Holographic Phase Transition in AdS Spacetime with Global Monopole

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Abstract. With motivation by holography, in this paper we attempt to survey whether holographic Van der Waals phase transition can be observed in the Anti-de Sitter spacetime with global monopole. We offer a possibility to proceed with a numerical calculation in order to discussion on phase transition. Furthermore, we verify numerically the Maxwell's equal area construction. In addition, the effect of global monopole on phase transition is also presented.

1. Introduction

Due to a black hole possessing thermodynamic properties, it is natural to ask whether it can undergo a Van der Waals phase transition in the same manner as a ordinary liquid-gas thermodynamic system. It was found that the charged AdS black hole in the entropy-temperature plane presented an analogous Van der Waals phase transition in the canonical ensemble[1]. Recently, in the P-V plane, the Van der Waals phase transition has been explored in various of AdS backgrounds[2-7]. Very recently, entanglement entropy and two point correlation function have been used to investigate Van der Waals phase transition[8-16]. All the results showed that there existed a Van de Waals-like phase transition in these gravity backgrounds.

In the framework of holography, it is interesting to detect the phase structure of a Reissner-Nordström black hole with global monopole in AdS background. In the present work, we attempt to study whether the Van der Waals-like phase transition can be observed, and discuss on global monopole's effect on phase transition.

2. Holography Van Der Waals Phase Transition

First, Let us review a black hole with global monopole in AdS background. In 2016, Ahmed A K, Camci U and Jamil M presented a Reissner-Nordström anti-de-Sitter black hole with global monopole charge. The metric reads

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \eta^2 + \frac{r^2}{L^2} \tag{2}$$

where M is mass parameter, Q stands for the black hole's charge, and $L=(-\Lambda/3)^{-1/2}$ is the AdS radius. The temperature of the black hole with global monopole is

$$T=\frac{3r_H^4+L^2\left(-Q^2+r_H^2\left(1+\eta^2\right)\right)}{4L^2\pi r_H^3} \tag{3}$$

where r_H is the event horizon which is determined by $f(r_H)=0$ (r_H is the largest root). With the relationship $F=M-TS$ and $S=\pi r_H^2$ yields

$$F=\frac{3Q^2+r_H^2\left(1-\frac{r_H^2}{L^2}+\eta^2\right)}{4r_H} \tag{4}$$

Now, we begin to explore the global monopole black hole's critical behavior and phase transition in the temperature--entropy plane. From Eq. (3) and the expression of entropy, we can obtain the function $T(S,Q,\eta)$ by eliminating r_H

$$T(S,Q,\eta)=\frac{-L^2\pi^2Q^2+L^2\pi S+3S^2+L^2\pi S\eta^2}{4L^2\pi^{3/2}S^{3/2}} \tag{5}$$

Note that the phase structure of a global monopole black hole is not only related to electric charge Q, but also the global monopole parameter η. Based on the function $T(S,Q,\eta)$ above, the phase structure of the global monopole black hole can be detected. To do so, we need to find out the critical value of phase transition according to the following relation

$$\left(\frac{\partial T}{\partial S}\right)_Q=\left(\frac{\partial^2 T}{\partial S^2}\right)_Q=0 \tag{6}$$

Connecting the Eqs. (5) and (6), we get the critical charge Q_{Cr}, critical entropy S_{Cr} and critical temperature T_{Cr}

$$S_{Cr}=\frac{1}{6}\pi(L^2+L^2\eta^2) \tag{7}$$

$$Q_{Cr}=\frac{1}{6}L\left(1+\eta^2\right) \tag{8}$$

$$T_{Cr}=\frac{3\sqrt{\frac{3}{2}}\left(-\frac{1}{36}L^4\pi^2\left(1+\eta^2\right)^2+\frac{1}{6}L^2\pi^2\left(L^2+L^2\eta^2\right)\left(1+\eta^2\right)+\frac{1}{12}\pi^2\left(L^2+L^2\eta^2\right)^2\right)}{L^2\pi^3\left(L^2+L^2\eta^2\right)^{3/2}} \tag{9}$$

The heat capacity is

$$C_Q=T\left(\frac{\partial S}{\partial T}\right)_Q \tag{10}$$

According to the expressions (3)-(5), we may plot the related transition curves as follows.

