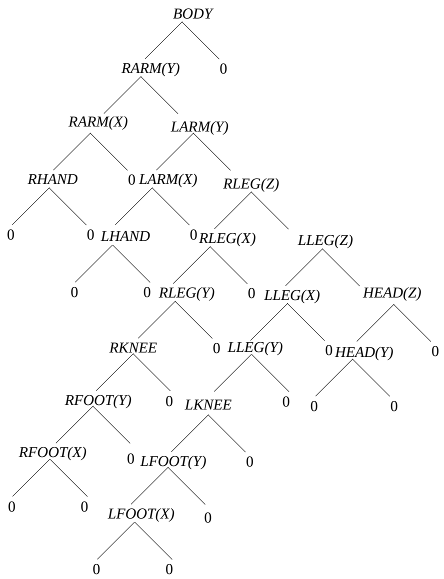

Figure 1. The family tree structure of robot

Table 1. Robot data list

ID	NAME	SISTER	CHILD
1	BODY	0	2
2	RARM(Y)	5	3
3	RARM(X)	0	4
4	RHAND	0	0
5	LRAM(Y)	8	6
6	LRAM(X)	0	7
7	LHAND	0	0
8	RLEG(Z)	14	9
9	RLEG(X)	0	10
10	RLEG(Y)	0	11
11	RKNEE	0	12
12	RFOOT(Y)	0	13
13	RFOOT(X)	0	0
14	LLEG(Z)	20	15
15	LLEG(X)	0	16
16	LLEG(Y)	0	17
17	RKNEE	0	18
18	LFOOT(Y)	0	19
19	LFOOT(X)	0	0
20	HEAD(Z)	0	21
21	HEAD(Y)	0	0

3. 3D Linear Inverted Pendulum Model Walking Parameters Setting

Similar to human walking, the robot's gait cycle can be simply divided into two-legged and one-legged periods. The process of shifting the center of gravity from the rear foot support area to the front foot support area during the dual foot support period. During the single leg support period, the center of gravity of the robot is moved forward by the support leg, which is pivoted from the rear of the support leg to the front of the support leg. Relative to the single-leg support period, the control of the two-legged support period is less difficult. In this paper, the walking cycle is regarded as the cycle of single-legged support period, and the exchange of the two legs is completed in an instant [6].

In the single-leg support period, reference [7] simplified the robot into a 3D linear inverted pendulum model as shown in Figure 2. The torso of the robot is simplified as a particle of all quality. The legs of the robot are simplified into a massless, telescopic connecting rod that connects the torso to the foot. The center of mass is moved forward by bending and stretching the legs. And telescopic force can be decomposed into three components of the coordinate system. As follows,

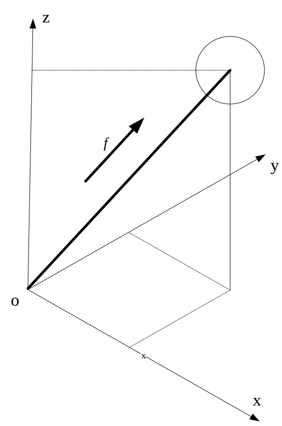

Figure 2. 3D linear inverted pendulum

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$$\begin{aligned} f_x &= (x / r) f \\ f_y &= (y / r) f \\ f_z &= (z / r) f \end{aligned} \tag{1}$$

Where r is the distance between the support point and the center of mass. The centroid is only affected by the forces of telescoping and gravity,it can be obtained through kinetic analysis.

$$\begin{aligned} M\ddot{x} &= (x / r) f \\ M\ddot{y} &= (y / r) f \\ M\ddot{z} &= (z / r) f - Mg \end{aligned} \tag{2}$$

Considering that the centroid constraint moves in a plane, the constraint plane is defined as follows

$$z = k_x x + k_y y + \tag{3}$$

Among them, k_x, k_y determine the plane tilt, z_c determines its height.

