

Optimal control for Malaria disease through vaccination

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Abstract. Malaria is a disease caused by an amoeba (single-celled animal) type of plasmodium where anopheles mosquito serves as the carrier. This study examines the optimal control problem of malaria disease spread based on Aron and May (1982) SIR type models and seeks the optimal solution by minimizing the prevention of the spreading of malaria by vaccine. The aim is to investigate optimal control strategies on preventing the spread of malaria by vaccination. The problem in this research is solved using analytical approach. The analytical method uses the Pontryagin Minimum Principle with the symbolic help of MATLAB software to obtain optimal control result and to analyse the spread of malaria with vaccination control.

1. Introduction

Malaria is a disease caused by an amoeba (single-celled animal) type of plasmodium where anopheles mosquito serves as the carrier. There are four malaria parasites which can infect human body, *plasmodium falciparum*, *plasmodium vivax*, *plasmodium ovale*, and *plasmodium malariae*. Malaria disease at one place can be different from another place, depending on the existence of the infected people, the malaria vector, the parasites, and the environmental factors [1]. Malaria disease is a contagious disease which generally suffered by people living in tropical or subtropical area. The disease may also affects people who live in the swamp area or unhealthy environment. Fever, cold, and cold sweat may appear as the symptoms. The malaria symptoms typically begin a few days after the infected mosquito bite the patient. The infection period depends on some factors such as infection rate, method and time of treatment, and the immune due to drugs, and other biological factors.

The spread of malaria disease which can be out of control requires an effort to study the spread pattern of the disease. Mathematical model can be applied to help predict and control the epidemic in the future. The malaria transmission can be approximated by mathematical model and simulated by applying the real problem case. The result is expected to be beneficial to resolve the malaria spread. Epidemic model of malaria in a population can consists of three groups of subpopulation, which are susceptible, infected, and recovered groups. Susceptible is a group of healthy people with a possibility to be infected by malaria disease. Infected is a group of infected people with a possibility to be susceptible again. Recovered represents a group of people who is recovered from the malaria disease and become immune to the disease. This system is known as SIR type epidemic model [2, 3].



The spread of the disease can be controlled through administering vaccination. Vaccination is conducted by giving an antigen compound which functions to improve the body's immune to viruses or disease. To determine the best strategy in administering vaccine to a population of malaria epidemic, the optimal control theory can be applied. The objective of this control through vaccination is to minimize the number of infected individuals as well as the cost [4].

This study concerns on investigating the malaria disease spread by employing mathematical model proposed by Aron and May [2]. Here, the model will be further developed by adding a control to the recovered population. This control aims to decrease the number of infected population through administering vaccine while minimizing the amount and cost of the vaccination. The model will be solve analytically with the symbolic help of MATLAB software.

2. Optimal Control Model Formulation

The dynamic system which will be discussed here is formulated in the system of ordinary differential equations consisting of three dependent state variables with respect to time, $x(t)$, $y(t)$, and $z(t)$. Here, $x(t)$ represents the number of people in susceptible group, $y(t)$ represents the number of people in infected group, and $z(t)$ represents the number of people in recovered group. The recovered population is assume to developed a partial immune as the result of their body's reaction when they were infected by the disease and recovered. The diagram of the population transfer among these group is presented in figure 1. In figure 1, h and ρ respectively

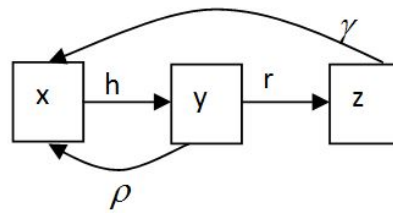


Figure 1. Malaria transmission diagram [2]

denote the transfer rates from susceptible to infected and from infected to susceptible. The transfer rates from infected to partially immune and from partially immune to the susceptible are respectively denoted by r and γ .