From Figure 1 and Figure 2, we know that the number of the global monopole black hole solution is related to the value of charged Q. Each curve corresponds to a different electric charge. Figs. 1 has

implication for phase transition. Here, in a temperature and entropy plane, we present the global monopole black hole's phase structure in a fixed charged ensemble. Evidently, a Van der Waals-like phase transition is clearly observed in the $T-S$ plane in Figure 2. In case of $Q>Q_{Cr}$, the temperature is monotonically larger with the increase of entropy, and corresponding system is thermodynamically stable since the heat capacity is positive. When Q reaches the critical value $Q=Q_{Cr}$, an inflection point arises and the heat capacity diverges, namely, there exists a second order phase transition at this point. when $Q<Q_{Cr}$, besides two stable branches with positive heat capacity, there is also an unstable branch with negative heat capacity, which corresponds to a first order phase transition. According to Maxwell's equal-area law, this unstable part need to be replaced with an isotherm $T=T^{SC}$. The subcritical temperature T^{SC} can be got from the plot of the free energy with respect to the temperature in Figure 3.

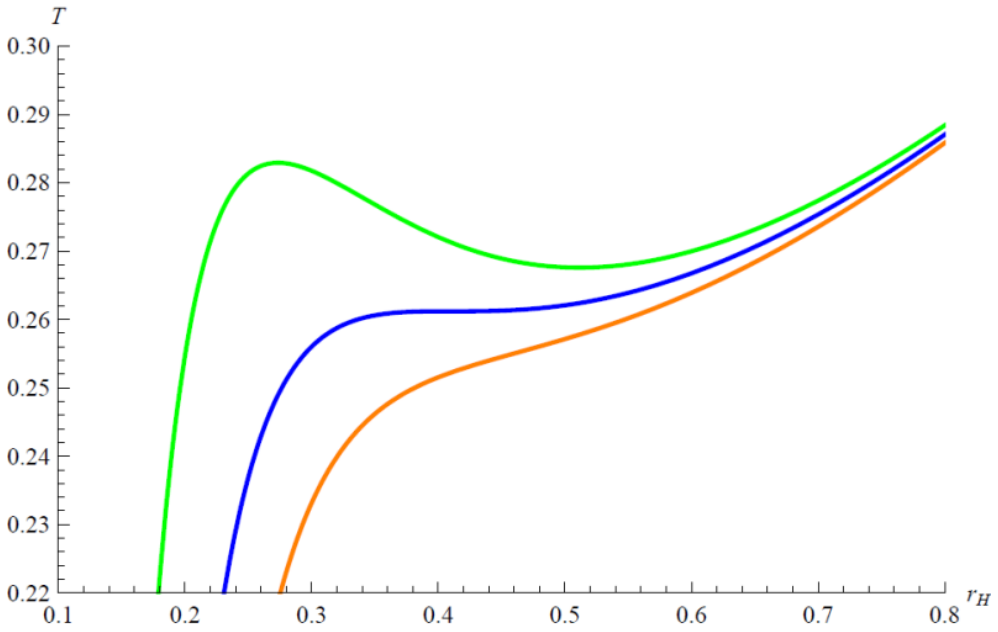

Figure 1. Plot of the temperature T versus the horizon r_H for $Q=0.14<Q_{Cr}$ (top), $Q=Q_{Cr}$ (intermediate), and $Q>Q_{Cr}$ (bottom). The parameters are taken as $L=1$ and $\eta=0.1$.

