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Triangle Number of Laurent polynomial for the closed braids σ^{2j}

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Abstract. In various computations, the triangle numbers help inductively to override a pattern from different structures increasingly. However, different structures of a particular entity effect the structure of presentation, including the order of numbers that characterize it. The structure of uniforms knots based on a particular woven pattern is followed by the behavior of the structure which in this case is represented by the presence of triangle numbers. This paper describes the existence of the triangle number based on the closed braids σ^{2j}.

1. Introduction

Like Pascal's triangle number, there is an arrangement of the triangle number that serves to solve an equation inductively [1]. The arrangement of this triangle number is based on the arrangement of certain object patterns that are mathematically modelled so that they can explicitly explain the pattern arrangement [2]. In everyday life the structure of the object can be considered to reinforce the arrangement of the object pattern, for example the fabric of the rope in addition to beautify also strengthens the relationship between the rope composers such as the hair braid to strengthen the thin hair [3]. Although at a glance, such arrangement appear similar but structurally between one pattern arrangement with another are mutually different or equal, and this difference can be explained mathematically [4].

Mathematically, a braid is the basic pattern of the links of strings or the hair braid. Each braid may turn into a knot in mathematical, i.e. a closed braid, and a braid has a specific identifier formula [5]. When pattern of braid regularly increase, in general the knots will follow increase of the pattern. If the pattern of braid regularly can be formulated based on the number pattern, the arrangement of the knots forms an array of numbers. This paper describe the composition of number pattern of the related closed braids.

2. Related Works and Motivation

A braid has been modelled mathematically [3]: A system called a *woven pattern* is two parallel lines in $\mathcal{R}^3$ in the same direction, each is said to be a top frame and the bottom frame, there are n different points p_i and q_j, $i, j = 1, \ldots, n$, they located on lines L_1 and $L2$, respectively. The adjacent points in a line have the same distance, and then one point of p_i is connected by the curve c_i just to a point q_j, where i and j should not be different [6, 7].

A braid is a weave together with a deformation operation (let's call it intuitively as a sweep), resulting in no weave of the same high such that (i) L_1 and L_2 stays parallel during the sweep, (ii) there are no slices between the curves during the sweep, and (iii) the curve remains normal during sweeping, i.e. the projected curve c_i is moving from p_i to q_j and take the distance from L_1 always increase. Next, intuitively it is stated that σ_i as notation for a woven geometric shape in which the curve c_i via c_{i+1} exactly once on it, whereas c_j for $j \neq i, i+1$ directly connects the points p_j to q_j. Instead, σ_i^{-1} is a webbing where the curve c_i via c_{i-1} exactly once on it, whereas c_j, $j \neq i-1, i$ are direct linking points p_j with q_j [8]. Therefore, for n-order braid, n is positive integer, we have generators $\sigma_1, \sigma_2, \ldots, \sigma_{n-1}$ of braid, and for a collection of n-order braids, we obtain an algebraic definition of the braid group B_n whereby the definition of a group presentation by generators and relations as follows.

Definition 1. The Artin braid group B_n is the group generated by $n-1$ generators σ_i, $i = 1, \ldots, n-1$ and the braid relations $\sigma_i\sigma_j = \sigma_j\sigma_i$ for all $i, j = 1, 2, \ldots, n-1$ with $|i-j| \geq 2$, and $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$ for $i = 1, 2, \ldots, n-2$.

Suppose B_n contains σ_i^{2j}, $j = 1, 2, \ldots, k$. or a group B_2 is generated by a single generator σ_1 only without a relation such that the group B_2 contains σ_1^{2j}. Thus in any braid possible contains the cyclic i.e. $\sigma^2, \sigma^4, \ldots$, or there is an infinite cyclic group of braid.

