

Analysis of Power System Low Frequency Oscillation Based on Energy Shift Theory

Junfeng Zhang¹, Chunwang Zhang^{2,*} and Daqing Ma³

¹Training Center of Jilin Electric Power Company

²Control Center of State Grid Changchun Power Supply Company

³Jilin Electric Power Company

*Corresponding author

Abstract. In this paper, a new method for analyzing low-frequency oscillation between analytic areas based on energy coefficient is proposed. The concept of energy coefficient is proposed by constructing the energy function, and the low-frequency oscillation is analyzed according to the energy coefficient under the current operating conditions; meanwhile, the concept of modal energy is proposed to analyze the energy exchange behavior between two generators. Not only does this method provide an explanation of low-frequency oscillation from the energy point of view, but also it helps further reveal the dynamic behavior of complex power systems. The case analysis of four-machine two-area and the power system of Jilin Power Grid proves the correctness and effectiveness of the proposed method in low-frequency oscillation analysis of power system.

Keywords: Low-frequency oscillation; Energy function; Modal analysis

1. Introduction

With the development of large interconnected power systems, inter-area oscillations are becoming an increasing concern. This kind of dynamic behavior is usually caused by minor disturbances, sometimes even without obvious reasons but mainly due to the weak inter-device damping [1-3]. Inter-area oscillations can be analyzed by the relative acceleration of generator rotor [4] caused by periodic energy exchange. The eigenvalue analysis method obtains the oscillation mode by linearizing the state equation of the system and effectively calculates the quantitative information of the oscillation mode so that it can be used to determine the optimal installation location and the parameter design of the controller [5]. Other methods, such as spectrum analysis based on power system signals and the Prony method for analyzing low-frequency oscillation [6], can identify these oscillatory forms from a time-domain perspective but cannot represent the exchange mode of energy in the network. As power systems become more complex, new forms of inter-area oscillations begins to appear, possibly combining two or more inter-area patterns, which requires further analysis of the system dynamics [7]. In recent years, several other analytical techniques have also been introduced into the power system, including linear and nonlinear methods [8 - 10]. In addition, there are also energy-based methods [9]. All of these methods have achieved some success in the analysis of low-frequency oscillation of power system.

In this paper, a method using energy shift to analyze the low-frequency inter-areal oscillation is proposed, based on which, the kinetic energy function and modal energy are introduced, and the energy relationship between the kinetic energy shift characteristics of key generators and related modes under small interference is also studied. This method can be equivalent to a physical kinetic model and can be



used with other techniques such as Prony and Fourier analysis [19]. The four-machine two-area examples in the references [20] and the actual calculation examples of Jilin Power Grid are used. The research results include the analysis of the generator units involved in the key mode and the analysis of the energy exchange affecting the system controller. Compared with eigenvalue analysis, the results show that the method is correct and feasible.

2. Kinetic Energy Function

2.1. State Space

The system can be expressed as follows in the form of a linear state equation under small disturbance behavior:

$$\dot{x} = Ax + Bu \quad (1)$$

where A and B are real constant matrices; $x^{(i)}$ is an n-dimensional state variable; A is $n \times n$ an order state matrix; B is $n \times r$ is an order control or input matrix; u is a feed forward matrix.

Under the stable framework with small disturbance, the general solution of rotational equation of the system is [11]

$$x(t) = \sum_{j=1}^n (v_j^T x^0) w_j e^{\lambda_j t} \quad (2)$$

where λ_j ($j = 1, \dots, N$) is the eigenvalue, w_j, v_j are left and right eigenvectors, respectively. By normalizing the eigenvectors, when $\lambda_i \neq \lambda_j, (w_i, v_j) = 0$; when $\lambda_i = \lambda_j, (w_i, v_j) = 1$.

