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Model of Decision Making through Consensus in Ranking Case

Gim Tarigan, Open Darnius

Departement of Mathematics, University of Sumatera Utara

Email: gintarigansibero@gmail.com, open@usu.ac.id

Abstract. The basic problem to determine ranking consensus is a problem to combine some rankings those are decided by two or more Decision Maker (DM) into ranking consensus. DM is frequently asked to present their preferences over a group of objects in terms of ranks, for example to determine a new project, new product, a candidate in a election, and so on. The problem in ranking can be classified into two major categories; namely, cardinal and ordinal rankings. The objective of the study is to obtin the ranking consensus by appying some algorithms and methods. The algorithms and methods used in this study were partial algorithm, optimal ranking consensus, BAK (Borde-Kendal)Model. A method proposed as an alternative in ranking conssensus is a Weighted Distance Forward-Backward (WDFB) method, which gave a little difference i ranking consensus result compare to the result oethe example solved by Cook, et.al (2005).

1. Introduction

In making decision, sometimes one has to select one among some alternative options, meaning that at least there are two or more alternative options to be chosen. In pratice there might be two or more Decision Maker (DM) involved in making a decision based on one or more considerations or certain criteria. The Consensus Ranking (CR) Method computes an overall ranking that most closely represents the ranking of thr majority of judges. Some advantages of the CR method are the it is intuitive for understanding, easy to verify, and difficult to manipulate. The main idea in ranking consensus how to decide a rank that satisfied the preference of all DM. A decision making in the problem of ranking consensus simply lies in formulating the right choice among a variety of DM's preference after an evaluation or assessment they made based on the given criteria.

In this life, humans are almost at all times involved in decision making, even in their daily lives which contexts may not be too complicated. When one makes a decision, there is a process running in his brain that determines the decision made. If the decision is made for simple things such as which clothes to choose, one does not need to formulate according to a regulation or model that needs to be analyzed mathematically. However, if what needs to be decided is complicated and highly risky, it requires a further study. The determination of ranking consensus based on some subjective preferences of Dm is a quite rapidly growing problem and is studied by some mathematics experts. Determining ranking among some objects with quantitative criteria is probably easy, but it is not easy for the objects with ordinal qualitative criteria. In order to be more objective to determine the ranking of the objects with qualitative criteria, it usually requires more than one adjudicator as the decision making team. This team, in the process, gives their individual ranking based on each preference before

formulating final result of the ranking. A decision maker frequently needs an aid equipment such as scientific, logical and structured analysis. The means of analysis is a decision making model implemented to make a decision of complicated problem by combining a group of ranking in order to obtain a ranking consensus or compromise rank which has been proposed individual preference have been proposed by many researches such as Kemeny and Snell (1962), Kendall (1962), and Inada (1969).

This ranking consensus problem can be categorized into two major primary groups; namely, Cardinal and Ordianal problems. The formulation of Cardinal ranking occurs when a decision maker is able to express the preference level of an alternative over the other alternatives with one utility function. On the other hand, Ordinal ranking does not require a certain preference level. A complete ordinal making form n alternatives is set to be integers (1, 2, ..., n). This study focused on ordinal ranking. The interest of ordinal ranking representation and formulation requires minimum amount of information, where each decision maker only decides one preference over the other preference. The simplest consensus development of ordinal ranking is majority method.

Borda, J.C (1781) proposed “sign method” to develop consensus on opinions by determining the mean of the ranking allocated by the decision maker to each alternative and the winning alternative is the one with the lowest mean. The same version of this model is represented by Kendall (1962). Kendall is the first person to study ordinal ranking in statistical framework with problem evaluation approach. Kendall’s solution is to arrange the alternative rankings according to the number of equivalent rankings. Kendall organizes ranking by means of sign method. Borda-Kendall (BK) technique is the most widely used ranking consensus method in the pratice due to its simple calculation. Cook and Seifor (1982) further studied BK technique and proposed “minimum variation” method to determine ranking consensus. The popular method to develop consensus is to define the distance function on the group of all rankings and later determine the closest ordinal ranking which may mean a minimum distance.