Let σ^{2j} be basic pattern of the knots, or the closed braids based on σ^{2j}, so there are a normal knots, that is unprime knot, whereby a prime knot is a knot with one string. Thus, σ^{2j} generate the knots possible consist of two or more string. Based on the shape of the cross in braids and trivial knot (the circle) we get

$$\ell P(L_+) + \ell^{-1} P(L_-) + mP(L_0) = 0 \tag{1}$$

where $P(L)$ is a polynomial of oriented link, it has ℓ and m are two variables of integer coeficient, and L_+, L_-, and L_0 are three oriented links corresponding with overlink, underlink, and unlink [9]. For example, for knot s_2 based on pattern σ^2 Eq. (1) be

$$P(s_2) = \ell m^{-1} - \ell m + \ell^3 m^{-1} \tag{2}$$

3. An Approach

The set of numbers for triangular shapes has a triangular frame composition based on entries [1] like Fig. 1. This entry is based on a basic braid arrangement with a pattern $\sigma^{2j} \in B_2$, $j = 1, 2, \ldots, n$, i.e. a finite collection of disjoint curves in $\mathcal{R}^3$. Each closed curve is a component of link, and a link of one component of knot [5]. It is expressed as follows [9].

Definition 2. Let $P(L)$ as a unique way to determine a knot. P is a function as follows

$$P : \cup_n \mathcal{L}_n \to B[\ell^{\pm}, m^{\pm}]$$

where ℓ and m as $u = -(\ell + \ell^{-1})m^{-1}$, $\ell \in \cup_n \mathcal{L}_n$ such that $P(\alpha L) - u^{cL-1}$, L has component cL and B is braid.

Theorem 1. Laurent polynomial $P(L)$ determine a unique way so that equivalent oriented links have same polynomial if and only if $P(unknot) = 1$, and if L_+, L_-, L_0 are three oriented

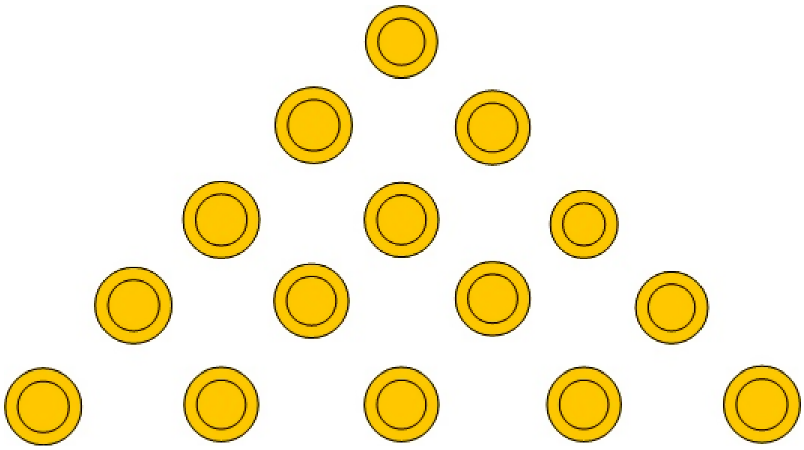

Figure 1. Frame of triangle number entries

link then Eq. (1).

Intuitively, a triple links of L_+, L_-, and L_0 are as two links and one unlink: First two links are the picture of unknot that have positive and negative crosses, respectively, but the last link as based on Theorem 1 is $\ell + \ell^{-1} + mP(L_0) = 0$. Thus, the last of triple links has

$$P(L_0) = -(\ell + \ell^{-1})m^{-1} \tag{3}$$

Thus, to derive the polynomial of a knot is based on Eq. (1), and we have a sequence of polynomial based on braid patterns σ^{2j}, and this inductively will build a general form of polynomial [10].