Kinetic Energy Function

The kinetic energy (KE) of a system consisting of a total of ng generators is defined as follows [12]:

$$KE = \frac{1}{2} \sum_{k=1}^{ng} M_k \omega_k^2 = \frac{1}{2} x_{\omega}^T M x_{\omega} \quad (3)$$

where $M = \text{diag}[M_1, \dots, M_{ng}]$ is the time inertia constant, x_k is the kth machine's relative speed.

By deriving the small disturbance energy function at the equilibrium point:

$$KE = KE^0 + \Delta KE = \frac{1}{2} \sum_{k=1}^{ng} M_k (\omega_k^0 + \Delta \omega_k)^2 = \frac{1}{2} x_{\omega}^T M x_{\omega} \quad (4)$$

where in the SRF $\omega_k^0 = 0 \quad k = 1, \dots, ng$

2.2. Energy Coefficient

2.2.1. Modal Energy. The kth machine's relative speed is obtained by equation (3):

$$x_{\omega k} = \Delta \omega_k(t) = \sum_{j=1}^n (v_j^T x^0) w_{kj} e^{\lambda_j t} \quad k = 1, \dots, ng \quad (5)$$

Equation (5) shows that the speed shift of the kth machine can be expressed as a linear combination of n modes.

Therefore, the kinetic energy shift of the kth machine can be expressed as follows:

$$\begin{aligned} \Delta KE = & \frac{1}{2} M_k \left\{ \left[\sum_{j=1}^n w_{kj} v_j^T x^0 e^{\lambda_j t} \right]^2 + \right. \\ & \left. \sum_{p=1}^n \sum_{q=p+1}^{n-1} (w_{kp} v_p^T x^0) (w_{kq} v_q^T x^0) e^{(\lambda_p + \lambda_q)t} \right\} \end{aligned} \quad (6)$$

Where

$$P_{kj}(\lambda_j, t) = [w_{kj} v_j^T x^0]^2 e^{2\lambda_j t} \quad (7)$$

$$P_{kpq} = (w_{kp} v_p^T x^0)(w_{kq} v_q^T x^0) e^{(\lambda_p + \lambda_q)t} \quad (8)$$

where P_{kj} is the kinetic energy coefficient, similar to the participation factor [11, 13], indicating the correspondence or participation size of the kth machine and the jth mode in the time-domain response. P_{kpq} is defined as the kinetic energy model coefficient that indicates the correspondence or participation size of the energy shift formed by the p and q modes in the time-domain response of the kth machine. The kinetic energy shift for one of the machines in equation (6) consists of two parts: one is the modal energy of a single mode and the other is the modal energy of two related modes.

2.3. Modal Energy

The state vector can be linearized with eigenvalues:

$$x = \sum_{j=1}^n (w_j^T x) v_j = \sum_j (v_j^T x) w_j \quad (10)$$

Therefore, the kinetic energy function (KE) can be expressed as [15]:

$$\Delta KE = \frac{1}{2} \left\{ \left[\sum_{i=1}^n (w_i^T x) v_i \right]^T M \left[\sum_{j=1}^n (v_j^T x) w_j \right] \right\} = \sum_{j=1}^n E_j \quad (11)$$

where E_j is the modal energy of the jth mode, expressed by orthogonal eigenvalues:

$$E_j = \frac{1}{2} x^T w_j v_j^T x = \frac{1}{2} \alpha_j e^{\lambda_j t} \left[\sum_{r=1}^n w_{jr} x_{or} \right] \quad (12)$$

where $\alpha_j = v_j^T x^0$. So substituting equation (5) into equation (12) gives:

$$E_j = \left\{ \sum_{m=1}^n [v_j x^0 w_{jm} e^{\lambda_j t}]^2 + \sum_{p=1}^{n-1} \sum_{q=p+1}^n (v_p^T x^0 w_{jp})(v_q^T x^0 w_{jq}) e^{(\lambda_p + \lambda_q)t} \right\} \quad (13)$$

Equation (13) represents the modal energy in the jth mode. Modal energy can obtain the distribution of energy in the network, and provide effective information for the installation of the controller.

