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Two-Dimensional Adhesive Contact Problem for Graded Coating Based on a New Multi-Layer Model

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Abstract. In this paper, the contact problem with adhesion for graded coating indented by a cylindrical indenter is considered. A multi-layered model is developed to model functionally graded material (FGM) coating with arbitrarily varying elastic modulus. By using the transfer matrix method and Fourier integral transform technique, the problem is reduced to a Cauchy singular integral equation. The numerical calculation is conducted to achieve the effect of the gradient index of the FGM coating and the adhesive parameter on the pull-off force and the relation between the normal load and the maximum indentation depth. The results show that the gradient index of coating can effectively change the behavior of adhesive contact.

1. Introduction

At the micro and nano scale, adhesion has an impact effect on the contact behavior of the coating surface. In order to study the microscopic phenomena of adhesion, contact, deformation and friction in the atomic and molecular scale of the solid surface, several adhesive theories have been developed. Johnson et al. [1] proposed the JKR adhesive contact theory by using the relationship between elastic energy and surface energy. The DMT model which considers the adhesion outside the contact region was suggested by Derjaguin et al. [2]. Maugis [3] based on the Dugdale model established a MD theory that model the transition between the JKR and DMT regimes. When the functionally graded material (FGM) as a coating or transition layer, it can reduce the stress concentration due to the material mismatch [4]. Chen et al. [5, 6] solved the adhesive contact problem of the FGM coating in which elastic modulus vary as power law function. They obtained a series of closed analytical solutions of contact problem with adhesion. Guo and its collaborators [7] investigated an axisymmetric frictionless adhesive contact problem of the FGM layer with power law gradient. They obtained a series of closed analytical solutions for double Hertz contact problem by using the JKR adhesive contact model [8]. Because the controlling equation which describes the mechanical properties of FGM coating is complex, the research on the adhesive contact problem of FGM coating still has difficulties and further study is needed.

Motivated by the previous research works on contact problems with adhesion, the mechanical behavior of the two-dimensional adhesive contact problem for graded coating with arbitrarily varying modulus is investigated in this paper. The FGM coating which is bonded to half-space is indented by a cylindrical indenter. Based on the fact that an arbitrary curve can be approached by a series of continuous but piecewise curves, the FGM is divided into several sub-layers and in each sub-layers the shear modulus is assumed to be exponential function. The gradient index of the FGM coating and the adhesive parameter on the pull-off force and the relation between the normal load and the maximum indentation depth are analyzed.

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2. Solution Of Two-Dimensional Adhesive Contact Problem For Functionally Graded Materials

Consider the contact problem for FGM coating indented by a cylindrical punch as shown in Figure 1a.

Functionally grade coating is perfectly bonded to the half-space. A functionally graded material coating of thickness h is divided into N sub-layers. The shear modulus varies exponentially in each sub-layers and is continuous at the sub-interfaces, i.e.,

$$\mu(y) \approx \mu_j(y) = \bar{\mu}_{j-1} e^{\gamma_j (y+(j-1)\bar{h})}, -(j-1)\bar{h} > y > -j\bar{h}, j = 1,2,...N \tag{1}$$

where $\bar{\mu}_{j-1}$ equal to the real value of the shear modulus at the sub-interfaces, $\bar{h} = h/N$ is thickness of the sub-layer. In the present paper, we assume that the Poison's ratios for both coating and half-space are a constant with the same value. The stresses and displacements are continuous at the sub-interfaces, which states

$$\sigma_{xyj} = \sigma_{xyj+1}, \sigma_{yyj} = \sigma_{yyj+1}, u_{xj} = u_{xj+1}, u_{yj} = u_{yj+1} \tag{2}$$

Using a Maugis type of adhesion model, the surface traction (Fig. 1b) is expressed as

$$\sigma = \sigma_1(x) - \sigma_0, \quad (x \le c) \qquad \sigma(x) = 0, \quad (a < x < c) \tag{3}$$

where $\sigma_1(x)$ represents the contact stress caused by the rigid indenter and the adhesive tensile stress σ_0 is a constant.

According to Ref. [9], the normal surface displacement can be expressed as

$$u_{y0}(x) = -\frac{\alpha_1}{\pi}\int_{-c}^{c} \ln|x-t|\sigma(t)dt + \frac{1}{\pi}\int_{-c}^{c}\sigma(t)I(x,t)dt \tag{4}$$

where,

$$I(x,t) = \int_0^\infty \left[m_{22}(s) - \frac{\alpha_1}{s} \right] \cos[s(x-t)]ds. \tag{5}$$

and $m_{22}(s)$ is a variable associated with materials parameter of functionally graded coating while $\alpha_1 = (v-1)/\mu_0$.