4. New Triangle Number

Based on Eq. (1) for each knot has triple link of basic patterns in braid σ^{2j}, $j = 1, \ldots, n$. Thus for σ^2 there are knot s_2 with triple links in braids as follows

$$s_2 \rightarrow (\sigma^0 = 1), (\sigma^2), (\sigma^1), \tag{4}$$

the knot s_4 has triple links as follows

$$s_4 \rightarrow (\sigma^2), (\sigma^4), (\sigma^3), \tag{5}$$

the triple links of knot s_6 be

$$s_6 \rightarrow (\sigma^4), (\sigma^6), (\sigma^5), \tag{6}$$

and so on, so for knot s_{2h} we have

$$s_{2h} \rightarrow (\sigma^{2(h-1)}), (\sigma^{2h}), (\sigma^{2h-1}). \tag{7}$$

see Table 1.

By using Theorem 1, we have $P(s_0) = P(unknot)$ like Eq. (3) or $P(s_0) = -\ell m^{-1} + \ell^{-1}m^{-1}$, while for $P(s_2)$ is Eq. (2),

$$P(s_4) = -\ell^3 m^{-1} + 3\ell^3 m - \ell^3 m^3 - \ell^5 m^{-1} + \ell^5 m \tag{8}$$

Table 1. Knot with two strings

Knot	$\to$	$+2$	$+1$	$+$
s_2	σ^0	σ^2	σ^1	2
s_4	σ^2	σ^4	σ^3	2
s_6	σ^4	σ^6	σ^5	2
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$2 \cdot \ldots$
s_{2h}	$\sigma^{2(h-1)}$	σ^{2h}	σ^{2h-1}	2