Due to the movement of the center of mass on the restraining surface, the resultant force should be in the restraining surface of the center of mass, So the constraint surface normal vector and resultant vector vertical.As follows,

$$\begin{bmatrix} f(x/r) & f(y/r) & f(z/r) - Mg \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ -1 \end{bmatrix} = 0 \tag{4}$$

From the above solution, and according to equation (3), the following expression is obtained.

$$f = \frac{Mgr}{z_c} \tag{5}$$

Substituting equation (5) into equation (2), the equation of motion of the center of mass on the constraint surface is obtained as follows:

$$\begin{aligned} \ddot{x} &= \frac{g}{z_c} x \\ \ddot{y} &= \frac{g}{z_c} y \end{aligned} \tag{6}$$

Solving the differential equation by Laplace transform, and the trajectory of x, y can be got. Then combined with the constraint surface equation, the trajectory of the centroid can be determined.

$$\begin{aligned} x(t) &= x(0)\cosh(t/T_c) + T_c \dot{x}(0)\sinh(t/T_c) \\ y(t) &= y(0)\cosh(t/T_c) + T_c \dot{y}(0)\sinh(t/T_c) \\ z &= k_x x + k_y y + z_c \end{aligned} \tag{7}$$

Where $T_c \equiv \sqrt{z/g}$ is a constant of the ratio of gravitational acceleration to centroid height. x(0) and x&(0) are the x-direction displacements and velocities of the centroid at time zero, respectively, y(0) and y&(0) are the y-direction displacements and velocities of the centroid at time zero, respectively, the initial conditions. After the initial conditions have been established, the trajectory of the mass center can be determined. Change the initial conditions, you can get different curves. Since the robot's center of mass is always forward in the forward direction and swings left and right in the lateral direction, the initial values of x and y are set as follows. Simulation results shown in Figure 3:

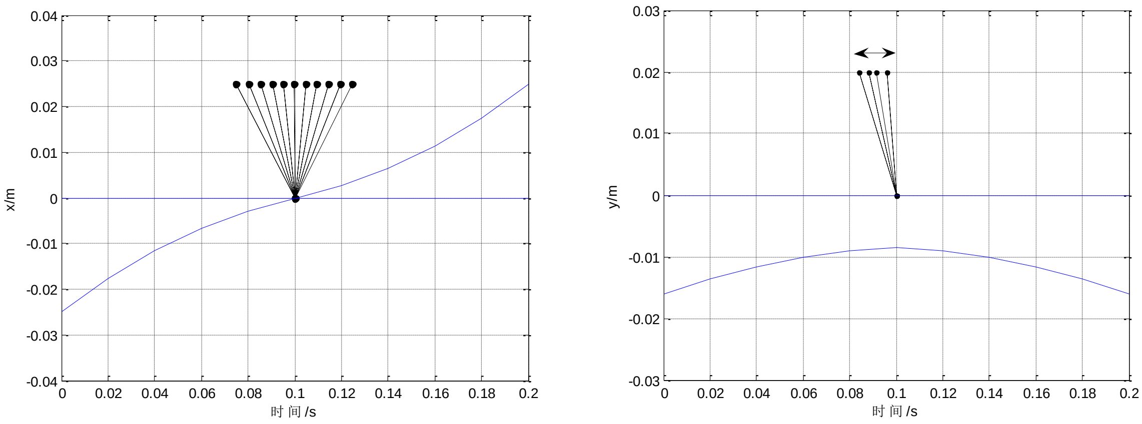

Figure 3. Inverted pendulum motion in x and y directions

The figure shows that in the x-direction, the centroid can continue to move forward beyond the support point, and the centroid can not cross the support point in the y-direction. After reaching a certain point, the centroid moves in the opposite direction.

4. Generation of Three-Dimensional Walk Pattern

In walking movement, the three-dimensional linear inverted pendulum can be regarded as a combination of two-dimensional linear inverted pendulum in two planes of xoz and yoz [8].

