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Dynamic Models Increase Understanding of Geometry Through Proof

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Abstract. This research paper is intended to explain and development dynamic models in enhancing the understanding of geometry in understanding the concepts that relates to geometry.This research is done by making steps or structure of workmanship that facilitate students in understanding and working on problems on geometry through proof. Students perform four proofs namely two-column proof, paragraph proof, proof trees, flow proof. The study on relevant literature search and the previous research results show theoretically that dynamic models are highly relevant models for understanding geometric concepts through proof. So that students can understand and have the ability to prove.

1. Introduction

Mathematical proof is a convincing demonstration of the formula, the theorem is true, with the aid of logic and mathematics. Mathematical proof is one of the aspects that must be considered in the mathematics learning in schools recommended from NCTM (2000). The few or many experiences of students in preparing a first junior high school proof will have an impact on the ability to prove when attending college at the first level, as stated by Moore (1994) that one of the reasons why students encounter difficulties in proof is the experience in constructing Evidence is limited to proof of school geometry. Accordingly, based on the results of a study conducted by Sabri (2003) on the concept of mathematical proof of prospective teachers it is suggested that the junior secondary school curriculum should prepare better students in mathematical proof learning. This suggests that the concept of proof of junior high students is very weak.

Based on the above description, the researcher wants to improve the understanding of the students' geometry through mathematical proof and proofing by students with dynamic model. So as to improve the competence of students' ability in solving problems on geometry material through proof.

Dynamic model is an abstraction and simplification of a complex system, but attempted to represent the system properly. Furthermore, based on the dynamic model obtained, simulations of policy scenarios are based on logically developed assumptions (Sterman, 2000). Dynamic models describe the behavior of the system over time.

This study aims to develop a learning model that can overcome difficulties and improve the thinking stage of junior secondary students. So as to improve the competence of the ability of junior high students to develop optimally and to increase students' interest to familiarize themselves with using proof in solving geometry problems with dynamic models.

2. Understanding Geometry

These are all the required expertise required for all other branches of mathematics, including science.

2.1 Troubleshooting Steps in Mathematics

According to Poyla (1973) there are 4 steps that need to be implemented to solve the math problem that is:

1). Understand and deepen the problem, 2). Finding a strategy, 3). Using strategies to solve problems and 4). Review it again and reflect on the solution or initial state.

There is no chance of solving a problem if we cannot understand it first. This process requires not only the introduction of what to look for but also the different types of important information combined together to get answers.

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2.2 Dynamic Models

Dynamic model is an abstraction and simplification of a complex system, but attempted to represent the system properly. Furthermore, based on the dynamic model obtained, simulation of policy scenarios based on logically developed assumptions (Sterman, 2000). The use of system dynamics methodology is more emphasized to the objectives of enhancing the notion of how the behavior of the system arises from its structure over time.

3. Research Methods

This research has a purpose to know how dynamic model of increasing understanding of geometry through proof hence this research is literature research in field of education about mathematics especially related to dynamic model, improvement of geometry understanding and proof. Relevant research findings will be presented in the discussion section. The steps taken in this research are

a. Identify dynamic models of increasing understanding of geometry through verification.
b. Demonstrate dynamic models in increasing understanding of geometry through proof.

3.1 Proof Representations that Support Developing and Writing Proofs

The purpose of this section is to highlight four different ways that proofs can be represented in geometry and discuss how these various representations have the potential to facilitate proving.

As pointed out by Anderson (1983), successful attempts at proof generation can be divided into two major episodes " an episode in which a student attempts to find a plan for the proof and an episode in which the student translates that plan into an actual proof". The proof forms that we highlight include proof tree, two-column proof, flow proof, and paragraph proof.

3.1.1 Two-Column Proof

A two-column proof lists the numbered statements in the left column and a reason for each statement in the right column (Larson, 2001). The two-column form requires that students record the claims that make up their argument (in the statements column) as well as their justification for these claims (in the reasons column). However, the consecutively numbered steps of the proof may lead students to believe that the deductive process is more linear than it actually is.

3.1.2 Paragragh Proof

This form is more conversational than the other proof forms (Larson, 2001). Paragraph proofs are more like ordinary writing and can be less intimidating (EDC,2009). For this reason, they look more like an explanation than a structured mathematical device (EDC,2009). However the lack of structure could also be a detriment. In particular, some teachers have concluded that the paragraph form was not appropriate for high school students because students tended to leave out the reasons that justified their statement. As a result, students would often come to invalid conclusions (Cirillo,2008).

