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Analysis of portfolio optimization with lot of stocks amount constraint: case study index LQ45

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Abstract. To form an optimum portfolio (in the sense of minimizing risk and / or maximizing return), the commonly used model is the mean-variance model of Markowitz. However, there is no amount of lots of stocks constraint. And, retail investors in Indonesia cannot do short selling. So, in this study we will develop an existing model by adding an amount of lot of stocks and short-selling constraints to get the minimum risk of portfolio with and without any target return. We will analyse the stocks listed in the LQ45 index based on the stock market capitalization. To perform this analysis, we will use Solver that available in Microsoft Excel.

1. Introduction

Portfolio is defined as a collection of investments that can be composed of various types of assets, such as bonds and stocks. Portfolio optimization is a process of selecting the proportion of assets in a portfolio that make the portfolio better than others based on several criteria, such as: minimum risk and / or maximum return.

Currently, the LQ45 stock index that was launched in February 1997 [6] includes indicators of stocks in the capital market in Indonesia. LQ45 uses 45 preferred stocks with criteria determined by Indonesia Stock Exchange, including liquidity and market capitalization. According to " Buku Panduan Indeks Harga Saham Bursa Efek Indonesia (2010) [3], the value of transactions in the regular market is the main measure of liquidity and since January 2005, the number of trading days and the frequency of transactions as a measure of liquidity has been added. Stocks that enter into the LQ45 index will be evaluated once every 3 months and the replacement of stocks into the LQ45 index is done once every six months, i.e in early February and August.

The problem that investors often encounter in investing, especially with stock instruments, is how to optimize their portfolio. Of course, many criteria can be used to optimize the portfolio. In [3], several criteria are discussed to optimize a portfolio, including minimizing risk, maximizing return, minimizing risk with a certain return target, and maximizing return with a certain target return. In a previous study [4], we have modelled the problem of portfolio optimization consisting of LQ45 group shares with the criteria of minimizing risk and minimizing risk with a certain return target. Both criteria that we use are also added to the terms short-selling is not allowed.

In our previous portfolio model, if the portfolio consists of all shares, then the number of shares purchased based on the proportion of funds obtained can be a positive real number. In fact, retail investors in Indonesia can buy shares in units of lot, where 1 lot equal to 100 shares. Thus, we need to develop an existing model by including the amount of lot of stocks constraint that need to be purchased in order to minimize the portfolio risk, with or without the target return.

2. The Models

Suppose that in a portfolio there are n assets and r_{ij}denotes the percentage return of the asset-i in the period j with $i = 1,2,\dots,n$ and $j = 1,2,\dots,m$ and assumed $m > n$. Let y_i $(i = 1,2,\dots,n)$ denotes the proportion of investment for the asset-i with $\sum_{i=1}^{n} y_i = 1$. The risk of portfolio risk (V) on the Markowitz model [5] can be defined as the variance of the portfolio [1, 2], i.e.

$$V = \boldsymbol{y}^T Q \boldsymbol{y}$$

with

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \text{ and } Q = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

The matrix Q is the variance-covariance matrix of historical stock returns. Biggs [1] has been discussed how to optimize the portfolio with several criteria, including minimizing risk, maximizing return, minimizing risk with specific target returns, and maximizing returns with certain target returns. Some of the criteria were also added to the terms without short-selling.

In the previous study, we have discussed how to minimize the risk with and without a certain return target with the absence of short selling terms. Then, the model we have obtained, is applied to a portfolio of stocks included in the LQ45 index under various scenarios [4].

In this study, we will include the amount of lot of stocks constraint that need to be purchased so that the portfolio risk is minimum. The number of stock lots is a nonnegative integer. Then we will conduct a risk analysis and portfolio return consisting of several stocks with the best average return based on market capitalization whose shares are in the LQ45 group. The market capitalization value of a firm represents the price a person pays to buy the entire company. The value of capitalization can be calculated by multiplying the share price by the number of shares outstanding. We divide the stocks into LQ45 index groups into three groups based on their capitalization value. Group I are companies with a capitalization value greater than 100 trillion, Group II are companies with a capitalization value of 21 trillion to 100 trillion, and Group III are companies with a capitalization value smaller than or equal to 20 trillion. We also analyse the risk and return of a portfolio consisting of several stocks with the best average returns for each group based on this market capitalization value. The purpose of our study is to provide learning for retail investors in preparing an optimal portfolio in which this portfolio consists of only stocks.