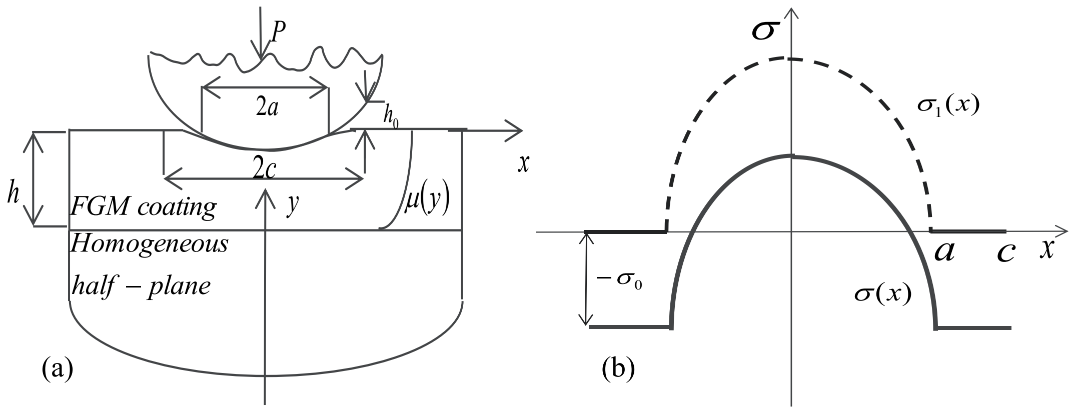

Figure 1. Functionally graded coated half space subjected to normal and tangential line concentrated force (a) and the linear multi-layered model for the functionally graded coating (b)

Substitution of (3) into equation (4), we can obtain

$$u_{y0}(x) = -\frac{\alpha_1}{\pi}\int_{-a}^{a} \ln|x-t|\sigma_1(t)dt + \frac{1}{\pi}\int_{-a}^{a}\sigma_1(t)I(x,t)dt - \frac{\alpha_1}{\pi}\int_{-c}^{c} \ln|x-t|(-\sigma_0)dt + \frac{1}{\pi}\int_{-c}^{c}(-\sigma_0)I(x,t)dt, \ -c \le x \le c, \tag{6}$$

The stamp profile for cylindrical punch is given by

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$$u_{y0} = -\delta_0 + x^2/2r\,, \tag{7}$$

where δ_0 is the maximum indentation and occurs at the centre of contact region.
The derivation of equation (6) with respect to x yields

$$\frac{\alpha_1}{\pi}\int_{-a}^{a}\frac{\sigma_1(t)}{t-x}dt + \frac{1}{\pi}\int_{-a}^{a}\sigma_1(t)Q(x,t)dt - \frac{\alpha_1}{\pi}\int_{-c}^{c}\frac{\sigma_0}{t-x}dt - \frac{1}{\pi}\int_{-c}^{c}\sigma_0 Q(x,t)dt = \frac{x}{R}, -a \le x \le a \tag{8}$$

where

$$Q(x,t) = \partial I(x,t)/\partial x = -\int_{0}^{\infty}\left[sm_{22}(s) - \alpha_1\right]\sin\left[s(x-t)\right]ds \tag{9}$$

In order to solve the equation (8), the geometric relation for the gap between the point $x = c$ and the $x = a$ should satisfy the following expression

$$\begin{aligned} u_{y0}(c) - u_{y0}(a) = -\frac{\alpha_1}{\pi}\int_{-a}^{a}\ln\left|\frac{c-t}{a-t}\right|\sigma_1(t)dt + \frac{1}{\pi}\int_{-a}^{a}\sigma_1(t)\left[I(c,t) - I(a,t)\right]dt \\ -\frac{\alpha_1}{\pi}\int_{-c}^{c}\ln\left|\frac{c-t}{a-t}\right|(-\sigma_0)dt + \frac{1}{\pi}\int_{-c}^{c}(-\sigma_0)\left[I(c,t) - I(a,t)\right]dt = \frac{c^2 - a^2}{2R} - h_0 \end{aligned} \tag{10}$$

where the work of adhesion for Maugis Model is $w = \sigma_0 h_0$. The adhesion region c can be determined by equation (10) after the contact region is given.

