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Portfolio optimization by using linear programing models based on genetic algorithm

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Abstract. In this paper, we discussed the investment portfolio optimization using linear programming model based on genetic algorithms. It is assumed that the portfolio risk is measured by absolute standard deviation, and each investor has a risk tolerance on the investment portfolio. To complete the investment portfolio optimization problem, the issue is arranged into a linear programming model. Furthermore, determination of the optimum solution for linear programming is done by using a genetic algorithm. As a numerical illustration, we analyze some of the stocks traded on the capital market in Indonesia. Based on the analysis, it is shown that the portfolio optimization performed by genetic algorithm approach produces more optimal efficient portfolio, compared to the portfolio optimization performed by a linear programming algorithm approach. Therefore, genetic algorithms can be considered as an alternative on determining the investment portfolio optimization, particularly using linear programming models.

1. Introduction

Investing in stocks is very attractive to many investors, even if investors have to face risks in their investments. The strategy in dealing with these risky investments is to form a portfolio of several stocks [14]. In the formation of a portfolio, investors always want to maximize the expected return with a certain level of risk they are willing to bear, or look for a portfolio that offers the lowest risk with a certain rate of return [15,16]. This portfolio characteristic, referred to as an efficient portfolio. To establish an efficient portfolio, it must adhere to assumptions about how investors behave in making investment decisions to be taken [14]. One of the most important assumptions is that all investors do not like risk (risk averse). Investors like this if faced with two investment options that offer the same returns with different risks, will tend to choose investments with lower risk [3]. While the optimal portfolio, is a portfolio selected an investor of the many options that exist in the collection of efficient portfolio [2,15]. Surely the portfolio selected by the investor is a portfolio that corresponds to the investor's preference, to the return or risk that it is willing to bear [3].

In the formation of a portfolio, an important thing to be noticed by investors is how to allocate capital resources, and the applicable provisions in investing, all of which are limited. This is with the aim that the portfolio is formed, resulting in an optimal portfolio. To find the solution is can be done by using linear programming (LP) optimization model. Such ways have been widely practiced by previous researchers. For example Mansini et al. [10], reviewing various portfolio optimization models that can be solved by LP. Presented in the literature of real features that have been modeled, and the solution approach to the resulting model. In most cases the model is integer linear programming (MILP). Mbah and Onwukwe [9], conducting an analysis of linear programming methods for the issue of portfolio selection and optimal financial investment in economic development. Liu [7], conducted a study that presents three possible models for the problem of portfolio selection with minimum transaction lots. As well as designing the appropriate genetic algorithm to get the solution. El hachlou et al. [1] optimizing stock portfolios using classification and genetic algorithm. Kapiamba et al. [4], performing a comparison between simulated annealing and genetic algorithm in optimal portfolio selection. Sadaf and Ghodrati [11], undertook a development of genetic algorithm methods for the selection and optimization of stock portfolios.

Based on the above description, there is a very interesting thing to investigate, that is how to optimize investment portfolio involving the use of linear programming model and genetic algorithm. Therefore, in this paper, research on investment portfolio optimization using linear programming model based on genetic algorithms is conducted. As the object of his research are some stocks that are traded on the capital market in Indonesia. The goal is to obtain a proportion of investment capital allocation, in order to obtain an optimal portfolio. The models used to achieve these objectives are discussed in the following sections.

2. Model of linear programming on investment portfolios and genetic algorithms

In this section we aim to discuss the models used in portfolio optimization, using linear programming based on genetic algorithms. The discussions include: investment portfolio, linear programming model, and genetic algorithm. It starts with a discussion of the investment portfolio as follows.

2.1. Portfolio investment in stocks

In this section it is intended to describe the investment portfolio in stocks. Investments in shares can be understood as a means of investment, either directly or indirectly, aimed at obtaining certain benefits, as a result of investments made [14]. In every investment decision on stock, as a rational investor, his attention is always aimed at the return of his investment. Investors will choose investments in stocks that give the highest expected return (return). However, investments in stocks that are carried contain an element of uncertainty, so investors should consider the risk factor [3].