In [4], we have discussed two models, they are

1. Minimize the risk of portfolio without a target return and short selling is not allowed.
   Model01

$$\min V = y^T Q y = \sum_{i=1}^{n} \sigma_i^2 \, y_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma_{ij} \, y_i y_j$$

   with constraints

$$\sum_{i=1}^{n} y_i = 1$$

   and $y_i \geq 0$ for every i.

2. Minimize the risk of portfolio with a particular target return and short selling is not allowed.
   Model02

$$\min V = y^T Q y = \sum_{i=1}^{n} \sigma_i^2 y_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma_{ij} y_i y_j$$

with constraints

$$\sum_{i=1}^{n} y_i = 1$$

$$\sum_{i=1}^{n} \bar{r}_i y_i = R_p$$

and $y_i \geq 0$ for every *i*. The $\bar{r}_i$ is the mean return for each asset.

Both models are transformed by utilizing penalty functions. Then, the model we have obtained, is applied to a portfolio consisting of stocks included in the LQ45 index with various scenarios [4].

The next problem is the purchase of shares must be in lot (1 lot = 100 shares). The number of shares *i* need to buy in forming a portfolio is

$$\frac{y_i M}{P_i}$$

with P_i: the current price of stock *i* and *M* denoting the amount of funds to be invested. Thus, the number of stock to buy (in lots) is

$$z_i = \frac{y_i M}{100 P_i} \Leftrightarrow y_i = 100 \frac{z_i P_i}{M}$$

with z_i: the number of lots of stock *i*. Since this lot number must be a nonnegative integer ($\mathbb{Z}^+$)), then **Model01** is modified to

Model01M

$$\min V = \boldsymbol{y}^T Q \boldsymbol{y}$$

with constraints (short selling is prohibited)

$$100 \sum_{i=1}^{n} z_i P_i = M$$

$$z_i \in \mathbb{Z}^+$$

In a similar way, **Model02** is modified to

Model02M

$$\min V = \boldsymbol{y}^T Q \boldsymbol{y}$$

with constraints (short selling is prohibited)

$$100 \sum_{i=1}^{n} z_i P_i = M$$

$$\frac{100}{M} \sum_{i=1}^{n} \bar{r}_i z_i P_i = R_p$$

$$z_i \in \mathbb{Z}^+$$

In both **Model01M** and **Model02M**, all entries of y_i's are replaced with $100 \frac{z_i P_i}{M}$. Since the result is a positive integer, the two models are not transformed as we did in [4]. However, we use the Solver which is a Microsoft Excel tool that is quite easy to use.

3. The Results

In this study, we will determine the number of shares in the LQ45 index to be selected along with the number of lots of z_i that need to be purchased from each of these stocks so that portfolio has the least risk. We use historical stock price data from July 1, 2015 to June 30, 2016. We download this data on July 10, 2016 at yahoo finance [7]. From this data we calculate the average return of each stock. To form a variance-covariance matrix Q, the formula [1, 2] is used:

$$\sigma_{ii} = \sigma_i = \frac{1}{m}\sum_{j=1}^{m}\left(r_{ij} - \bar{r}_i\right)^2$$

and

$$\sigma_{ij} = \frac{1}{m}\sum_{k=1}^{m}\left(r_{ik} - \bar{r}_i\right)\left(r_{jk} - \bar{r}_j\right)$$

with $\bar{r}_i$ representing the average return of the stock i. The parameters we use for the following results are $M =$ Rp 100 million and the accuracy of each constraint is 10^{-5}.

From the historical data of stocks price, we get that there are 14 stocks whose average return is negative. Therefore, the 14 stocks we do not use in the formation of portfolio. Thus, we only use 31 shares of 45 stocks in the LQ45 group.

The following is given two tables of selected portfolio consisting of stocks in the LQ45 index. Table 1 and Table 2 are each portfolios formed using **Model01M** and **Model02M**. From both tables it is seen that the target return constraints play a role in the formation of portfolio. In Table 2, the portfolio consists of more stocks (13 stocks) than Table 1 (only 7 stocks). For a portfolio with a 0.02% return target provides greater risk than a portfolio with no target return.