3.1.3 Proof Trees

The proof tree is an outline for action, where the action is *writing* the proof. Anderson (1983) described the proof tree as follows :

> The students must either try to search forward from the givens trying to find some set of paths that converge satisfactorily on the statement to be proven, or must try to search backward from the statement to be proven, trying to find some set of dependencies that lead back to the givens (p.194)

The proof tree could also be a useful tool to scaffold the work of determining what the given premises are or what conclusion can be proved.

3.1.4 Flow Proof

A flow proof uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by Arrows (Larson, 2001) and separated into different "branches". The flow proof helps students to brainstorm , working through the most difficult parts of solving a proof : (1) understanding the working information-analyzing the given and the diagram- and (2) knowing what additional information is needed to solve the proof-analyzing what is being proved (Brandell, 1994). A disadvantage to this proof form might be that students are not required to supply reasons that justify their statements in the way that the "Reason" column of the two-column proof forces them to do.

3. Discussion of research results

Nine different problems (presented in no particular order) that illustrate how students may be provided with opportunities to expand their role in the process of proving. Here's the discussion of 9 student problems in proving:

1. In problem 1, the student is provided with a conjecture (the diagonals of a rectangle are congruen) and a corresponding diagram and asked to write the "Given" and the "Prove" statements.
2. In Problems 2, the student is provided with the "Given" and the "Prove" statements but is asked to draw the diagram.
3. In Problem 3, when provided with a particular theorem, the student is asked to do all three of these tasks (write the "Given", the "Proven" and draw the diagram).
4. Problem 4 is similar to the first three in that students are invited to determine the "Given", but this time they also provided with the statement to be proved as well as the proof of that statement. Problem 4 is similar to the "fill in" type proofs that we have seen in some textbooks (Serra : 2008) except that rather than having students fill in the statements or reasons, they are filling in the premises.
5. Next, in Problem 5, students are asked to draw a conclusion or determine what could be proved when provided with particular "Given" conditions and a corresponding diagram.
6. In Problem 6, students are asked to determine what auxiliary line might be drawn in order to construct the proof that two angles are congruent. This is not a common problem posed to students because, typically, teachers either construct the auxiliary lines for their students or a hint is provided in the textbook that helps students determine where this line should be drawn (Herbs & Brach : 2006).
7. Problem 7 is unique in the sense that the student is asked what could be proved, but the givens are ambiguous. Leaving the problem more open-ended affords students opportunities to write conjectures. It is expected that the student will consider two different cases corresponding to whether the quadrilateral is concave or convex. In both cases the student could argue that the remaining pair of sides are congruent to each other.
8. Finally, in Problems 8 and 9, students have the opportunity to take part in analyzing proofs. In Problem 8, a paragraph proof is provided, and students are asked to find the error.
9. Problem 9, students are provided with a proof and asked to determine what theorem was proved.

In this section, we proposed nine problems that illustrate how teachers could increase their student's involment in proving by having them make reasoned mathematical conjectures, use conjectures to set up a proof, and evaluate mathematical proofs by looking for errors and determining what was proved. In the next, we address the issue of supporting students in proving by commenting on multiple proof representations.

4. Conducting Proof

In the study of mathematics, especially about the learning of geometry where in solving the problem one on mathematics.

The four steps above are recognized by different names: recognize, plan, do, check again (*see, plan, do, check*). In this way the student will have a more comprehensive understanding of the student's answers to the problem and make the student more confident in the truth.

The following is an example of a question in which the student performs a proof with the empirical experience of the researcher on some students.

