Optimization Model for Capacity Management and Bed Scheduling for Hospital

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Abstract. Hospital is a very important institution to provide health care for people. It is not surprising that nowadays the people's demands for hospital is increasing.. However, due to the rising cost of healthcare services, hospitals need to consider efficiencies in order to overcome these two problems. This paper deals with an integrated strategy of staff capacity management and bed allocation planning to tackle these problems. Mathematically, the strategy can be modeled as an integer linear programming problem. We solve the model using a direct neighborhood search approach, based on the notion of superbasic variables.

Keywords: Capacity management, bed allocation planning, modeling, integer programming, neighborhood search..
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1. Introduction

The obvious thing for most people in this planet is to have good health. The legal institution which provide and maintain good health in any country is hospital. Therefore, it is understandable that nowadays the people's demand for hospitals is increasing. One of the the main cause of this rise lies in the ageing populations who are putting heavy demands for health care. The direct impact of this condition is the number of hospitals are increasing. This case also occurs for the people who live in big city, such as, Medan city of North Sumatera province, Indonesia. However, there are still more people from Medan to go to other countries, such as, Malaysia and Singapore, to get better health services. Undoubtedly, the urgent need to tackle this situation is to improve health service performance of hospitals.

All operations related to the health service performance in hospitals are limited in terms of capacity. Therefore, in order to fulfill the patients' demand for health care, the hospitals management should have a planning and controlling the capacity of the operations [1], [2], [3].

Capacity planning decisions are concerned primarily to ensure that the institution has the capabilities to respond to the level of demand experiences, especially to health care industry because not only it relates to the management of highly specialized and costly resources (i.e., nurses, doctors, and advanced medical equipment), but also it makes a difference between life and death in critical conditions [4]. Public hospitals have, in general, more demand for health services than available capacity. Therefore it is important to forecast and manage demand with good precision, in order to adjust capacity or take alternative courses of action for example transfer demand to other facilities. Demand forecasting and management is part of a larger design that intents to provide a systemic solution to global hospital management. Such solution is commonly based on the design of a general process structure for hospitals and which defines the management processes that are needed to optimize the use of resources in doing so and to ensure a predefined service level for patients. The general process structure allowed us to determine the key processes where implementation of new practices would generate most value [5] [6].

For inpatient care facilities in a hospital, this requires information on bed capacity and nursing staff capacity, on a daily as well as annual basis. Quantitative models can be used to calculate capacity needs for different planning purposes and for short, medium and long term planning issues. Although several useful models are described in the international literature [7], [8], [9] many of them are difficult to apply in practice because they require a great deal of data and clerical work [7].

To be able to apply capacity management in practice, models must fulfill different functions: "annual staff planning," "roster scheduling support," and "daily assignment of nurses to wards [10-11]." In addition, "strategic decisions" are sometimes mentioned as a separate planning level [7, 12, 13]. Models based on mathematical optimization techniques from operations research are generally focused on short-term scheduling [14, 15]. Models that do integrate different planning horizons (daily, periodical (1–2 months), and annual) are for example described by [16] and [17]. These models contain connected models for periodical staff planning and daily scheduling. However, models incorporating operational planning issues with tactical and strategic decisions or operational scheduling support with annual staff planning were not found in the literature. Sitepu and Mawengkang [18] use stochastic optimization model to solve hospital nursing – staff management problem.

In general, capacity models are aimed at calculating the number of nurses needed, whereas capacity management models should ideally also give insight into opportunities for improving capacity usage.

Regarding to bed resources capacity planning most approach reported in the literature fall into the use of mathematical programming and simulation. Li et al., [11] present a mathematical programming model approach. They propose a multi-objective decision aiding model for solving allocation of beds problem in a hospital. Their model is based on queuing theory and goal programming (GP). In order to obtain some essential characteristics of access to various departments within a hospital, a waiting line model is explored. The results are used to construct a goal programming frame work, taking account of targets and objectives related to customer service and profits from the hospital manager and all department heads. [19] create a mathematical model based on a dynamic dispatching approach for bed resources allocation considering hospitalization demands, bed capacity and income. The objectives of their model are to maximize income and to minimize the costs to use supplementary beds. Another goal programming model approach for solving re-allocation bed in a hospitals was proposed by [20]. The constraints considered in their model are total number of beds, nursing work hours, waiting time, the definite bed allocation to patient, and the definite bed allocation to ward. An integer linear programming model approach is featured by [14] to solve a hospital bed management problem constrained with budgetary cuts. They implemented their result in French hospitals.