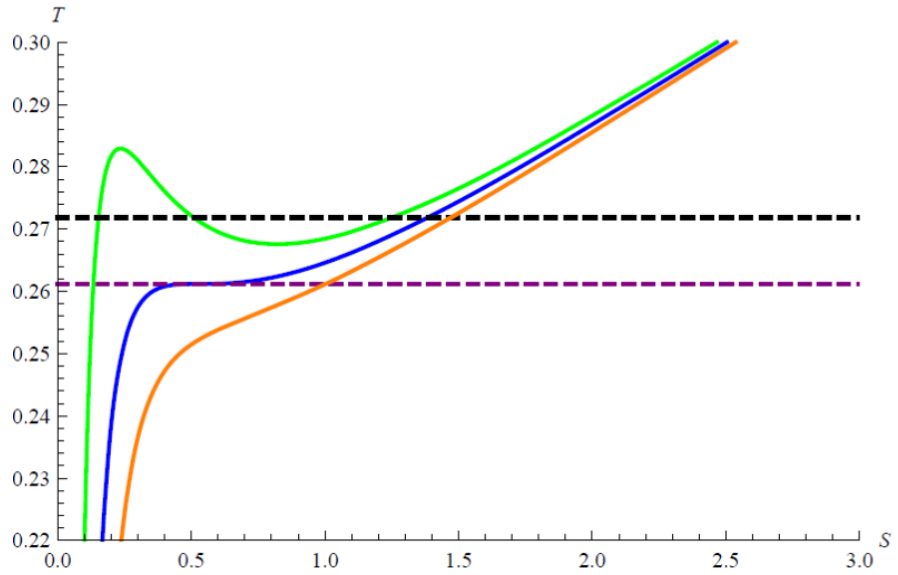

Figure 2. Plot of the temperature T versus the entropy S for $Q=0.14<Q_{Cr}$ (top), $Q=Q_{Cr}$ (intermediate), and $Q=0.16>Q_{Cr}$ (bottom). Above black dash line corresponds to the first order phase transition temperature $T^{SC}=0.2718$. Below purple dash line corresponds to the second order phase transition temperature $T_{Cr}=0.2612$ the parameters are taken as $L=1$ and $\eta=0.1$.

The plot in Figure 3 shows that the relationship between the temperature and free energy for different Q. In case of $Q<Q_{Cr}$, a classic swallowtail structure is always observed, which is responsible for the first order phase transition in Figure 2. We indicate the transition temperature T^{SC} by a red dashed line in Figure 3, which is the horizontal coordinate of the junction. In Figure 4, we find that an inflection point emerges, and it just corresponds to the inflection point of the second order phase transition as is presented in the middle curve in Fig 2. Furthermore, the longitudinal coordinate of an inflection point just coincides with the critical temperature T_{Cr} in Eq. (9).

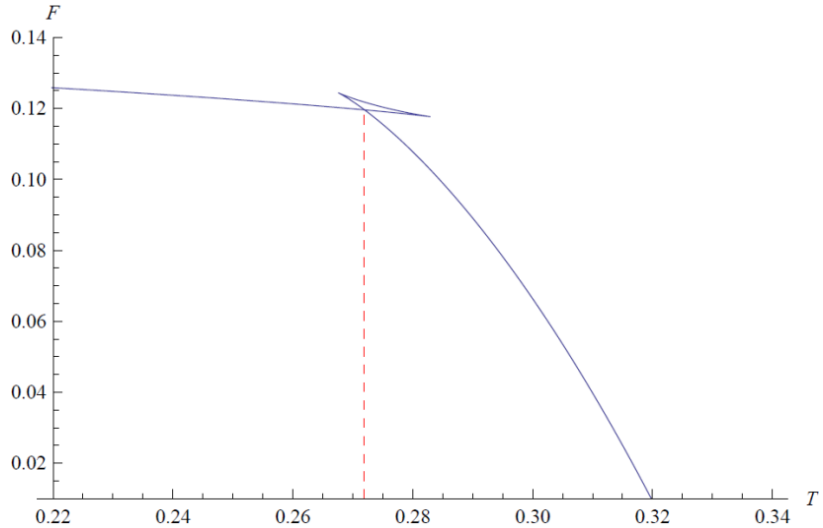

Figure 3. Plot of the temperature T versus the energy F for $Q=0.14<Q_{Cr}$. The red dash curve corresponds to the first order phase transition temperature $T^{SC}=0.2718$ The parameters are taken as $L=1$ and $\eta=0.1$.