The applied force P is calculated according to the contact stress $\sigma(x)$,

$$p = \int \sigma(x)dx = \int_{-a}^{a}\sigma_1(x)dx - \int_{-c}^{c}\sigma_0 dx. \tag{11}$$

The maximum indentation is the difference between the normal displacement at the point $x = 0$ and the point $x = c$ and expressing the result, i.e.

$$\delta_0 = u_{y0}(c) - u_{y0}(0) = \frac{c^2}{2R} - h_0 \tag{12}$$

Making the following normalizations

$$x = a\zeta, t = a\eta(-a \le t \le a), -1 \le (\eta,\zeta) \le 1,\ c = ma \tag{13}$$

$$\lambda = \frac{2\sigma_0}{\left(\pi wK^2/R\right)^{1/3}},\quad f = \frac{2\sigma_1}{\left(\pi wK^2/R\right)^{1/3}},\quad A = \frac{a}{\left(\pi wR^2/K\right)^{1/3}},\quad H = \frac{h}{\left(\pi wR^2/K\right)^{1/3}},\quad 2hs = z \tag{14}$$

where $K = \frac{4}{3}\frac{E}{1-v^2} = \frac{8}{3}\frac{\mu_1}{1-v} = -\frac{8}{3\alpha_1}$, the eqs. (8) and (10) can be written as

$$\begin{aligned} \frac{1}{\pi}\int_{-1}^{1}\frac{-f(\eta)}{\eta-\zeta}d\eta + \frac{1}{\pi}\frac{A}{H}\int_{-1}^{1}(-f(\eta))Q_1(\zeta,\eta)d\eta + \frac{\lambda}{\pi}\ln\left(\frac{m-\zeta}{m+\zeta}\right) + \\ \frac{-\lambda}{\pi\alpha_1}\int_{0}^{\infty}\left[\frac{z}{2h}m_{22}\left(\frac{z}{2h}\right) - \alpha_1\right]\left\{\frac{2\sin(zA\zeta/(2H))\sin(zAm/(2H))}{z}\right\}dz = \frac{3}{4}A\zeta \end{aligned}, \tag{15}$$

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$$\begin{aligned}&-\int_{-1}^{1}\ln\left|\frac{m-\eta}{1-\eta}\right|f(\eta)d\eta+\frac{1}{\alpha_1}\int_{-1}^{1}f(\eta)\left[I_1(m,\eta)-I_2(1,\eta)\right]d\eta+\lambda m\int_{-1}^{1}\ln\left|\frac{m-mt}{1-mt}\right|dt\\&-\frac{2H\lambda}{A\alpha_1}\int_0^\infty\left[\frac{z}{2h}m_{22}(\frac{z}{2h})-\alpha_1\right]\left[\frac{\sin(zAm/H)-2\cos(zA/(2H))\sin(zAm/(2H))}{z^2}\right]dz=\frac{-3\pi A}{8}(m^2-1)+\frac{3}{2\lambda A}\end{aligned} \tag{16}$$

The Erdogan-Gupta method [10] will be used to convert Eq. (15) to a system of linear algebraic equations. After the dimensionless contact stress $f(x)$ is obtained by solving the system of linear algebraic equations, the size ratio of the adhesion/contact region m is determined by satisfy Eq. (16).

3. Numerical Results and Discussions

In this Section, it is assumed that the shear modulus of coating varies in the power manner

$$\mu(y)=\mu_0+(\mu_h-\mu_0)(y/h)^n \tag{17}$$

where n is a positive constant characterizing the gradual variation of the shear modulus. Throughout the paper, the Poisson's ratios for both coating and half space are 0.33.

Firstly, the minimum dimensionless contact pressures for different values of N (numbers of sub-layer) are shown in Table 1. The parameters $\mu_0/\mu_h=0.25$, $A=1.45$, $n=0.5$, $\lambda=1$ are chosen for calculation. In Table 1, the results with $N=12$ or $N=14$ may be considered sufficiently accurate. So we choose $N=12$, that is, divide the coating into twelve sub-layers.

Table 1. Effect of N on the minimum dimensionless contact pressure with $\mu_0/\mu_h=0.25$, $A=1.45$ $n=0.5$

N	$\lambda=1$	$\lambda=5$
6	2.257	1.646
8	2.316	1.702
10	2.353	1.738
12	2.373	1.760
14	2.395	1.783

Fig. 2 shows the relation between the applied load P vs. A (a) and the relation between the applied load P vs. δ (b) for some selected values of n with $\lambda=5$ and $\mu_0/\mu_h=0.25$. It can be seen from Fig. 2a, the required pull-off force increases as the gradient index n decreases when the contact radius is small. While the contact radius is large, the normal load required to produce the same contact radius increases as the shear modulus gradient decreases. In Fig. 2b, when the maximum indentation depth is negative, it indicates that the work of adhesion causes the upward displacement on the surface of graded coating. When the applied force P is the tensile force and the indentation is positive, to produce the same indentation, the normal load increases with the decrease of gradient index n. While the indentation δ is negative, in order to create the same indentation, the normal load decrease with the decrease of gradient index n.