In terms of simulation technique for solving bed allocation problem, the works in the literatures belong to the use of simulation only and a combined simulation-optimization. Due to the inherent uncertainties in the problem, it is reasonable to use simulation technique only. In particular, the use of simulation we should be able to get useful insights on bed allocation. These results can be found in [15], [21], and [22]. Laker et al.[23] use discrete event simulation for solving bed allocation problem in the emergency department of a hospital. An interesting review of using simulation in healthcare is addressed in [24]. However the use of only simulation may not reach the best solution, particularly when the problems involve combinatorial nature. It is not surprising that some literatures propose a combine strategy simulation and optimization. [25] propose a simulation optimization approach for solving resource allocation in an emergency department. The use of multi-objective optimization combined with simulation to tackle the bed allocation problem can be found in [12].

2. Problem Description

Increasing demand for healthcare through hospitals created heavy challenges to their managers and decision makers. The challenges involve high costs, limited budget, and limited resources. Most of hospitals in Medan are encountered with some pressures, such as, shortages of qualified healthcare professionals, limited hospital equipments and facilities, increasing operational costs.

Capacity planning, for hospitals in particular, is concerned with making sure of balancing the quality of health care delivered with the cost of providing that care. Such planning involves predicting the quantity and particular attributes of resources required to deliver health care service at specified levels of cost and quality. The most fundamental measure of hospital capacity planning is the number of inpatient beds accordingly the number of doctors and the number of nurses. Hospital bed capacity decisions have traditionally been made based on target occupancy levels. Certain units in the hospital, such as, intensive care units (ICUs) are often run at much higher utilization levels because of their high costs.

The other important thing that should be considered in order to enhance the service performance of hospitals is waiting time, due to the bed capacity allocation system. Alternative strategy to overcome this situation is to have a well coordinated hospital capacity management along with bed allocation system.

3. Mathematical Model

The decision problem, for our problem, is to maximally coordinate the utilization of multifold resources within the hospital. In our case the multifold resources are doctors, nurses, beds and rooms.. In this case the most appropriate model to be created is a linear integer programming problem.

Notations

Sets

I Set of departments

J Set of doctors type

K Set of nurses type

L Set of available beds

R Set of rooms

E Set of medical equipments

H Set of technicians

Decision variables

XD_{ij} : Number of type $j \in J$ doctors in department $i \in I$

XN_{ik} : Number of type $k \in K$ nurses in department $i \in I$

XDA_{ij} : Number of type $j \in J$ doctors added in department $i \in I$

XNA_{ik} : Number of type $k \in K$ nurses added in department $i \in I$

XT_{ih} : Number of type $h \in H$ technicians in department $i \in I$

XB_{il} : Number of beds $l \in L$ in department $i \in I$

XBA_{il} : Number of beds $l \in L$ added in department $i \in I$

XM_{ie} : Number of medical equipment $h \in H$ in department $i \in I$

Binary variables

Z_{ir}^{l} Equals to 1, if bed $l \in l$ are allocated for room $r \in R$ in department $i \in I$, equals to 0 otherwise.

Y_{r}^{i} Equals to 1 if room $r \in R$ is used in department $i \in I$ Equals to 0 otherwise

Parameters

bd_{ij} : Cost of $j \in J$ type doctors in department $i \in I$

bs_{ik} : Cost of type $k \in K$ nurses in department $i \in I$

bsa_{ik} : Cost of type $k \in K$ nurse-aids in department $i \in I$

bt_{hi} : Cost of type $h \in H$ technicians in department $i \in I$

bt_{il} : Cost of operating beds $l \in L$ in department $i \in I$

bw_{il} : Waiting cost for beds $l \in L$ in department $i \in I$

ba_{lr}^{i} : Cost for allocating bed $l \in L$ for room $r \in R$ if used in department $i \in I$

br_{i}^{r} : Cost to operate room $r \in R$ if used in department $i \in I$

bm_{er}^{i} : Cost to operate medical equipment $e \in E$ for room $r \in R$ if used at department $i \in I$

md_{ij} : Maximum number of doctor for each type can be allocated to each department

mn_{ik} : Maximum number of nurse each type can be allocated to each department

mna_{ik} : Maximum number of nurse-aids each type can be allocated to each department

mb_{il} : Maximum number of beds $l \in L$ can be allocated to each department

ρ_i : Maximum fund available for department $i \in I$

A. *Objective Function*

The objective of this problem is to find all the decision variables such that to minimize total operating cost.

Mathematically, the objective function can be written as follows.

$$\begin{aligned} \text{Minimize} \quad TC = & \sum_{i\in I}\sum_{j\in J} bd_{ij}(XD_{ij}+XDA_{ij}) + \sum_{i\in I}\sum_{k\in K} bs_{ik}(XN_{ik}+XNA_{ik}) + \sum_{i\in I}\sum_{h\in H} bt_{hi}XT_{hi} + \\ & \sum_{i\in I}\sum_{l\in L} bt_{il}(XB_{il}+XBA_{il}) + \sum_{i\in I}\sum_{l\in L} bw_{il}XBA_{il} + \sum_{e\in E}\sum_{r\in R}\sum_{i\in I} bm^{i}_{er}XM^{i}_{er} + \\ & \sum_{l\in L}\sum_{r\in R}\sum_{i\in I} ba^{i}_{lr}Z^{i}_{lr} + \sum_{i\in I}\sum_{r\in R} br^{i}_{r}Y^{i}_{r} \end{aligned} \tag{1}$$