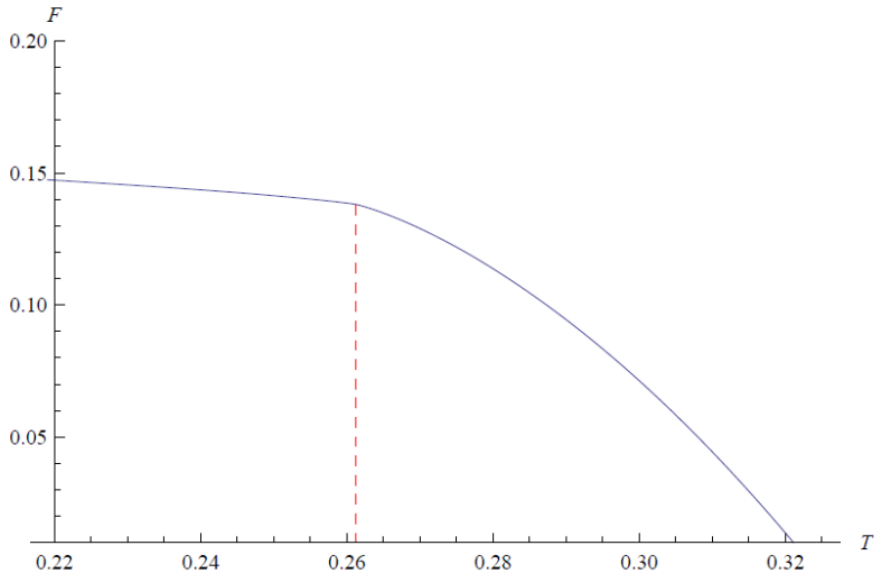

Figure 4. Plot of the temperature T versus the energy F for $Q = Q_{Cr}$. The red dash curve corresponds to the second order phase transition temperature $T_{Cr} = 0.2612$ The parameters are taken as $L = 1$ and $\eta = 0.1$.

Then, we discuss on how the global monopole η effects the phase transition. For different the global monopole parameter η, the related curves give a similar behavior not only in the $T - S$ plane but also in the $F - T$ plane. Here, we take $\eta = 0.3$ and $\eta = 0.1$ as for example, and compare their phase transitions in the $T - S$ plane. The related transition curves are plotted as follows

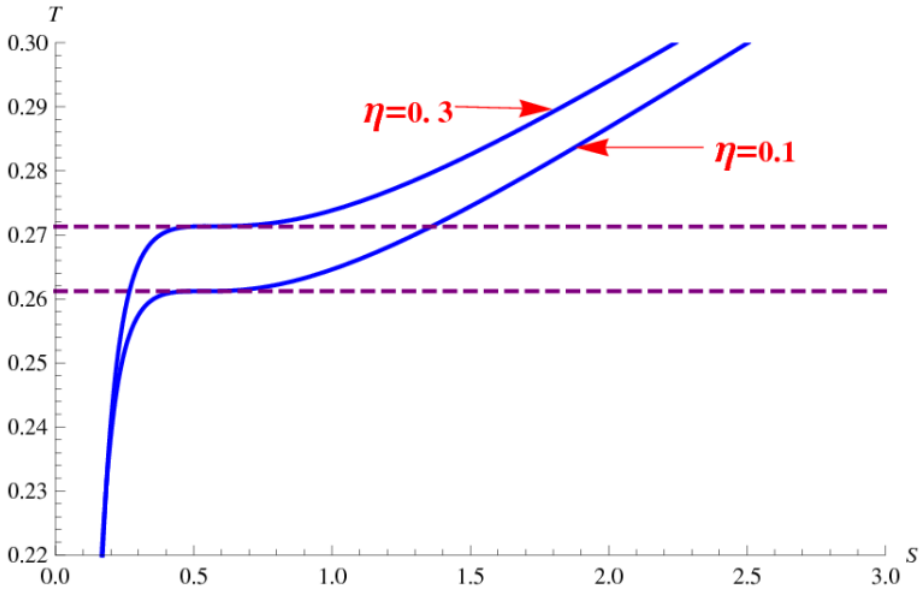

Figure 5. Plot of the temperature T versus the entropy S in case of $Q = Q_{Cr}$ for $\eta = 0.3$ (top) and $\eta = 0.1$ (bottom). The purple dash lines correspond to the second order phase transition temperature for different η. The parameters are taken as $L = 1$.

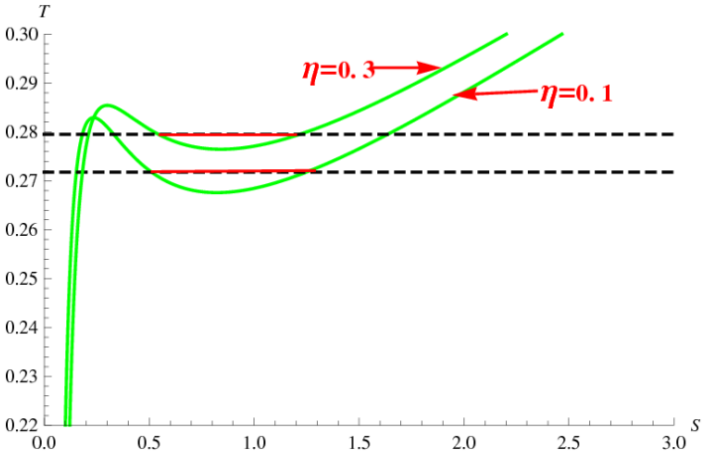

Figure 6. Plot of the temperature T versus the entropy S in case of $Q < Q_{Cr}$ for $\eta = 0.3$ (top) and $\eta = 0.1$ (bottom). The black dash lines correspond to the first order phase transition temperature for different η. The parameters are taken as $L = 1$.

