

An Integer Programming Model for Multi-Echelon Supply Chain Decision Problem Considering Inventories

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Abstract. In this paper we address a problem that is of significance to the industry, namely the optimal decision of a multi-echelon supply chain and the associated inventory systems. By using the guaranteed service approach to model the multi-echelon inventory system, we develop a mixed integer programming model to simultaneously optimize the transportation, inventory and network structure of a multi-echelon supply chain. To solve the model we develop a direct search approach using a strategy of releasing nonbasic variables from their bounds, combined with the “active constraint” method. This strategy is used to force the appropriate non-integer basic variables to move to their neighbourhood integer points.

Keywords: Mixed Integer Linear Programming, Supply chain, Multi-Echelon, Inventory.

1. Introduction

Most companies nowadays are organized into networks of manufacturing and distribution sites that procure raw materials, process them into finished goods, and distribute the finished goods to customers. The goal is to deliver the right product to the right place at the right time for the right price. These production-distribution networks are what we call “supply chains.”

Supply Chain Management has attracted companies’ attention as they realize the potential cost benefits of integrating decisions with other members of their supply chain. The primary cost factors within a supply chain can be put into the categories of production, transportation and inventory. The signature of supply chain management is the integration of activities. Effective supply chain members invariably integrate the wishes and concerns of their downstream members into their operations while simultaneously ensuring integration with their upstream members.

It is widely accepted that supply chain and its management are the logical progression of developments in logistics and logistics management respectively (Cooper and Ellram, 1993; Bowersox and Closs, 1997; Kent and Flint, 1997; Ganeshan et al., 1998).

Inventory optimization remains one of the key challenges in supply chain management. Typically, large amounts of working capital are tied up in today’s supply chains, restricting the opportunities for growth that are essential for a company’s success in competitive markets. However, researches have shown that inventories have a high opportunity to reduce them within supply chain, and hence increasing competitiveness factors and reduce production costs.

From industrial aspects inventory management is a very important problem, however most of the models in the literature consider inventory management and supply chain network design separately, even though they are closely interrelated (Viau et al., 2007). This is due to the fact that



minimizing transportation and inventory costs are considered as being two contradictory objectives. Nevertheless, there are some related works on supply chain optimization that take into account the inventory costs, but they do not mention any detailed inventory management policy (Daskin et al., 2002; Shen et al., 2003; Sourirajan et al., 2007; You and Grossman, 2008; You and Grossman, 2010). In these models, the safety stock level is treated as a parameter, and can be considered as a lower bound of the total inventory level (Rodriguez et al., 2013; Bok et al., (2000); Verderame et al., (2008); Varma et al., (2007); Schulz et al., (2006), or it is considered as an inventory target that would lead to some penalty costs if violated (Jackson and Grossmann, 2003).

Most of the literature put focuses on inventory structure single stage integrated with supply chain design. Daskin et al (2002), Shen et al (2003) and Darnius and Mawengkang (2014) present a joint location-inventory model, which extends the classical uncapacitated facility location model to include nonlinear working inventory and safety stock costs for a two-stage supply chain network, so that decisions on the installation of distribution centers (DCs) and the detailed inventory replenishment decisions are jointly optimized. To simplify the problem, inventories in the retailers are neglected, and they also assume that all the DCs have the same constant replenishment lead time, and the demand at each customer has the same variance-to-mean ratio. You and Grossmann (2008) proposed a mixed-integer nonlinear programming (MINLP) approach to study a more general model based on the one developed by Daskin et al (2002) and Shen et al (2003), relaxing the assumption on identical variance-to-mean ratio for customer demands.

Supply chain management in the process industries has long been used as a tool to define production and distribution policies, as well as product allocation. This is the case of Cohen and Lee (1988) who described the modelling of a supply chain composed of raw material vendors, primary and secondary plants (each one with inventories of raw materials and finished products), distribution centres, warehouses and customer areas. Later, Cohen and Moon (1991) used supply chain optimisation to analyse the impact of scale, complexity (the operating costs are a function of the utilisation rates and number of products being processed in each facility) and weight of each cost factor (e.g. production, transportation and allocation costs) on the optimal design and utilisation patterns of the supply chain systems.

Timpe and Kallrath (2000) described an MILP model, which combined production, distribution and marketing and involved plants and sales points, to cover the relevant features required for the complete supply chain management of a multi-site production network. Jayaraman and Pirkul (2001) developed a Capacitated Plant Location Problem (CPLP) type model for planning and coordination of production and distribution facilities for multiple commodities, comprising raw materials suppliers, production sites, warehouses and customer areas. The authors followed a holistic approach to the supply chain, resulting in a deterministic, steady-state, multiechelon problem. Park (2005) considered both integrated and decoupled production and distribution planning problem, consisting of multiple plants, retailers, items over multiple periods. The author proposed mixed integer optimisation models and a two-phase heuristic solution to maximise the total net profit.