Ronald [5] suggested that the immune is achieved and developed over time, then there is the rate of immune development $r(t)$ with its initial value is defined as $r(0) = 0$. When a person is exposed to the disease, there is a delay for a person to achieve an immune. The immune tends to asymptotically decrease to a certain value r_∞ . Therefore, the rate of change of the immune system is presented as follow

$$\frac{dr}{dt} = \sigma t(r_\infty - r), \quad r(0) = 0. \quad (1)$$

If σ denotes twice of the rate, the equation (1) becomes $\frac{dr}{dt} = 2\sigma t(r_\infty - r)$. Hence, we have

$$r(t) = r_\infty(1 - e^{-\sigma t^2}).$$

Also, we define a control variable representing the amount of vaccine given to the recovered

population. Hence, the SIR model with the addition of control variable $u(t)$ is given as

$$\begin{aligned}\frac{dx}{dt} &= -hx + \rho y + \gamma z - ux, \\ \frac{dy}{dt} &= hx - \rho y - r_{\infty}(1 - e^{-\sigma t^2})y, \\ \frac{dz}{dt} &= r_{\infty}(1 - e^{-\sigma t^2})y - \gamma(h)z + ux.\end{aligned}\tag{2}$$

The parameter h represents the transfer rate from susceptible to infected, ρ is the transfer rate from infected to susceptible, γ is the transfer rate from recovered to susceptible, r_{inf} is the transfer rate from infected to recovered, and u is the amount of vaccine which is being administered where $u = \{u(t) : 0 \leq u \leq 1\}$.

Furthermore, we define an objective equation which minimizes the vaccination cost:

$$\min J(u) = \int_{t_0}^{t_f} y(t)^2 + u(t)^2 dt,\tag{3}$$

where t_0 is the initial time and t_f is the final time. The optimal control system will be solved to determine an optimal value of $u(t)$ such that $J(u(t)) = \min\{J(u(t)) : u \in U\}$ where $U = \{u(t) : 0 \leq u \leq 1, 0 \leq t \leq t_f\}$.

3. Results and Simulations

3.1. Analytical solution and simulation of malaria disease spread

The system (2) and (3) will be solved analytically. Here, we assign the parameters' values $h = 1.99$, $\rho = 0.074$, and $\gamma = 0.05$. Forming the Hamiltonian and applying the Pontryagin Minimum Principle yields the following optimal state and costate equations,

$$\begin{aligned}\frac{dx}{dt} &= 0.074y - 1.99x + 0.05z - 1.0\lambda_1x^2 + \lambda_3x^2, \\ \frac{dy}{dt} &= 1.99x - 0.187y, \\ \frac{dz}{dt} &= 0.113y - 0.05z + \lambda_1x^2 - 1.0\lambda_3x^2, \\ \frac{d\lambda_1}{dt} &= \lambda_1(x(\lambda_1 - 1.0\lambda_3) + \lambda_1x - 1.0\lambda_3x + 1.99) - 1.99\lambda_2 - 1.0(\lambda_1 - 1.0 * \lambda_3)(\lambda_1x - 1.0\lambda_3x) - \\ &\quad 1.0\lambda_3(x(\lambda_1 - 1.0\lambda_3) + \lambda_1x - 1.0\lambda_3x), \\ \frac{d\lambda_2}{dt} &= 0.187\lambda_2 - 0.074\lambda_1 - 0.113\lambda_3 - 1.0, \\ \frac{d\lambda_3}{dt} &= 0.05\lambda_3 - 0.05\lambda_1.\end{aligned}\tag{4}$$

Based on the equation (4), the optimal control solution will be determined to investigate the optimal strategy to control the transmission of the malaria disease.

Figure 2 displays that the number of the susceptible population decreases while the number of infected and recovered population increases over time. The optimal control also increases over time. The optimal control strategy obtained from the Hamiltonian's result for the population in the system is $x = 0.0001$, $y = 0.6958$ dan $z = 0.3041$. Therefore, the strategy solution for optimally control the system is by giving the amount of vaccination $u = 3173$ to the population in the system.