It is obvious that we can observe the effect of global monopole η on the phase transition from Figure 5 and Figure 6. The critical temperature T_{Cr} increases with global monopole η increasing. Moreover, as global monopole η increases, the unstable scale become smaller.

Next, In order to further verify the Van der Waals like phase transition, we turn to check Maxwell equal area law for the first order phase transition and the corresponding statement can be written as

$$A \equiv \int_{S_i}^{S_a} T(S,Q,\eta)dS = T^{SC}(S_a - S_i) \equiv A' \tag{11}$$

where $T(S,Q,\eta)$ is defined in Eq. (5), S_i and S_a are the smallest and largest roots of the equation $T(S,Q,\eta) = T^{SC}$. Now, we take $\eta = 0.1$ as an example. We can get $A' = 0.2983$ at right side of Eq. (11), and obtain $A = 0.2984$ by integrating left side of Eq. (11). Namely, A equals to A' in our numeric accuracy. So the Maxwell's equal area construction holds in the $T - S$ plane.

For the global monopole black hole's second order phase transition, we confirm it by calculating the critical exponent of the heat capacity. Near the critical point, setting $S = S_{Cr} + \xi$ and expanding the Hawking temperature in small ξ, we can obtain

$$\begin{aligned}
T(S,Q,\eta) &= \frac{-L^2\pi^2Q^2 + L^2\pi S_{Cr} + 3S_{Cr}^2 + L^2\pi S_{Cr}\eta^2}{4L^2\pi^{3/2}S_{Cr}^{3/2}} \\
&+ \frac{\xi}{8L^2\pi^{3/2}S_{Cr}^{5/2}}\left[2S_{Cr}(L^2\pi + 6S_{Cr} + L^2\pi\eta^2) - 3\left(-L^2\pi^2Q^2 + L^2\pi S_{Cr} + 3S_{Cr}^2 + L^2\pi S_{Cr}\eta^2\right)\right] \\
&+ \frac{\xi^2}{32L^2\pi^{3/2}S_{Cr}^{7/2}}\left[24S_{Cr}^2 - 12S_{Cr}\left(L^2\pi + 6S_{Cr} + L^2\pi\eta^2\right) + 15\left(-L^2\pi^2Q^2 + L^2\pi S_{Cr} + 3S_{Cr}^2 + L^2\pi S_{Cr}\eta^2\right)\right] \\
&+ \frac{\xi^3}{192L^2\pi^{3/2}S_{Cr}^{9/2}}\left[-126S_{Cr}^2 + 90\left(L^2\pi + 6S_{Cr} + L^2\pi\eta^2\right) - 105\left(-L^2\pi^2Q^2 + L^2\pi S_{Cr} + 3S_{Cr}^2 + L^2\pi S_{Cr}\eta^2\right)\right]
\end{aligned} \tag{12}$$

Using Eqs. (9) and (12), we find

$$T-T_{Cr}=\frac{\left(S-S_{Cr}\right)^3}{192L^2\pi^{3/2}S_{Cr}^{9/2}}\Big[-126S_{Cr}^2+90\left(L^2\pi+6S_{Cr}+L^2\pi\eta^2\right)$$

$$-105\left(-L^2\pi^2Q^2+L^2\pi S_{Cr}+3S_{Cr}^2+L^2\pi S_{Cr}\eta^2\right)\Big] \tag{13}$$

Thus, from Eqs. (10) and (13), we have $C_Q \sim \left(T-T_{Cr}\right)^{-2/3}$, namely the critical exponent of the second order phase is −2/3, which is consistent with the mean field theory.

3. Conclusion

To conclude, in this paper, employing the relation between the temperature and entropy, we plot isocharges of the global monopole in a fixed charge ensemble, and discuss on the effect of the global monopole on Van der Waals phase transition in the $T-S$ plane. To further verify the phase transition, the equal area law is checked in this plane, and the critical exponent for the second order phase transition is also calculated. The result shows that, for the Reissner-Nordström anti-de-Sitter black hole with global monopole, the Van der Waals phase transition can also be presented in the $T-S$ plane.

4. References

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