The objective of our model is to construct an integrating production, inventory and routing decisions within two-echelon context that minimizes total costs while meeting the following requirements: decide how many warehouses are needed, where to locate the opened warehouses, and how to allocate products from plants to warehouse and then to customers; find out how much to keep in inventory at any time period; and create how to build the vehicles' routes starting from opened warehouse to customers and back to that warehouse. In general distribution problems can be considered as a combinatorial optimization model. Therefore the model is formulated as a Mixed Integer Program (MIP). A direct feasible search approach is developed for solving the model.

2. Model formulation

This model deals with the distribution of several products N from a single manufacturer to a set of candidate warehouse (W) and then to a set of retailers, I .

To build a model of the problem some assumptions are needed.

- (i) The retailers deal with deterministic demand.
- (ii) The fleet of vehicles are homogen with the same capacity.

- (iii) Out-of-stock situations never occur.
- (iv) The route considered only from warehouse to retailers..
- (v) The inventory holding cost for warehouses is the same for all candidate warehouses.
- (vi) Inventory level at retailers is limited by constraint, namely: physical capacity at retailers site.

Let $G = (V, E)$ be an undirected network where V is a set of nodes comprised of a subset I of m potential warehouse sites and a subset $J = V/I$ of n customers. E is a set of route connecting each pair of nodes in V . We define a feasible route as a route in which a vehicle starts from a candidate warehouse, visits a number of retailers, and comes back to the same warehouse. As such, a feasible route does not pass by more than one warehouse. As a result, the number of possible feasible routes will be $W \times 2^I$, where W and I are the number of warehouses and retailers, respectively. Feasible routes and the associated parameters (α_{ir} and β_{rw} , as explained later) are required to define the model and, therefore, generated before solving the model. We assume vehicle capacity to be larger than the maximum customer demand at any time period. Moreover, in any time period, each vehicle travels at most on one route, and customers are visited at most once.

The objective is to minimize total cost incurred in the inventory-location problem. The total cost consists of three components. They are as follows:

- (i) warehouse fixed-location cost: the cost to establish and operate a warehouse;
- (ii) retailer unit-inventory holding cost: the cost to store products at retailer; and
- (iii) routing cost: the cost associated with transportation the goods from plants to warehouse and the cost from warehouse to retailers.

Sets

- O = Set plants, $O = 1, \dots, |O|$
- N = Set of products, $N = 1, \dots, n$
- W = Set of candidate warehouses $W = 0, \dots, |W|$
- I = Set of retailers, $I = 0, \dots, |I|$
- V = Set of nodes, $V = O \cup W \cup I$
- T = Set of time periods $T = 1, \dots, |T|$
- R = Set of all feasible routes
- K = Set of homogeneous vehicles $K = 1, \dots, |K|$

Parameters

- F_w = Fixed cost of opening and operating warehouse $w \in W$
- C = Vehicle capacity
- d_{ijt} = Demand of product $j \in N$ at customer $i \in I$ in time period $t = 1, \dots, T, \dots, T + \tau_{\max} - 1$
- u_{ijt} = Upper bound inventory level of product $j \in N$ at customer $i \in I$ in time period $t \in T$,
 $u_{it} = \left(\sum_{t < t + \tau_{\max}} d_{it} \right)$
- h_{ijt} = Inventory holding cost of product $j \in N$ at customer $i \in I$ in time period $t \in T$
- I_{ij0} = Inventory level of product $j \in N$ at customer $i \in I$ at the beginning of time period $t = 1$
- $\beta_{rw} = \begin{cases} 1 & \text{if route } r \in R \text{ visits warehouse } w \in W; \\ 0 & \text{otherwise} \end{cases}$
- $c1_{ow}$ = Transportation cost from node plant $o \in O$ to warehouse $w \in W$
- $c2_{wi}$ = Transportation cost from node warehouse $w \in W$ to retailer $i \in I$
- c_r = Transportation cost of route $r \in R$

Decision Variables

$$X_{ij} = \begin{cases} 1 & \text{if warehouse } i \in W \text{ is served by plant } j \in O; \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand of retailer } i \in I \text{ is served by warehouse } j \in W; \\ 0 & \text{otherwise} \end{cases}$$

I_{it} = Inventory level of product $j \in N$ at customer $i \in I$ at the end of time period $t \in T$

$$\theta_{rt} = \begin{cases} 1 & \text{if route } r \in R \text{ is selected in time period } t \in T; \\ 0 & \text{otherwise} \end{cases}$$

$$m_w = \begin{cases} 1 & \text{if warehouse is opened at location } w \in W; \\ 0 & \text{otherwise} \end{cases}$$

a_{jirt} = Quantity of product $j \in N$ delivered to customer $i \in I$ by route $r \in R$ in time period $t \in T$

3. The Model

The problem can be formulated as a MIP problem which has a mathematical form as follows.