Constraints:

$$XD_{ij} + XDA_{ij} \le md_{ij}, \qquad \forall i \in I, \forall j \in J \tag{2}$$

Constraints (2) state that the number of doctor should not be greater the maximum number of doctors can be allocated to each department.

$$XN_{ik} + XNA_{ik} \le mn_{ik}, \qquad \forall i \in I, \forall k \in K \tag{3}$$

$$XD_{ij} + XDA_{ij} \ge XT_{ih}, \qquad \forall i \in I, \forall j \in J, \forall h \in H \tag{4}$$

$$XB_{il} + XBA_{il} \le mb_{il}, \qquad \forall i \in I, \forall l \in L \tag{5}$$

$$\sum_{l\in L} Z^{i}_{lr} \le Y^{i}_{r}, \qquad \forall i \in I, \forall r \in R \tag{6}$$

Constraints (6) are to make sure that beds are allocated to the available room.

$$\begin{aligned} & \sum_{i\in I}\sum_{j\in J} bd_{ij}XD_{ij} + \sum_{i\in I}\sum_{k\in K} bs_{ik}XN_{ik} + \sum_{i\in I}\sum_{h\in H} bt_{ih}XT_{ih} + \sum_{i\in I}\sum_{l\in L} bt_{il}XB_{il} + \\ & \sum_{i\in I}\sum_{l\in L} bw_{il}XBA_{il} + \sum_{i\in I}\sum_{e\in E}\sum_{r\in R} bm^{i}_{er}XM^{i}_{er} + \sum_{i\in I}\sum_{r\in R}\sum_{l\in L} ba^{i}_{rl}Z^{i}_{rl} + \sum_{i\in I}\sum_{r\in R} cr^{i}_{r}Y^{i}_{r} \le \sum_{i\in I}\rho_i \end{aligned} \tag{7}$$

Constraints (7) state that the budget spent should not be greater than the fund provided.

$$\begin{gathered} XD_{ij}, \mathrm{XDA}_{ij}, \mathrm{XN}_{ik}, \mathrm{XNA}_{ik}, \mathrm{XT}_{ih}, \mathrm{XB}_{il}, \mathrm{XBA}_{il}, XM^{i}_{er} \ge 0 \quad \text{and integer} \\ \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{K}, \forall \mathrm{l} \in \mathrm{L}, \forall \mathrm{e} \in \mathrm{E}, \forall \mathrm{h} \in \mathrm{H} \end{gathered} \tag{8}$$

$$Z^{i}_{lr}, \mathrm{Y}^{i}_{r} \in \{0,1\} \qquad \forall i \in I, \forall l \in L, \forall r \in R \tag{9}$$

Expressions (8) and (9) are the nature of decision variables.
It can be seen that the model is in the form of a large scale integer programming problem.

4. Feasible Neighborhood Heuristic Search

Branch-and-bound approach is a general method for solving linear integer programming problem. However, for large-scale problems such a procedure would be prohibitively expensive in terms of total computing time, and frequently the algorithm terminates without solving the problem. We have adopted the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

This may be implemented within the structure of a program by marking all integer variables at their bounds at the continuous solution as nonbasic and solving a reduced problem with these maintained as superbasic..

The procedure may be summarized as follows:

Step 1 : Solve the problem ignoring integrality requirements.

Step 2 : Obtain a (sub-optimal) integer feasible solution, using heuristic rounding of the continuous solution.

Step 3 : Divide the set I of integer variables into the set I_1 at their bounds that were nonbasic at the continuous solution and the set I_2, $I = I_1 + I_2$.

Step 4 : Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step 5 : At the solution obtained in step 4, examine the reduced costs of the variables in I_1. If any should be released from their bounds, add them to set I_2 and repeat from step 4, otherwise terminate.

The above summary provides a framework for the development of specific strategies for particular classes of problems. For example, the heuristic rounding in step 2 can be adapted to suit the nature of the constraints, and step 5 may involve adding just one variable at a time to the set I_2.

At a practical level, implementation of the procedure requires the choice of some level of tolerance on the bounds on the variables and also their integer infeasibility. The search in step 4 is affected by such considerations, as a discrete step in a super basic integer variable may only occur if all of the basic integers remain within the specified tolerance of integer feasibility.

In general, unless the structure of the constraints maintains integer feasibility in the integer basic variables for discrete changes in the superbasic, the integers in the set I_2 must be made superbasic. This can always be achieved since it is assumed that a full set of slack variables is included in the problem.

5. Conclusions

This paper presents an optimization model of capacity management problem of a hospital which integrate bed allocation planning.. The optimization model is a large scale mixed integer linear program. We then propose a feasible neigfhbourhood integer search for solving the model.

6. References

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