3. Steps to Calculate the Energy Exchange Process

(1) The steps to calculate the energy exchange mode and determine the control strategy of the controller through the related information are as follows:

(2) Form the state matrix A in equation (1) and linearize it at the equilibrium point of the small disturbance operating condition to calculate the eigenvalues and eigenvectors.

(3) For each machine, calculate their KE coefficients by equations (8) and (9).

(4) Calculate the amplitude and phase angle of the kinetic coefficients and kinetic mode coefficients to determine the oscillating fleet.

(5) Sort the machine kinetic energy coefficient according to the size, and analyze the coefficient that takes larger energy participation in the same mode.

(6) For the key modes with obvious energy oscillations, the controller can be designed with relevant information.

The above approach provides a new way to calculate the energy exchange in the network, based on which the relevant controller can be designed.

4. Example Analysis

4.1. Four-machine Two-area

In order to verify the effectiveness of the proposed method in multi-machine system research, a simulation experiment is carried out on a four-machine two-area system, as shown in Fig.1. Sixth-order model generator is used, while a third-order excitation system is added; each node is a constant impedance load. The system's parameters and generator parameters are detailed in the references [20].

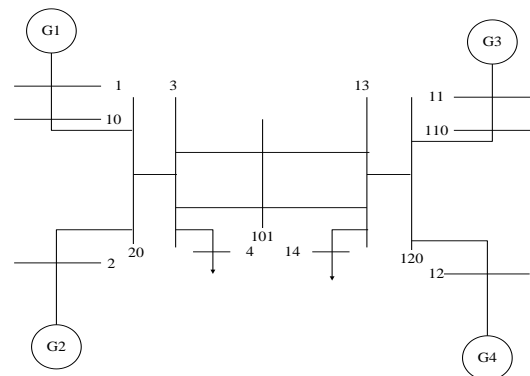


Figure 1.four machine two area study system wiring diagram

For the 4-machine system, three electromechanical modes are derived from eigenvalue analysis and energy variation. The results of eigenvalue analysis are given in Table 1, which are respectively the inter-area oscillation mode: Mode 1 (0.61HZ), inter-area mode: Mode 2 (1.32HZ), and Mode 3 (1.37HZ). Table 1 also shows the degree of participation of the generators. Table 2, Table 3, and Table 4 give the relevant results of the energy variation method. Table 2 shows each eigenvalue energy mode of each generator, where it is clear to see the participation size of the generator in the oscillation. The participation of 0.0066, 0.0052, 0.0063, and 0.0081 is one order of magnitude smaller than the others, so weak participation is shown in the oscillation; Table 3 gives the phase angle of the energy mode of the corresponding generator of each mode. The relationship of oscillating generators can be clearly seen. Mode 1 is the oscillation of G1 and G2 relative to G3 and G4; Mode 2 is the oscillation of G1 relative to G2; Mode 3 is the oscillation of G3 relative to G4. Through the above analysis of kinetic energy coefficient, the oscillation form of generators can be derived, which is basically consistent the result of eigenvalue method. Table 4 mainly shows the data related to the modal energy in order to evaluate the relationship of the generators involved in the composite oscillation from the energy point of view. It can be seen from Table 4 that 0.074 is the maximum modal energy, which means that the oscillation energy of G1 in Mode 1 and Mode 2 is relatively large. In other words, G1 plays a dominant role in Mode 1 and Mode 2.

The energy analysis shows that the dominant form of energy mode exists in the inter-area contact line, which is Mode 1. Other energy modes include two inter-area modes, which can reflect the key information of the system and can provide energy reference for further installation of PSS.