1. Problem
   Show two right triangles are congruent if the hypotenuse and a leg of one are congruent to the corresponding parts of the other. Given : Right ΔABC with are right angle at C and Right ΔDEF with are right angle at F.
2. How is the problem solved?
   For this matter you can also use a theory that you know. For this problem is the use of congruence in the triangle. You can check with an image size. This means that you first need to think about it.
3. Purpose of achievement
   After understanding the problem then draw the triangle in accordance with what is known in the matter, then see the properties of triangle angles and the size of each side of both triangles.
4. Settlement process
   Plan for problem solving to verify and Learning Outcomes is students are able to solve problems and verify mathematical problems
5. Stages of Teaching
   - Remind again how to solve problems in the first mathematics and ask students to write something they know when they encounter a problem (draw two triangles by naming each corner from the known in the matter, then look at the size on each side and angle On both triangles). And develop a plan that can be used or that have been d sorted to solve the problem.
   - Run a plan and discuss what they think is more useful and appropriate in solving the problem.
   - Discussing the skills and abilities they have to figure out an unknown size or explanation. This might help students mention what they already know and get from their settlement.
   - While students solve problems ask and ask students to explain their reasoning. If students are unable to provide further answers. Then you may want to suggest more directly by using the nature of congruence and give solution.
6. Expansion of the problem
   Can you find a side that has the same length?
   If a side is the same length can you determine that each corner also has the same magnitude on the two triangles?
7. Solution
   On this issue everything has to be done by proving that the two triangles are congruent. Then there are four ways to prove that the two triangles are congruent by using dynamic model stages.

To Prove : $\Delta ABC \cong \Delta DEF$.

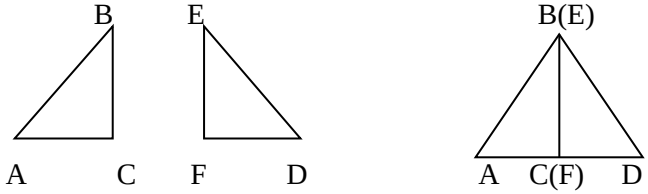


Figure 1. Two Triangles

Plan : Move the two given triangles together so that $\overline{BC}$ coincides with $\overline{EF}$, forming an isosceles triangle. The given triangles are proved congruent by using Theorem 1 and SAA.

A. A two-column proof lists the numbered statements in the left column and a reason for each statement in the right column.

Table 1. Two-Column Proof

Statements	Reasons
1. $\overline{BC} \cong \overline{EF}$	1. Given
2. Move triangles ABC and DEF together so that $\overline{BC}$ coincides with $\overline{EF}$, A and D are on opposite sides of $\overline{BC}$.	2. A geometric figure may be moved without changing its size or shape. Equal lines may be made to coincide.
3. $\angle C$ and $\angle F$ are right angles	3. Given
4. $\angle ACD$ is a straight angle	4. The whole equals the sum of its parts
5. $\overline{AD}$ is a straight line segment.	5. The sides of a straight angle lie in a straight line.
6. $\overline{AB} \cong \overline{DE}$	6. Given
7. $\angle A \cong \angle D$	7. If two sides of a triangle are congruent the angles opposite these sides are congruent.
8. $\Delta ADC \cong \Delta BCD$	8. SAA.

B. A paragraph proof describes the logical argument with sentences. It is more conversational than a two-column proof.

Since ΔABC dan ΔDEF is two isosceles triangles with $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$ Then move ΔABC and ΔDEF so that $\overline{BC}$ coincides with $\overline{EF}$, A and D are on opposite sides of $\overline{BC}$. If $\angle C$ and $\angle F$ are right angles then $\angle ACD$ is a straight angle and $\overline{AD}$ is a straight line segment. If two sides of a triangle are congruent then $\angle A \cong \angle D$ the angles opposite these sides are congruent. Thus, $\angle ADC \cong \angle BCD$. Because SAA, Thus $\Delta ABC \cong \Delta DEF$.