Expected return is basically the weighted average of various historical returns, with the probability of each stock return as its weighting factor. Expected return is reflected by the mean of the profitability probability distribution. While the risk of an investment is measured by the amount of variance or standard deviation from expected return [15,16]. The greater the degree of spread (variance), illustrates that investments are becoming increasingly risky. A common strategy used in risky investment conditions, is to form a portfolio of some stocks [2]. The essence of forming a portfolio is to allocate funds to various investment alternatives on stocks, so the risk of investing in stocks as a whole can be minimized, and the profit rate of return can be maximized 13].

To minimize the risk and maximization of the expected return can be done using linear programming, as discussed in the following sections.

2.2. Linear programming model on investment portfolio

This section aims to discuss the formulation of linear programming model for optimization of investment portfolio in stocks.

Suppose that R_i is the i-th random variable of stock return, $i = 1,...,N$ with N is the number of stocks, and μ_i is the return expectation of R_i. Suppose also that W_0 is the initial capital owned by the investors, and π is the return desired by investors. Meanwhile, notation δ_i is the maximum value that investors want to invest in stocks i, $i = 1,...,N$ with N is the numbers of stocks. It is assumed that short selling is not allowed, thus the proportion $w_i \geq 0$, $i = 1,...,N$ with N is the number of stocks. Thus, suppose that the short selling is denoted by following equation (1) [17].

$$S = \{\mathbf{w'} = (w_1,...,w_N): \sum_{j=1}^{N} w_j \mu_j \geq \pi W_0,\ \sum_{j=1}^{N} w_j = W_0,\ 0 \leq w_j \leq \delta_j,\ j = 1,...,N\}. \tag{1}$$

Konno's formulated the investment portfolio optimization model as given in equation (2) [17].

$$\text{Minimum} \quad L_1 = E\left|\sum_{j=1}^{N} w_j R_j - E\left(\sum_{j=1}^{N} w_j R_j\right)\right|, \tag{2}$$

Subject to $\mathbf{w} \in S$.

Since the objective function is nonlinear, then according to Konno's and Yamazaki's methods, the model is constructed in the following way [17]:

$$\text{Minimum} \quad \frac{1}{T}\sum_{t=1}^{T} z_t, \tag{3}$$

$$\text{Subject to} \quad z_t \geq -\sum_{j=1}^{N} w_j (R_{jt} - \mu_j),\ t = 1,...,T,$$

$$z_t \geq \sum_{j=1}^{N} w_j (R_{jt} - \mu_j),\ t = 1,...,T,$$

$$\mathbf{w} \in S.$$

Here μ_j is the return expectation of j-th stocks, R_{jt} is the return of j-th stocks during periode t. Note that in this model there is no need to estimate the variance-covariance matrix.

Meanwhile, according to Cai's, the above investment portfolio optimization can be formulated as follows [17]:

$$\text{Minimum} \quad L_\infty(\mathbf{w}) = \underset{j}{\text{Maksimum}}\, E\,|w_j R_j - w_j \mu_j|, \tag{4}$$

Subject to $\mathbf{w} \in S$.

This model can also be transformed into the following linear form [17]:

$$\text{Minimum} \quad z, \tag{5}$$

$$\text{Subject to} \quad w_j q_j \leq z,\ j = 1,...,N,$$

$$\mathbf{w} \in S.$$

where $q_j = E\,|R_j - \mu_j|$, $j = 1,...,N$, is the absolute deviation of expectations of R_j from the mean return. Obviously, that if the distribution of each variable R_j given is random, this function is explicitly defined. Historical data can also be used to estimate μ_j and q_j. Model of L_∞ and related techniques are easy to be manipulated and applied practically. In addition, optimal portfolio selection does not contain correlation between stocks, which is the same as Konno's model and the constant number for this model is determined by the number of stocks.