4.1. Gait Unit

In the single leg support period, the centroid conforms to the two-dimensional linear inverted pendulum motion in the x and y directions. From [9] we can see that the trajectory of the centroid is a hyperbola in the constrained plane. Based on its symmetry, the end position of the walking unit can be determined. Re-use of the symmetry of the walking unit, we can see the initial conditions and termination conditions are symmetrical. In the x-direction, the corresponding termination condition is $(\bar{x},\bar{v}_x)$, if the initial condition is $(-\bar{x},\bar{v}_x)$, since the centroid can cross the support point. In the y-direction, the corresponding termination condition is $(-\bar{y},-\bar{v}_y)$, if the initial condition is $(-\bar{y},\bar{v}_y)$, since the centroid can not cross the support point. The relationship between the two is:

$$\begin{aligned} \bar{v}_x &= \bar{x}(C+1)/(T_c S) \\ \bar{v}_y &= \bar{y}(C-1)/(T_c S) \end{aligned} \tag{8}$$

$$C \equiv \cosh\frac{T_p}{T} \qquad S \equiv \sinh\frac{T_p}{T}$$

After the initial conditions are determined, the motion trajectory can be determined based on the differential equation of two-dimensional linear inverted pendulum motion. The trajectory is generated by the pedestrian unit, and the same pedestrian units are connected according to the periodicity of the pedestrian and the initial conditions are alternately changed.

4.2. Gait Parameters

In the process of robot walking, landing point (support point) changes in turn, we use the step size and step width to represent and describe the changes in the location, see Table 2 below and Figure 4 below.

Table 2. Step and step width of the value

n	1	2	3	4	5	6	7
$S_x^{(n)}$	0.0	0.10	0.10	0.10	0.10	0.10	0.0
$S_y^{(n)}$	0.08	0.08	0.08	0.08	0.08	0.08	0.08

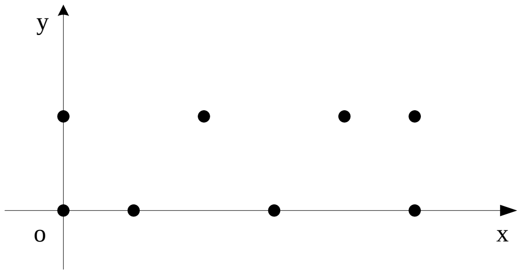

Figure 4. The location of the robot landing site

Here, $s_x^{(n)}$ is a step in the forward direction, $s_y^{(n)}$ is a step width in the left-right direction, and (n) is a step in the superscript. With the left foot support point as the origin of coordinates, the n-th foothold position $(p_x^{(n)}, p_y^{(n)})$ can be expressed as follows:

$$\begin{bmatrix} p_x^{(n)} \\ p_y^{(n)} \end{bmatrix} = \begin{bmatrix} p_x^{(n-1)} + s_x^{(n)} \\ p_y^{(n-1)} + (-1)^n s_y^{(n)} \end{bmatrix} \tag{9}$$

Where $(p_x^{(0)}, p_y^{(0)})$ is the position of the first supporting foot, the walking parameters of the n-th step are shown as follows,

$$\begin{bmatrix} \overline{x}^{(n)} \\ \overline{y}^{(n)} \end{bmatrix} = \begin{bmatrix} s_x^{(n+1)} / 2 \\ (-1)^{n+1} s_y^{(n+1)} / 2 \end{bmatrix} \tag{10}$$

The above equation is the coordinate of the reference coordinate system, that is, the support point as the origin. At the same time, the n-th walking parameter is determined by the n+1 step and step width. This condition is used in many precontrols. According to the symmetry of the gait unit we can know the walking speed of the walking unit [7].

$$\begin{bmatrix} \overline{v}_x^{(n)} \\ \overline{v}_y^{(n)} \end{bmatrix} = \begin{bmatrix} (C+1)/(TcS)\overline{x}^{(n)} \\ (C-1)/(TcS)\overline{y}^{(n)} \end{bmatrix} \tag{11}$$