Some other researchers use integer programming and goal programming to solve ranking consensus problem. Ali et.al (1986) presented integer distance function. Cook et.al (2006) used an extensive experiment simulation to compare integer programming by means of heuristic procedure. Iz and Jellassi (1991) used goal programming to measure the individual preference of the group members through Ordinal ranking scheme.

Multiple Criteria is also commonly used it formulate and solve the problem of ranking consensus. Cook and Krees (1991) propose a weighted ordinal ranking model in which each set of a alternatives is ordianlly ranked over a set of criteria. The criteria are:

1. The urgency of weigth allocated to each criteria.
2. The urgency of various positions at which an alternative can be placed, and
3. The precision in which a decision maker is able to differentiate the alternatives over a certain criteria. the clonclusion is that the optimal ranking consensus method cannot be emphasized on the decision maker. It is better to understand the strength and weakness of each method and allow the decision maker to select a certain method. The main problem in his study emerged when a k group of decision makers are asked to organize the ranking of n objects as the alternatives, and how to combine the decision makers’ making into are one ranking consensus, or the so called ranking compromise.

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2. Method

2.1. Consensus Ranking Concept

Consensus ranking concept in the ordinal qualitative criteria is meant basic framework which leads to problem approach that can mathematically be analogized in order to facilitate understanding in searching for the alternative of solution.

This concept began with an assumption that there was a group of N object which would be ranked by the K persons as the team of decision makers (DM) in the ranking, and a ranking method was done partially in which each DM was given as many (n≤ N) objects randomly, uaing replacement method. After that, each DM freely ranked n object they owned based on individual preferences to be recommended for a consensus ranking with skenario which called Weighted Distance Forward Backward (WDFB).

2.1.1. Weighted Distance Forward Backward (WDFB) Method

Weighted Distance Forward Backward (WDFB) method suggested as a method for determining ranking consensus method consists in three scenarios. Defining distance in ranking was made by giving value of one for a distance of two adjascent object in a single ranking; for example, in the ranking arrangement of three object, a,b, and c was two. The first scenario in this method was begin by determining the weight toward the distance in ranking with the following formula:

$$\frac{n+1-i}{\sum_{i=1}^{n} i} \quad (1)$$

The weigth is used to calculate a Object Ranking Value (ORV). ORV was calculated for each DM by multiplying the distance between a pair of object in ranking forward and the weight of the object ranking distance. ORV was then added for each dominating object of the entire Dm and is donated as Foeward Object Ranking Value (FORV). The second scenario was also begun by determining ranking weight according to the weight formula in the first scenario, but with minus mark as follows:

$$-\frac{n+1-i}{\sum_{i=1}^{n} i} \quad (2)$$

ORV was the calculated by multiplying the distance between a pair of object in a rank backward and the weight of the object rank distance. ORV was then added for each dominating object of the entire DM and is donated as Backward Object Ranking Value (BORV). In the last scenario, FORV was added with BORV as the basic for ranking consensus object by putting the right order called Basic Value of Consensus Ranking Object (BVCRO) from maximum to minimum. This was what it called Consensus Ranking.

3. Result and Discussion

In order to implement the WDFB method as it was suggested to be method of ranking consensus, an example completed by Cook, et.all (2005) was taken, and the usage of the main algorithm of optimal ranking consensus its compromised ranking was 2 > 1 > 4 >6 > 3 > 5. The WDFB method proposed in this paper solvet the example, and it is purposely done compare their result.