Table 1. Selected portofolio using Model01M

Stock	Lots
ADHI	83
WSKT	25
AKRA	19
MYRX	353
PTBA	3
LPKR	68
BBCA	16
$V = 0.017\%$	
$R = 0.092\%$	

Table 2. Selected portofolio using Model02M

Stock	Lots	Stock	Lots
WKST	159	TLKM	3
PWON	89	SCMA	1
CPIN	57	SMRA	1
HMSP	1	PTPP	1
BBTN	56	MYRX	3
ANTM	10	SRIL	1
ICBP	7		
$V = 0.026\%$			
$R_p = 0.02\%$			

Next, we divide the 31 stocks by market capitalization. Market capitalization is total market value of all of company's outstanding stocks. This value is calculated by multiplying a company's stocks outstanding by the current market price of one stock. We group the stocks in the LQ45 index into 3 groups based on market capitalization, that is, Group 1 is the company with market capitalization larger than 100 trillion Rupiahs (9 stocks), Group 2 is the company with market capitalization between 20 and 100 trillion Rupiahs (15 stocks), and Group 3 is the company with market capitalization under 20 trillion Rupiahs (7 stocks).

Similar as we do before, the following five tables will be displayed which state the number of lots needed to form the portfolio by using the **Model01M** and **Model02M** consisting of small, medium and big market capitalization stocks, respectively. Nevertheles, there is no feasible solution for selected portfolio using the Model02M for big market capitalization.

Table 3. The selected portfolio consists of small cap stocks using Model01M

Stock	Lots
ANTM	2
BBTN	59
MYRX	355
PTBA	2
PTPP	58
SRIL	40
SSMS	73
WIKA	82
$V = 0.016\%$	
$R = 0.074\%$	

Table 4. The selected portfolio consists of small cap stocks using Model02M

Stock	Lots
ADHI	247
ANTM	14
BBTN	125
MYRX	33
SRIL	305
$V = 0.06\%$	
$R_p = 0.2\%$	

Table 5. The selected portfolio consists of medium cap stocks using Model01M

Stock	Lots
ADRO	105
AKRA	9
ICBP	15
JSMR	25
LPKR	117
PWON	40
WSKT	123
$V = 0.019\%$	
$R = 0.136\%$	

Table 6. The selected portfolio consists of medium cap stocks using Model02M

Stock	Lots
ADRO	6
BSDE	27
ICBP	18
PWON	148
SMRA	2
WSKT	216
$V = 0.06\%$	
$R_p = 0.2\%$	

Table 7. The selected portfolio consists of big cap stocks using Model01M

Stock	Lots
BBCA	14
BBRI	54
UNVR	1
TLKM	48
$V = 0.033\%$	
$R = 0.055\%$	

A moderate investor (who can accept the risk with the middle level) can form a portfolio without a target return with reference to Table 5. In this table, the investor can expect a return of 0.136% with the risk of 0.019%. The expected return is greater than the portfolio of Table 7 and slightly smaller than the portfolio Table 3. Whereas if an investor wants to expect a return of 0.2%, then the investor can form a portfolio of Table 4 or Table 6 which both provide the same risk, i.e. 0.06%.

4. Conclusions and Further Research

From the above discussion, we can conclude that the portfolio consisting of LQ45 index with market capitalization of between Rp 20 and Rp 100 trillion is relatively better (in terms of less risk and bigger returns) than other portfolios. For further research, we will include the cost of stock purchase transactions in the formation of the portfolio. In addition, portfolio rebalancing also seems to be considered to keep the portfolio set up by investors to provide a return that matches their expectations.

References

[1] Biggs M C B 2005 *Nonlinear optimization with financial applications* (London: Kluwer Academic Publisher).
[2] Biggs M C B and Kane S J 2009 A global optimization problem in portfolio selection *Computational Management Science* **6** (3) 329-345.
[3] Bursa Efek Indonesia 2010 *Buku panduan indeks harga saham Bursa Efek Indonesia*.
[4] Chin L, Chendra E, and Sukmana A 2015 Analysis of portfolio optimization consisting of stocks in the LQ45 index *ICMAME Proc. Sanata Dharma University*.
[5] Markowitz H 1952 Portfolio selection *The Journal of Finance* **7** (1) 77-91.
[6] www.idx.co.id.
[7] http://finance.yahoo.com