The objective function is to minimize installation cost of warehouses, and all transportation costs.

$$\min \sum_{w \in W} f_w m_w + \sum_{t \in T} \left(\sum_{r \in R} c_r \theta_{rt} + \sum_{j \in N} \sum_{i \in I} h_{jit} I_{jit} \right) + \sum_{i \in O} \sum_{j \in W} c1_{ij} X_{ij} + \sum_{i \in O} \sum_{j \in I} c2_{ij} Y_{ij} \quad (1)$$

$$\text{Subject to} \quad \sum_{r \in R} \theta_{rt} \leq 1 \quad \forall t \in T \quad (2)$$

$$\sum_{i \in O} X_{ij} = m_j; \quad \forall j \in W \quad (3)$$

$$\sum_{j \in W} Y_{ji} = 1; \quad \forall i \in I \quad (4)$$

$$\sum_{i \in I} a_{jirt} \leq C \theta_{rt} \quad \forall j \in N, r \in R, t \in T \quad (5)$$

$$I_{ijt-1} + \sum_{r \in R} a_{jir} a_{jirt} = d_{jit} + I_{jit} \quad \forall j \in N, i \in I, t \in T \quad (6)$$

$$I_{ijt} \leq u_{ijt} \quad \forall i \in I, t \in T, j \in N$$

$$(7) \quad \theta_{rt} \leq \sum_{w \in W} \beta_{rw} m_w \quad \forall r \in R, t \in T$$

$$(8) \quad \sum_{r \in R} \theta_{rt} \leq |K| \quad \forall t \in T$$

$$(9) \quad X_{ij} = \{0,1\}; \quad \forall i \in O, \forall j \in W \quad (10)$$

$$Y_{ij} = \{0,1\}; \quad \forall i \in I, \forall j \in W \quad (11)$$

$$\theta_{rt} \in \{0,1\} \quad \forall r \in R, t \in T \quad (12)$$

$$m_w \in \{0,1\} \quad \forall w \in W \quad (13)$$

$$\alpha_{jrt}, I_{jit} \geq 0 \quad \forall j \in N, i \in I, r \in R, t \in T \quad (14)$$

Constraints (2) guarantee that a customer is visited once at most in any time period. Constraints (3) state that if warehouse j is installed then it should be served by only one plant i . Constraints (4) describe that each demand from retailer j should be served only by warehouse i . Constraints (5) account for the vehicle capacities. Inventory balance equations are represented in Constraints (6). Constraints (7) is to ensure that the inventory level at a customer never exceeds the total demand in the next consecutive time periods. Constraints (7) guarantee that routes start and end with open warehouses only. Constraint (9) limit the maximum number of routes at any time period to be no greater than the number of vehicles. Finally, Constraints (10) to (13) for the binary variables, and constraints (14) ensure that quantities to be shipped to customers and inventory levels are non-negative.

4. The Algorithm

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Calculate

$$v_i^T = e_i^T B^{-1}$$

this is a pricing operation

Step 3. Calculate $\sigma_{ij} = v_i^T a_j$

With j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\sigma_{ij}} \right| \right\}$$

I. For nonbasic j at lower bound

$$\text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{-\sigma_{ij}}$$

$$\text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}}$$

$$\text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$$

$$\text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$$

II. For nonbasic j at upper bound

$$\text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{-\sigma_{ij}}$$

$$\text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}}$$

$$\text{If } \sigma_{ij} > 0 \text{ and } \delta_{i^*} = 1 - f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$$

$$\text{If } \sigma_{ij} < 0 \text{ and } \delta_{i^*} = f_i \text{ calculate } \Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$$

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased from LB or decreased from UB. If none go to next i^* .

Step 4. Calculate

$$\alpha_{j^*} = B^{-1} \alpha_j^*$$

i.e. solve $B \alpha_{j^*} = \alpha_j^*$ for α_{j^*}

Step 5. Ratio test; there would be three possibilities for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

If j^* lower bound

Let

$$\begin{aligned} A &= \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\} \\ B &= \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\} \\ C &= \Delta \end{aligned}$$

the maximum movement of j^* depends on: $\theta^* = \min(A, B, C)$

If j^* upper bound

Let

$$\begin{aligned} A' &= \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\} \\ B' &= \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\} \\ C' &= \Delta \end{aligned}$$

The maximum movement of j^* depends on: $\theta^* = \min(A', B', C')$

Step 6. Exchanging basis for the three possibilities

1. If A or A'

- $x_{B_{i'}}$ becomes nonbasic at lower bound $l_{i'}$
- x_{j^*} becomes basic (replaces $x_{B_{i'}}$)
- x_{i^*} stays basic (non-integer)

2. If B or B'

- $x_{B_{i'}}$ becomes nonbasic at upper bound $u_{i'}$
- x_{j^*} becomes basic (replaces $x_{B_{i'}}$)
- x_{i^*} stays basic (non-integer)

3. If C or C'

- x_{j^*} becomes basic (replaces x_{i^*})
- x_{i^*} becomes superbasic at integer-valued

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Do integer lines search to improve the integer feasible solution

5. Conclusion

In this paper, we have presented a mixed integer programming (MILP) model that determines the optimal network design, transportation and inventory levels of a multi-echelon supply chain with the presence of customer demand to be fulfilled from the chosen warehouse. To solve the resulting MILP problem efficiently for large scale instances, a direct search algorithm was proposed.

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