Fig. 3 shows the relation between the applied load P vs. A (a) and the relation between the applied load P vs. δ for some selected values of the adhesion parameter λ when the stiffness ratio $\mu_0/\mu_h=0.25$ and $n=0.5$. It can be seen from Fig. 3a that the JKR approximation becomes valid as $\lambda=5$ and the Hertz approximation is reached as $\lambda=0.1$ [11]. For a homogeneous half-space, the pull-off force for the JKR model is approximately equal to $1.802(\pi w^2KR)^{1/3}$ [12], but the pull-off force for compliant substrate ($\mu_0/\mu_h=0.25$)is greater than $1.802(\pi w^2KR)^{1/3}$. As adhesion parameter λ increases, the required pull-off force increases. When the contact radius is large, the normal load decreases with the adhesion parameter λ increasing in order to produce the same contact radius. It can be seen from Fig. 3b that the normal load increases with the increase of the maximum indentation depth δ when δ is positive. When the indentation depth δ is negative, the relation between the indentation and applied force is complicated. With the decreasing of adhesion parameter λ, the larger applied force is needed to gain the same indentation.

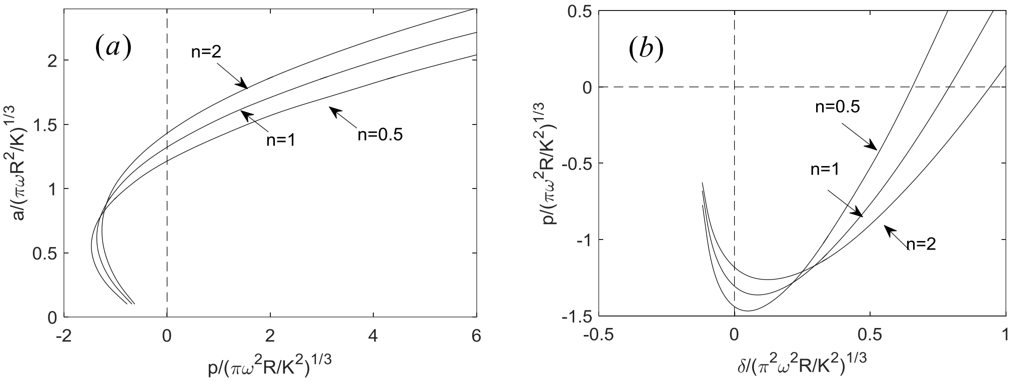
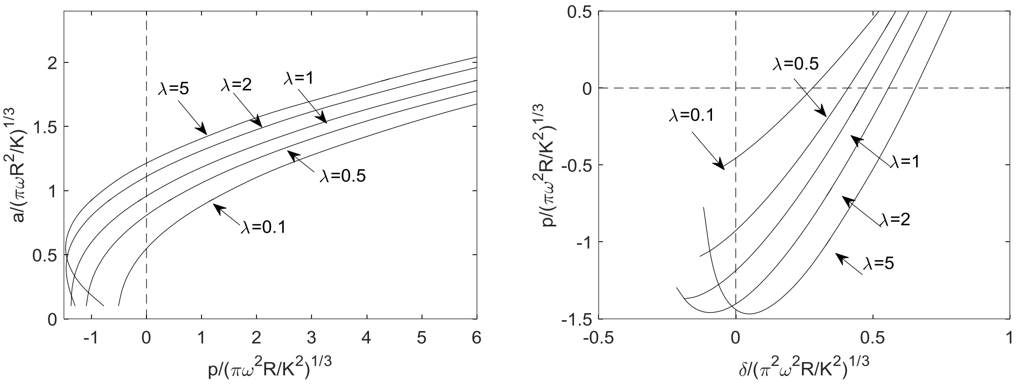


Figure 2. The effect of the shear modulus gradient n on the relations of P vs. A (a) and P vs. δ (b) with $\lambda = 5$, $\mu_0 / \mu_h = 0.25$.



(a) (b)

Figure 3. The effect of the adhesion parameter λ on the relations of P vs. A (a) and P vs. δ (b) with $n = 0.5$ and $\mu_0 / \mu_h = 0.25$.

4. Results

The present paper investigated the two-dimensional adhesive contact problem for FGM coating bonded to the homogeneous half-space under a cylindrical indenter. A new multi-layer model is applied to simulate the FGM coating with the shear modulus varying in the power manner. The problem is formulated by using the singular integral equation. The numerical results show that

- The adhesion parameter λ has a great impact on the relation of applied force P versus contact radius A and applied force P versus the maximum indentation δ. The smaller adhesion parameter corresponds to the DMT model as well as the larger adhesion parameter corresponds to the JKR model.
- The contact region and the maximum indentation are greatly affected by the gradient index n of FGM coating. With the decrease of the gradient index, the pull-off force increases. The results imply that the performance of coating can be changed by adjusting material gradient of coating.

5. Acknowledgments

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