C. A Proof Tree

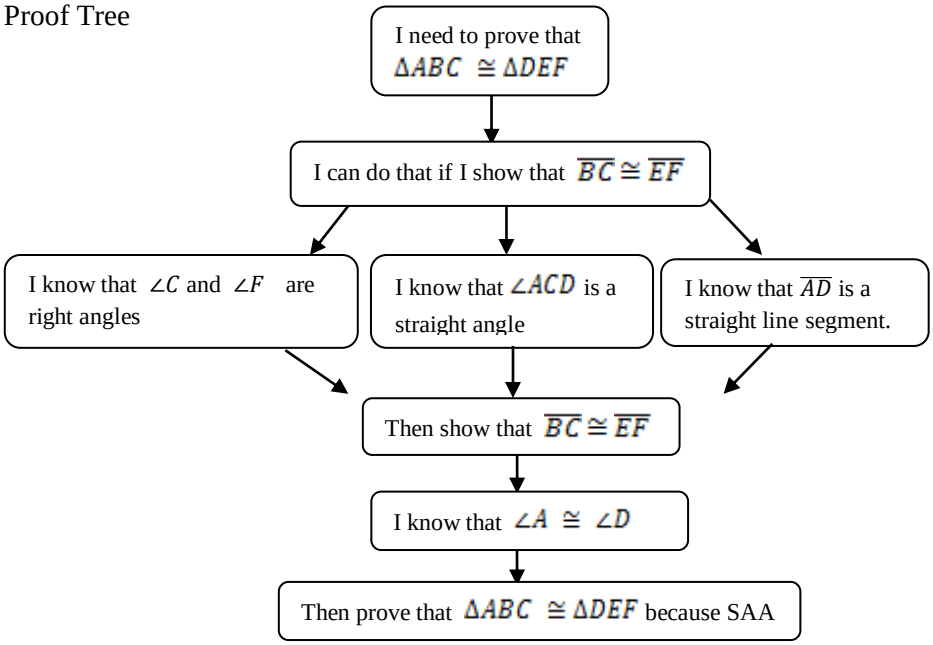


Figure 2. A proof tree

A proof tree is an outline or plan of action that specifies a set of geometric rules that allows students to get from the givens of the problem through intermediate levels of statemen, to the to-be-proven statement.

D. A Flow Proof

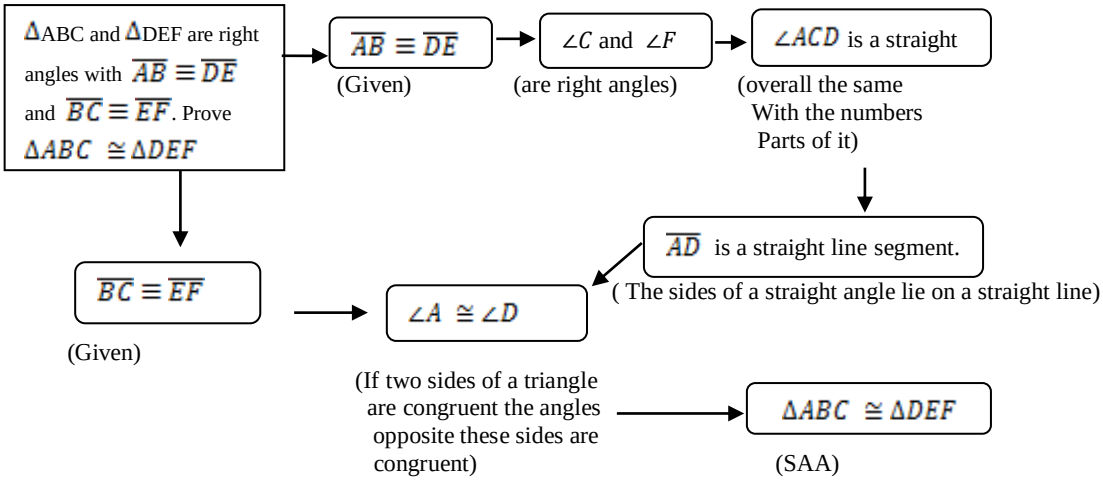


Figure 3. Flow Proof

A flow proof uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows. Depending on whether it is the plan or the proof itself, students may or may not choose to write the reasons beneath the statements.

5. Conclusion

After the researchers conducted a discussion of the data obtained from the results of the study, the researchers took the conclusion, namely:

1. Based on the empirical experience undertaken on the students, each of the evidences has a contribution to influence the increase in understanding of geometry in students. Of the four proofs performed with the dynamic model indicates that two-column proof is more understood by the students. Seen from the results some students do after learning the problem solving on geometry through the four proofs.
2. Teachers are fully aware of the purpose of this, teachers should pose problems to the students in order to give a greater role in proving. Teachers introduce problems for students to write down reasons, write statements to be proven, as well as ideas on diagrams. Students should be provided with opportunities to make excuses and evaluate mathematical and authentic opinions. Finally, teachers advance and provide different types of reasoning and methods on proof. Teaching the exercises so that students engage in solving problems, guessing, writing conditional statements to prove and then explanations and prove their allegations can assign students more opportunities to engage in mathematics.

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