4.3. The Relationship between CoM and ZMP

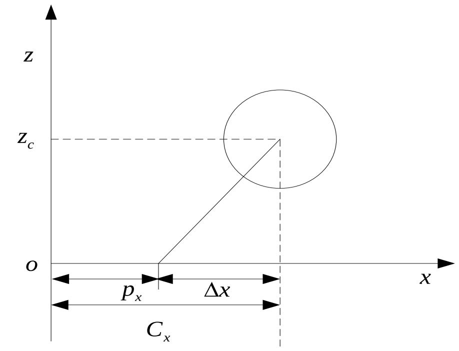

Figure 5. The relationship of CoP and ZMP

The robot is in the process of walking, support legs continue to alternate forward, with respect to the world coordinate system, the coordinates of the support leg constantly changes, the corresponding CoM coordinates also change, see Figure 5. A swing cycle, the robot do three-dimensional inverted pendulum motion, ZMP and CoM satisfy the differential equation [10] $\ddot{\Delta}x = \frac{g}{z_c}\Delta x$, in the formula $\Delta x = C_x - P_x$, the equation of motion for a linear inverted pendulum relative to the world coordinates is:

$$P_x = C_x - \frac{z_c}{g}\ddot{C}_x a \tag{12}$$

5. Swing Leg Ankle Trajectory Planning

In the single leg support period, the swing leg swings forward from the back of the support point to the support point, and when it reaches the ground, it is converted into a support leg. In the swing leg swinging process, must go through three key points, namely the initial point, the middle point and the end point. Using the three difference method [11], you can find the swing leg trajectory. The constraint equation of interpolation points:

$$X(t) = \begin{cases} -s & t = kT \\ 0 & t = kT + T/2 \\ s & t = kT + T \end{cases} \tag{13}$$

$$Z(t) = \begin{cases} h_{foot} & t = kT \\ H + h_{foot} & t = kT + T/2 \\ h_{foot} & t = kT + T \end{cases} \tag{14}$$

Where k is an integer, h_{foot} is the height of the ankle, H is the maximum height of the ankle, and s is the step size. Robotic Swing Legs Ankle trajectory planning, which primarily considers the impact and speed between the foot and the ground. If there is acceleration when the swinging leg falls, it will hit the ground and the swinging leg becomes a supporting leg at a speed of zero. To consider the maximum height of feet can be achieved, this can ensure that the robot has a certain obstacle avoidance capability. Due to the periodicity of the robot motion, the initial velocity and acceleration of the oscillating leg at time kT are both zero, and the velocity and acceleration of the oscillating leg at the instant of arrival (k+1)T are also zero, this minimizes the impact between the foot and the ground. Therefore, suppose that the first derivative and the second derivative of swinging legs x(t) and z(t) at moment kT and (k+1)T are both zero, that is:

$$\dot{X}(t)=\begin{cases}0 & t=kT\\ 0 & t=kT+T\end{cases} \tag{15}$$

$$\ddot{X}(t)=\begin{cases}0 & t=kT\\ 0 & t=kT+T\end{cases} \tag{16}$$

$$\dot{Z}(t)=\begin{cases}0 & t=kT\\ 0 & t=kT+T\end{cases} \tag{17}$$

$$\ddot{Z}(t)=\begin{cases}0 & t=kT\\ 0 & t=kT+T\end{cases} \tag{18}$$

According to the robot's own performance requirements, planning a single step s=0.06m, single-step cycle T=0.2s, the final swing leg ankle X-direction trajectory:

$$\begin{aligned}x(t)&=-0.06+150*t^3-1125*t^4\\ &+2250*t^5 \qquad t\in[0 \quad T_p]\end{aligned} \tag{19}$$

In order to effectively avoid the obstacle, planning the swinging leg bottom in the middle of the swing that is $t=\frac{T}{2}$ to reach the high point $H=0.01m$, The ankle height $h_{foot}=0.018m$, the final swing leg movement Z-direction:

$$\begin{aligned}z(t)&=0.018+144*t^3-2160*t^4\\ &+10800*t^5-18000*t^6 \qquad t\in[0 \quad T_p]\end{aligned} \tag{20}$$

6. Conclusion

This paper analyzes the robot's motion architecture and determines the relationship between the various joints. Then, the robot's motion is reduced to a three-dimensional linear inverted pendulum model, according to the initial test conditions and the relationship between CoM and ZMP, the trajectory of CoM is obtained. In the forward gait and lateral gait, According to the ZMP equation, the stability of the gait planning is verified, the effectiveness of the method is verified by simulation platform. To further improve the speed and stability of walking requires more in-depth study on planning methods and control methods.

7. Acknowledgement

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