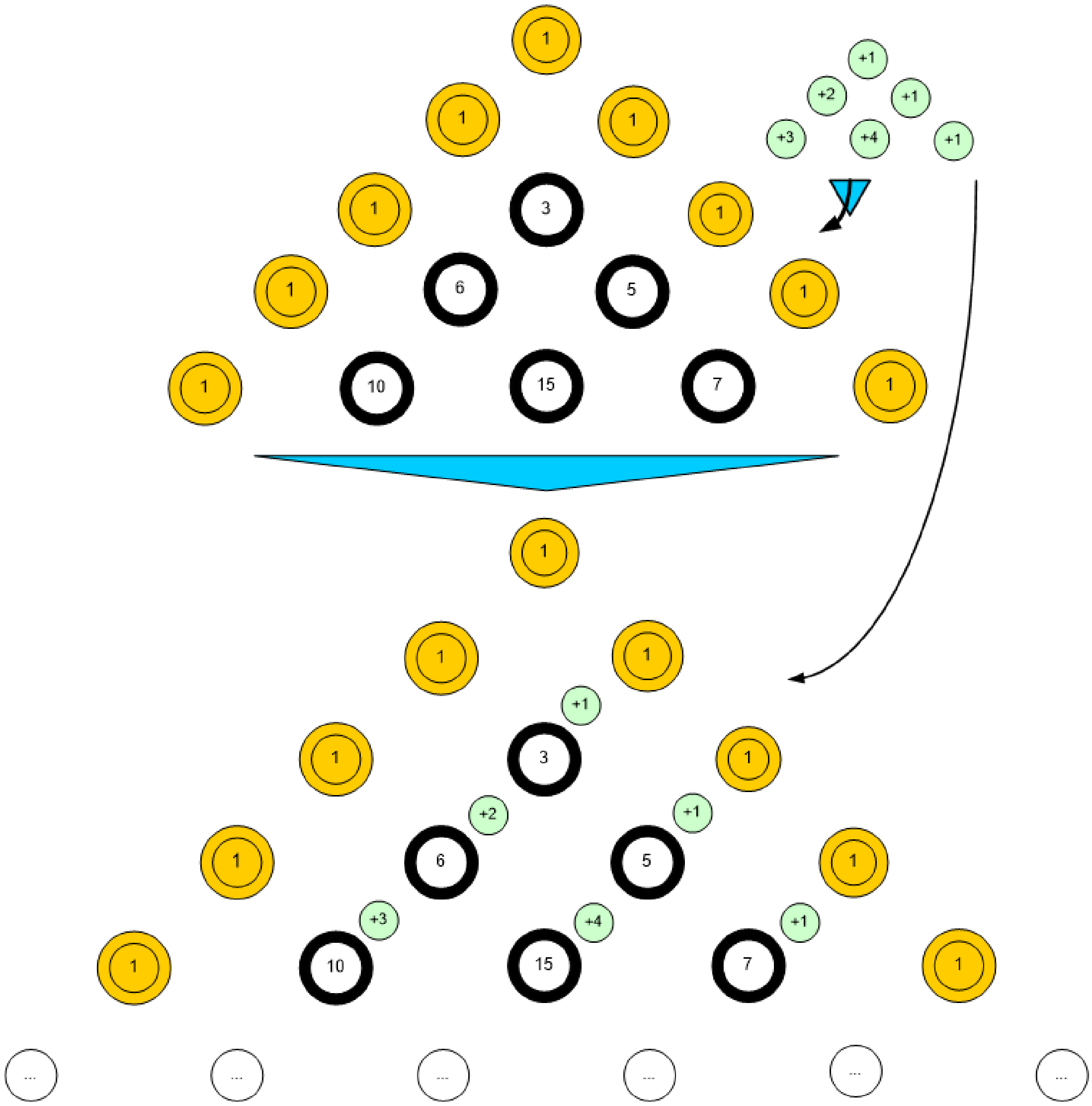

Figure 2. Triangle number of Laurent polynomials for σ_{2j}

and

$$P(s_6) = \ell^5 m^{-1} - 6\ell^5 m + 5\ell^5 m^3 - \ell^5 m^5 + \ell^7 m^{-1} - 3\ell^7 m + \ell^7 m^7. \tag{9}$$

Therefore, based on Eqs. (3), (2), (8), and (9) we have two composition of numbers, i.e. [−1], [−1]-[−1], [−1]-[3]-[−1], [−1]-[−6]-[5]-[−1] and [1], [1], [−1]-[1], [1]-[−3]-[1]. By ignoring the negative sign, the second composition follows the first composition pattern. Thus obtained the initial form of the triangle number as follows

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 3 & & 1 & \\ 1 & & 6 & & 5 & & 1 \end{array}$$

The outer numbers of the triangle numbers are 1, while the inner number varies: 3 come from $1+1$ and plus 1, 5 come from $1+3$ and plus 2, while 5 come from $1+3$ plus 1. Then if forwarded to the next line, $1 + 6$ along with 3 to 10, $6 + 5$ along with 4 to 15, and $5 + 1$ along with 1 to 7, and the next line item arrangement is $1, 10, 15, 7$ and 1. So there is a companion triangle number as follows

$$\begin{array}{ccccc} & & +1 & & \\ & +2 & & +1 & \\ +3 & & +4 & & +1 \end{array}$$

with the numbers on the next line are +4, +7, +5, and +1.Thus, generally the number triangle for knot s_{2h} as follows

$$P(s_{2h}) = \sum_{i=1}^{k} (-1)^{|i-k-1|} B_{hi} \ell^{2h-1} m^{2i-3} + \sum_{i=1}^{k-1} (-1)^{|i-(k-1)|} B_{(h-1)i} \ell^{2h+1} m^{2i-3} \tag{10}$$

see Fig. 2. Diagonal from up-left to down-right on right side has an addition value +1, diagonal from up-right to down-left on left side has an addition value +2 increasingly from +1, second diagonal from up-left to down-right on second right has addition value +2 increasingly from +2, while second diagonal from up-right to down-left on second left has addition value +3 increasingly from 2.

5. Conclusion

The triangle numbers of Laurent polynomials that typically illustrates the cyclic knot follows the generators pattern σ^{2j}. The triangle numbers forms certain structures based on the braid structure that regularly their weave increases. Besides these triangle numbers are different from other triangle numbers both from the composition and the structure, which is formed from two arrangement of triangle numbers that support each other's existence. The next study deals with the possibility of developing computationally to determine the number of more strings.

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