Furthermore, Teo's model formulates optimization of investment portfolio as follows [6]:

$$\text{Minimum} \quad H_\infty^T = \frac{1}{T}\sum_{t=1}^{T} \underset{j}{\text{Maksimum}}\, E\,|w_j R_{jt} - E(R_{jt}) w_j|,$$

Subject to $\mathbf{w} \in S$.

For this model, each capital asset pricing model (CAPM) between the market portfolio and each stock return is determined by the use of an optimization method. This model can be transformed into the following linear form [17]:

$$\text{Minimum} \quad \frac{1}{T}\sum_{t=1}^{T} z_t ,$$

$$\text{Subject to} \quad w_j a_{jt} \le z_t ,\ t=1,...,T ,\ j=1,...,N ,$$

$$\mathbf{w} \in S .$$

where $a_{jt} = E \,|\, R_{jt} - E(R_{jt}) \,|$, $t=1,...,T$, $j=1,...,N$. It is clear that the size of a constant is determined by the number of stocks and the number of periods. Therefore, if N and T become large, the computational speed of the completion of this model can be relatively slowly.

Recall the classic model of Markowitz's, as follows [13]:

$$\text{Minimum} \quad Var(\mathbf{w}) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} ,$$

$$\text{Subject to} \quad \mathbf{w} \in S .$$

The objective function in the Markowitz model is the quadratic form in the variance-covariance matrix. This means optimal allocation of portfolios can be solved using standard quadratic programming algorithms. The use of this quadratic programming algorithm needs to be assured that the elements of the variance-covariance matrix have an inverse. The allocation of portfolio proportions may be different from the previous three models, since the previous three models do not require the variance-covariance matrix estimation [17].

2.3. Genetic Algorithm

This section aims to discuss genetic algorithms in linear programming models for optimizing investment portfolios of stocks.

Genetic algorithms can be understood as a search method based on the mechanisms of scientific genetics and natural selection [5]. The mechanisms of scientific genetics reflect the ability of individuals to marry, and produce offspring that have characteristics similar to their parents. While the mechanism of natural selection states that living things can survive, if able to adapt to the environment [8]. Therefore, it is expected that the resulting offspring have the best combination of characteristics from their parents, and can sustain future generations.

The general structure of a genetic algorithm can be expressed as the following steps [12]:

a) Generating the initial population, this initial population is generated randomly so as to obtain an initial solution;
b) The population itself consists of a number of chromosomes that present the desired solution;
c) Forming a new generation, in forming a new generation used three operators, namely reproduction / selection, cross over and mutation;
d) Evaluation of the solution, this process will evaluate each population by calculating the fitness value of each chromosome, and evaluating it until the stop criteria are met. If the stopping criteria have not been met, it will be formed again new generation by repeating steps b).

Based on the general structure of the genetic algorithm, the genetic algorithm to solve the linear programming model of investment portfolio optimization issues such as equations (3), (4), and (5), can generally be composed as follows:

1) Determine the initial population. The N number of initial population, are generated randomly. This initial random population number is then converted into the form of decimal values $\mathbf{w}$, where $\mathbf{w} \in S$;

2) Evaluate chromosomes. The fitness values of chromosomes are the objective function values (3), (4), and (5). Based on fitness values, the smallest value is selected for the minimization program.
3) Calculation of population convergent percentage. Percent of population convergence p_c, is a percentage of the number of individuals who have the same fitness value and also the best. This p_c number is calculated using the following formula:

$$p_c = \frac{n}{pop} \times 100\%,$$

where n is the number of individuals who have the same fitness and also the most, and pop is the number of population.
4) Checking stopping condition. The process of genetic algorithm will stop when the generation counter has reached the number of defined generations c_g that is c_g =1000, or the convergent percentage of the population p_c reach the defined threshold limit that is $\theta = 90\%$.
   1) Chromosome selection. Selection process is done using roulette wheel selection. Due to the minimization program, it evaluates the fitness $eval(v_i)$, $i = 1,...,N$, done based on equations (3), (4), and (5), by the formula:

   $$eval(v_i) = \frac{1}{f(\mathbf{w})},$$

   where $f(\mathbf{w})$ is the fitness values referring to equations (3), (4), and (5).
   2) Crossbreeding. The new population of the selection results is cross-breeded, using the Single-Point Crossover (SPX) method.
   3) Mutation. Mutations of each generation are obtained by using $m \times pop\_size \times p_m$, with m as a number of mutation, pop_size is a population size, and p_m probability of mutation (its value is determined randomly).
4) Decoding. It is the process of encoding the genes in a chromosome to get its value back as it was, changing the coding to decimal values.

Furthermore, this genetic algorithm is used to analyze the case of investment portfolio optimization as follows.

3. Result and discussion

In this section it is aimed at modeling linear programming, and finding the optimal solution of the investment portfolio optimization problem. The discussions include: data being analyzed, linear programming modeling, and finding the optimal solution. It starts with the discussion of analyzed data as follows.

3.1. Data analyzed

This section is intended to explain the data being analyzed. The data analyzed in this paper is the closing price of stock from five companies listed on a stock market, with a period of one year (12 months), including stocks of: A, B, C, D, and E. Each stocks has a monthly return as given in Table 1. While the difference of monthly data to its average, are given in Table 2.

Toward these five stocks, investors provide budget of W_0 at IDR 100,000,000 to be invested. However, this investor expects a minimum monthly return π at 3% (or equivalent to IDR 3,000,000). Investors also want no stocks to be proportioned δ_i ($i = 1,...,N$) greater 75% than its budget (or equivalent to IDR 75,000,000).

Table 1. Montly stocks return

Month	A	B	C	D	E
t	R_{1t}	R_{2t}	R_{3t}	R_{4t}	R_{5t}
1	0.0062	-0.0153	0.0175	0.0068	-0.0220
2	0.0000	0.0308	0.0044	0.0023	0.0073
3	0.0272	-0.0408	-0.0433	0.0000	0.0073
4	0.0026	0.0253	0.0477	0.0114	-0.0037
5	-0.0635	0.0051	0.0179	-0.0023	0.0000
6	0.0188	0.0000	0.0000	-0.0023	-0.0253
7	0.0439	0.0261	0.0091	-0.0023	0.0180
8	0.0183	-0.0209	-0.0181	0.0023	0.0110
9	0.0079	0.0356	0.0135	-0.0181	0.0261
10	0.0044	-0.0149	-0.0269	-0.0045	0.0190
11	0.0023	0.0245	0.0139	0.0152	0.0254
12	-0.0062	-0.0049	0.0000	0.0158	0.0800
μ_j	0.0052	0.0042	0.0030	0.0020	0.0119

Table 2. The difference of return and expected return

Month	A	B	C	D	E
t	$R_{1t}-\mu_1$	$R_{2t}-\mu_2$	$R_{3t}-\mu_3$	$R_{4t}-\mu_4$	$R_{5t}-\mu_5$
1	0.00105	-0.01949	0.01449	0.01564	-0.03391
2	-0.00515	0.02655	0.00143	0.00259	-0.00466
3	0.02203	-0.04504	-0.04628	-0.04512	-0.00461
4	-0.00256	0.02110	0.04475	0.04591	-0.01560
5	-0.06867	0.00092	0.01496	0.01611	-0.01193
6	0.01364	-0.00422	-0.00298	-0.00182	-0.03725
7	0.03870	0.02189	0.00611	0.00727	0.00609
8	0.01312	-0.02516	-0.02108	-0.01992	-0.00096
9	0.00278	0.03141	0.01056	0.01172	0.01414
10	-0.00072	-0.01911	-0.02989	-0.02873	0.00712
11	-0.00285	0.02028	0.01092	-0.02773	0.01347
12	-0.01136	-0.00913	-0.00298	-0.00182	0.06811

Based on the return data from the five stocks and the information described above, optimization is done by using linear programming based on genetic algorithm, as follows.