The problem can be regarded as the chioice of the best n=4 proposals out of N=6 research proposals to be financed, and there are five DMs involved as assesor to obtain the ranking of the six proposal were ranked 1,2,3,4,5,and 6, the result of the ranking of each DM for the four ranked proposals would be as follows:

DMProposal	*Rangking*
1{1,2,3,5}	$1 \succ 3 \succ 2 \succ 5$
2{1,2,4,6}	$2 \succ 1 \succ 4 \succ 6$
3{3,4,5,6}	$4 \succ 3 \succ 5 \succ 6$
4{1,4,5,6}	$6 \succ 1 \succ 4 \succ 5$
5{1,2,3,6}	$6 \succ 2 \succ 3 \succ 1$

3.1. Implementation of WDFB Method

The settlement as the ranking consensus by using WDFB method was done according to the stages which had been designed in the scenario as follows:

Scenario I. Forward Weighted Ranking Distance: for ranking 1, 2, 3, and 4 consecutively it was found $\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$, while the distance among the objects in each Dm was determined in the example made by DM I as follows: the distance from object 1 to object 2 was 1, annotated with d (1,2) = 1, the distance form object 1 to object 3 was 2, annotated with d (1,3) = 2, and the distance from object 1 to object 5 was 3, annotated with d (1,5). Furthermore, the value of each ranking object is calculated by multiplying the distance with weights with in the forward ranking. In this example for the ranking made by DM1 of the object 1 was

ORB1=d(1,2)* $\frac{4}{10}$= $\frac{4}{10}$

ORB1=d(1,3)* $\frac{4}{10}$, = $\frac{8}{10}$

ORB1=d(1,5)* $\frac{4}{10}$= $\frac{12}{10}$.

The ranking value of object 2 and object 3 could be obtained as follows:

ORB2=d(2,3)* $\frac{3}{10}$ =$\frac{3}{10}$

ORB2=d(2,5)*$\frac{3}{10}$ = $\frac{6}{10}$, and

ORB3=d(3,5)*$\frac{2}{10}$ = $\frac{2}{10}$, so that it could be calculated for each DM with a similiar way.

Scenario II. Backward Weighted Ranking Distance: for ranking 1, 2, 3, and 4 consecutively, it was found $\frac{-4}{10}, \frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}$, while the distance among the objects in the ranking for each Dm could be determined. In the example made by DM 1, it was found that the distance from object 5 to object 3 was 1, annotated with d(5,3)=1, the distance form object 5 to object 2 was 2, annotated with d(5,2)=2, and the distance from object 5 to object 1 was 3, annotated with d(1,5). Furthermore, the value of each ranking object is calculated by multiplying the disatance with weights within the backward ranking. In this example for the ranking made by

DM1 of the object 5 are:

ORB5=d(5,3)*$\frac{-4}{10}$= $\frac{-4}{10}$

ORB5=d(5,2)*$\frac{-4}{10}$=$\frac{-8}{10}$

ORB5=d(5,1)*$\frac{-4}{10}$=$\frac{-12}{10}$. The ranking value of object 3 and object 2 could be calculated with the same method and it was found that

ORB3=d(3,2)*$\frac{-3}{10}$=$\frac{-3}{10}$

ORB3=d(3,1)*$\frac{-3}{10}$=$\frac{-6}{10}$

ORB2=d(2,1)*$\frac{-2}{10}$=$\frac{-2}{10}$, so that it could be calculated for each DM with a similiar way.

Scenario III: the result of the calculation of each DM was added up in order to obtain FORV and BORV. The total of FORB and BORV for each object was made as the making consensus ranking. Form the calculation of each DM, it could be found the Basic Value of Consensus Ranking (BVCR) as follows:

BVCRO1=1,4 BVCRO2=2,4

BVCRO3=0,7 BVCRO4=1,0

BVCRO5=-5,5 BVCRO6=0

So that the Consensus Ranking Proposal was P2, P1, P4, P3, P6, P5, and thus the proposal which was financed, based on the consensus, was Proposal Number 2,1,4, and 3. In terma of Cook, et.al example's notation, the result is 2 › 1 › 4 › 3. The difference lies on proposal3 instead of proposal 6.

4. Conclusion

WDFB Method with in the three proposed scenarios have slight differences in completion of consensus ranking problem in the investigated example of Cook, at.al (2005) using the main algorithm model optmal ranking consensus. This requires more in-depth studies that can be done in future studies. Can be suggested that this method can be modeled, and further can be done in a simulation study comparing it with existing methods.

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