Table 1. eigenvalue analysis results

Mode	Eigenvalue	Participation generator (participation factor)	Frequency	Damping
I	-0.0052 + 0.0385i	G1 (0.1246)	0.61	0.14
		G2 (0.1801)		
		G3 (0.1576)		
		G4 (0.1268)		
II	-0.0325 + 0.0828i	G1 (0.1881)	1.32	0.37
		G2 (0.1702)		
		G3 (0.0794)		
		G4 (0.0905)		
III	-0.0325 + 0.0860i	G1 (0.0812)	1.37	0.35
		G2 (0.0731)		
		G3 (0.1756)		
		G4 (0.2357)		

Table 2. kinetic energy coefficient analysis

Generator	Mode	Amplitude of kinetic coefficient
G1	20,22,24	0.0155, 0.0545, 0.0066
G2	20,22,24	0.0324,0.0548,0.0052
G3	20,22,24	0.0269,0.0063,0.0385
G4	20,22,24	0.0163,0.0081,0.0763

Table 3. energy coefficient analysis

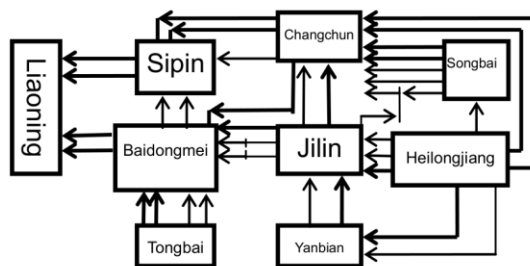
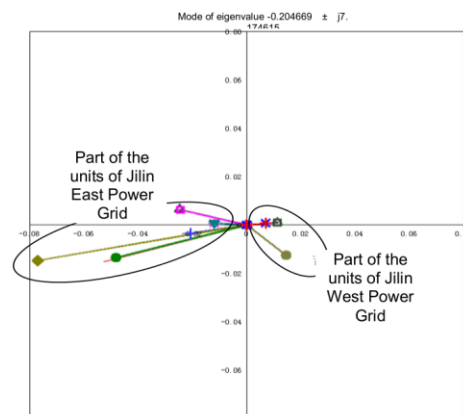
Mode	Frequency	Phase angle of energy coefficient
Mode 1	0.61	G1 (-0.0848), G2 (-0.0237) to G3 (1.5629), G4 (1.5351)
Mode 2	1.32	G1 (-0.629) to G2 (1.5461)
Mode 3	1.37	G3 (-0.827) to G4 (1.6964)

Table 4. energy calculation result of the amplitude

Generator	Energy mode	Amplitude of modal energy
G1	1,3	0.074
	1,4	0.0395
	2,3	0.0291
	2,4	0.0187
G2	1,2	0.0325
G3	3,4	0.0535

4.2. Jilin Power Grid Example

Jilin Power Grid is located in the central part of the Northeast Power Grid, connected with Liaoning Power Grid in the south, Heilongjiang Power Grid in the north, and Tongliao Power Grid of Inner Mongolia in the west. By the end of 2008, Jilin Power Grid had been interconnected with Liaoning Power Grid via 500kV Feng-Xu Line 1, Feng-Xu Line 2, and Sha-Li Line B, and 220kV Qing-Fu Line, Chang-Ju Line, Mei-Zhong Line 1, Zhong-Bai Line and Yun-Wo Line (220kV Wei-Wo Line is the only one located in Jilin Province but not connected with Jilin Power Grid; instead it is directly connected with Liaoning Power Grid), with Tongliao Power Grid of Inner Mongolia via Triple-circuit 220kV Bao-Ling Line, Tong-Li Line, and Dian-Shuang Line, and with Heilongjiang Power Grid via 500kV He-Nan Line 1, He-Nan Line 2, Yong-Bao Line, and Lin-Ping Line and 220kV Chang-Xin Line, Song-Wu Line, Jing-Ping Line, and Yu-Shuang Line.

**Figure 2****Figure 2** Jilin Power Grid system wiring diagram**Figure 3****Figure 3.** Oscillation modes between part of the units of Jilin East and West Power Grids**Table 5.** Eigenvalue calculation results

Frequency (Hz)	Attenuation damping ratio (%)	Electrical circuits correlation ratio
1.141875	2.8515	11.6236

Table 6. Modal energy calculation results

Mode	1	2	3	4	5	6
EJ	0.506976	0.25452	0.253434	0.266849	0.245892	0.278605

As can be clearly seen in Figure 3, there is a clear oscillatory behavior of Jilin West Power Grid relative to Jilin East Power Grid, with a frequency of 1.14, belonging to the low-frequency oscillation range.