3.2. Modeling of Konno's and Yamazaki's optimization methods

To optimize the investment portfolio in the above five stocks, this paper is done by using Konno's and Yamazaki's linear programming model. Referring to equation (3), using data in Table 1 and Table 2, the Konno's and Yamazaki's linear programming model is structured as given in (6).

$$\text{Minimum}\quad \frac{1}{12}\sum_{t=1}^{12} z_t = \frac{1}{12}(z_1+z_2+z_3+z_4+z_5+z_6+z_7+z_8+z_9+z_{10}+z_{11}+z_{12}), \tag{6}$$

$$\begin{aligned}
\text{Subject to:}\quad & z_1-0.00105w_1+0.01949w_2-0.01449w_3-0.01564w_4+0.03391w_5\geq 0\\
& z_1+0.00105w_1-0.01949w_2+0.01449w_3+0.01564w_4-0.03391w_5\geq 0\\
& z_2+0.00515w_1-0.02655w_2-0.00143w_3-0.00259w_4+0.00466w_5\geq 0\\
& z_2-0.00515w_1+0.02655w_2+0.00143w_3+0.00259w_4-0.00466w_5\geq 0\\
& z_3-0.02203w_1+0.04504w_2+0.04628w_3+0.04512w_4+0.00461w_5\geq 0\\
& z_3+0.02203w_1-0.04504w_2-0.04628w_3-0.04512w_4-0.00461w_5\geq 0\\
& z_4+0.00256w_1-0.02110w_2-0.04475w_3-0.04591w_4+0.01560w_5\geq 0\\
& z_4-0.00256w_1+0.02110w_2+0.04475w_3+0.04591w_4-0.01560w_5\geq 0\\
& z_5+0.06867w_1-0.00092w_2-0.01496w_3-0.01611w_4+0.01193w_5\geq 0\\
& z_5-0.06867w_1+0.00092w_2+0.01496w_3+0.01611w_4-0.01193w_5\geq 0\\
& z_6-0.01364w_1+0.00422w_2+0.00298w_3+0.00182w_4+0.03725w_5\geq 0
\end{aligned}$$

$$z_6 + 0.01364w_1 - 0.00422w_2 - 0.00298w_3 - 0.00182w_4 - 0.03725w_5 \geq 0$$
$$z_7 - 0.03870w_1 - 0.02189w_2 - 0.00611w_3 - 0.00727w_4 - 0.00609w_5 \geq 0$$
$$z_7 + 0.03870w_1 + 0.02189w_2 + 0.00611w_3 + 0.00727w_4 + 0.00609w_5 \geq 0$$
$$z_8 - 0.01312w_1 + 0.02516w_2 + 0.02108w_3 + 0.01992w_4 + 0.00096w_5 \geq 0$$
$$z_8 + 0.01312w_1 - 0.02516w_2 - 0.02108w_3 - 0.01992w_4 - 0.00096w_5 \geq 0$$
$$z_9 - 0.00278w_1 - 0.03141w_2 - 0.01056w_3 - 0.01172w_4 - 0.014146w_5 \geq 0$$
$$z_9 + 0.00278w_1 + 0.03141w_2 + 0.01056w_3 + 0.01172w_4 + 0.014146w_5 \geq 0$$
$$z_{10} + 0.00072w_1 + 0.01911w_2 + 0.02989w_3 + 0.02873w_4 - 0.00712w_5 \geq 0$$
$$z_{10} - 0.00072w_1 - 0.01911w_2 - 0.02989w_3 - 0.02873w_4 + 0.00712w_5 \geq 0$$
$$z_{11} + 0.00285w_1 - 0.02028w_2 - 0.01092w_3 + 0.02773w_4 - 0.01347w_5 \geq 0$$
$$z_{11} - 0.00285w_1 + 0.02028w_2 + 0.01092w_3 - 0.02773w_4 + 0.01347w_5 \geq 0$$
$$z_{12} + 0.01136w_1 + 0.00913w_2 + 0.00298w_3 + 0.00182w_4 - 0.06811w_5 \geq 0$$
$$z_{12} - 0.01136w_1 - 0.00913w_2 - 0.00298w_3 - 0.00182w_4 + 0.06811w_5 \geq 0$$
$$0.0052w_1 + 0.0042w_2 + 0.0030w_3 + 0.0020w_4 + 0.0119w_5 \geq 3{,}000{,}000$$
$$w_1 + w_2 + w_3 + w_4 + w_5 \leq 100{,}000{,}000$$
$$0 \leq w_1, w_2, w_3, w_4, w_5 \leq 75{,}000{,}000$$

3.3. *Optimal solution using genetic algorithm*

In this section it seeks to solve the linear programming model of investment portfolio optimization, set up in section 3.2. Completion is done using genetic algorithm optimization. The algorithm used is referring to what has been described in section 2.3. The genetic algorithm process is dismissed when the counter generation has reached the number c_g =1000, or percent of population conventions p_c has reached the number θ = 90%. Based on the optimization process using genetic algorithm, optimal solution obtained when the allocation of capital is as follows: w_1 = 14,340,000.00, w_2 = 8,970,000.00, w_3 = 6,760,000.00 , w_4 = 66,560,000.00 , and w_5 = 3,370,000.00. The allocation of such investment capital gets the values from z_1 = -80,080.58; z_2 = 527,066.25; z_3 = -584,574.40; z_4 = 982,734.59; z_5 = -503,510.46; z_6 = -103,807.88; z_7 = 1,116,909.92; z_8 = -88,601.07; z_9 = 766,340.08; z_{10} = -268,632.90; z_{11} = 957,989.80; dan z_{12} = 998,166.65. So it gets the minimum magnitude at $z = (1/12)\sum_{i=1}^{5} z_i$ = 310,000.00.

By considering the values of z_1 to z_{12}, it is shown that the income earned monthly, from January to December is not always the same, but varies even some of which are negative or causing loss. The largest positive income occurred in July, which was IDR1,116,909.92, and the smallest positive income occurred in May at IDR503,510.46. While the largest negative income occurred in March at IDR584,574.40, and the smallest negative income occurred in January at IDR80,080.58. The average income in one year amounted IDR310,000.00.

Although this formulation seems very simple because it does not require an estimate of variance-covariance matrix, it does however lead to a rather pleasant portfolio. This formulation will produce an unlikely solution, if all shares during a given month generate a negative return. If for example during August, even stock A has a negative income, just like the others, there will be no portfolio with positive income. One possibility to reach a viable solution is to let z have a negative value as well. For example, if stock a returns in May is -0.0635, it will usually change in the next month.

4. Conclusions

In this paper we have discussed about the investment portfolio optimization using linear programming model based on genetic algorithms. Based on the results of the discussion can be concluded as follows. The investment portfolio optimization model that has an objective function in the quadratic form can be transformed into a linear form based on Konno's and Yamazaki's methods. The process of optimizing linear programming model can be solved by using genetic algorithm. Such investment portfolio optimization techniques are applied for stock analysis: A, B, C, D, and E. The optimal portfolio is obtained when the capital allocation is as follows: w_1 = 14,340,000.00, w_2 = 8,970,000.00, w_3 = 6,760,000.00, w_4 = 66,560,000.00, and w_5 = 3,370,000.00. Optimizations result obtained that the income monthly, from January to December is not always the same, but varied even some of which are negative or bring loss. However, in this case, with a capital investment of IDR100,000,000.00, a positive income of IDR310,000.00 is earned. Such optimization techniques are one way that can be used in investment portfolio analysis.

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