It can be seen from Table 6 that the EJ energy of Mode 1 is the largest, which is the dominant mode of the East and West Grids under this operating condition, and Modes 2-6 are the local modes in this operating condition.

From the above results, it can be seen that there are obvious oscillating fleet in Jilin Power Grid, and oscillation behaviors exist between and the East and West Power Grids. Based on this result, PSS can be configured on the relevant units to increase the damping capabilities of the system for prevention

5. Conclusions

In this paper, a low-frequency oscillation analysis method based on the analysis of small-disturbance energy modes is proposed, where the key generators and tie lines in low-frequency oscillation are determined by the indexes of kinetic energy shift, and a way to understand the low-frequency oscillation phenomenon from the perspective of energy exchange is provided. In addition, KE analysis provides related information of each mode through the modal energy, which provides a reference for further optimization of PSS installation.

6. References

- [1] IEEE Power Engineering Society, Inter-area oscillations in power systems, IEEE Publication 95 TP 101, IEEE, 1995.
- [2] Liu Qu, Zhou Shuangxi, Feng Zhihong. Analysis and synthesis of power system stability. Proceedings of the CSEE, 1986, 6 (5): 24 ~ 30 .
- [3] A. Olwegard, Improvement of system stability in interconnected power systems, CIGRE Conf., (1982) Paper 32-17.
- [4] R.L. Cresap, J.F. Hauer, Emergency of a new swing mode in the western power system, IEEE Trans. Power Syst. PAS-100 (1981) 3017–3024.
- [5] Li Fang. Research on small-disturbance stable distributed parallel algorithm of power system [Doctoral Dissertation]. Beijing: China Electric Power Research Institute, 2001
- [6] Xiao Jinyu, Xie Xiaorong, Hu Zhixiang et al. Improved Prony algorithm for online identification of low frequency oscillations in power systems. Journal of Tsinghua University, 2004, 44 (7):
- [7] J.F. Hauer, C.J. Demeure, L.L. Scharf, Initial results in Prony analysis of power system response signals, IEEE Transact. Power Syst., (1990) 80-89.
- [8] C. Jing, J.D. McCalley, M. Kommareddy, An energy approach to analysis of interarea oscillations in power systems, IEEE Transact. Power Syst. 11 (2) (1996) 734–740.
- [9] E.Z. Zhou, Power oscillation flow study of electric power systems, Elect. Power Energy Syst 17 (2) (1995) 143–150.
- [10] V. Vittal, N. Bhatia, A.A. Fouad, Analysis of the inter-area phenomenon in power systems following large disturbances, IEEE/PES 1991, Winter Meeting, New York, Feb. (1991) paper WM 228-7 PWRs.
- [11] I.J. Pe'rez Arriaga, Selective modal analysis with applications to electric power systems, IEEE Transact. 101 (1982) 3117–3134.
- [12] T.R. Kane, D.A. Levinson, Dynamics: Theory and Applications, McGraw-Hill, New York, 1985.
- [13] A.M.A. Hamdam, Coupling measures between modes and state variables in power system dynamics, Int. J. Cont. 43 (2) (1986) 1029–1041.

- [14] S.K. Starret, A.A. Fouad, Nonlinear measures of mode-machine participation, IEEE Transact. Power Syst. 5 (1998) 389–394.
- [15] A.G.J. MacFarlane, Use of power energy concepts in the analysis of multivariable feedback controllers, Proc. IEEE 116 (8) (1969) 1449–1452.
- [16] Ni Yinxin, Chen Shousun, Zhang Baolin. Theory and analysis of dynamic power systems [M]. Beijing: Tsinghua University Press, 2002.