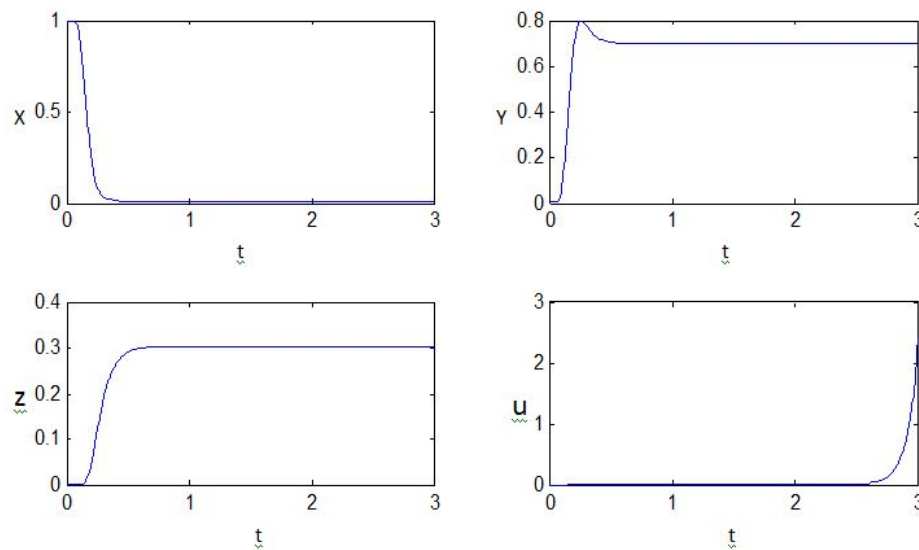


Figure 2. The graph of state and control variables for $h = 1.99, \rho = 0.074, \gamma = 0.05$.

3.2. Analytical solution and simulation for modified objective function with penalty

In this section, we modified the objective function by adding penalties to the infected and control variable in the performance index. The objective function with penalties can be written as

$$\min J(u) = \int_{t_0}^{t_f} \eta y(t)^2 + \frac{\alpha}{2} u(t)^2 dt, \quad (5)$$

where η is the infected state penalty and α is the vaccine administering control penalty. Here, the penalties' values should be chosen in the interval $(0, 1)$.

Some combinations of penalties' values are chosen to find the values which yield the best solution for the optimal control system. Figure 3 presents the optimal solution for $\eta = 0.9$ and $\alpha = 0.2$.

The rate of change of the population obtained from the model with penalties indicate a change with respect to time. Figure 3 displays that there are increase and decrease in the number of population over time in each group. The optimal control strategy obtained from this simulation is $x = 0.1109, y = 0.6864, z = 0.3136$. Therefore, the strategy solution for optimally control the system is by giving the amount of vaccination $u = 1198$.

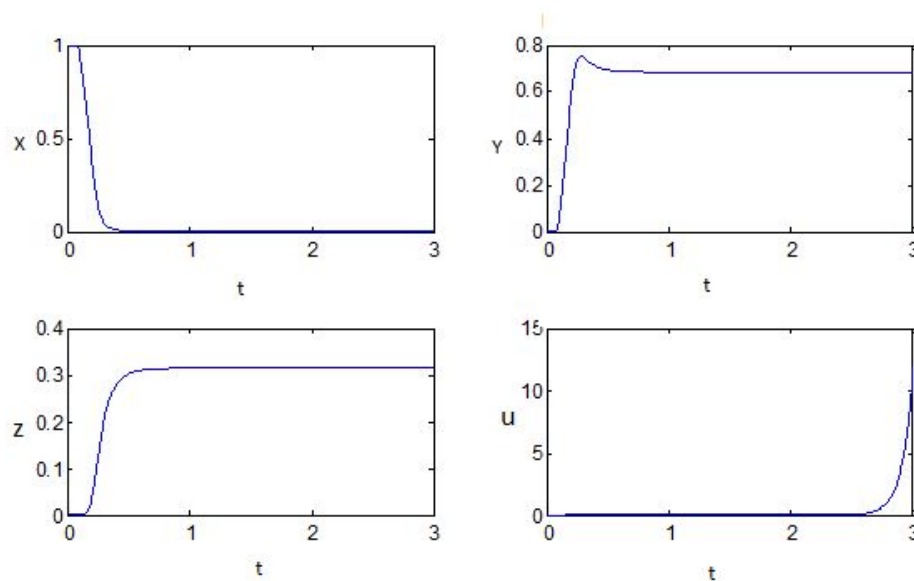


Figure 3. The graph of state and control variables for $h = 1.99, \rho = 0.074, \gamma = 0.05$ and penalties' values $\eta = 0.9, \alpha = 0.2$.

4. Conclusion

The transmission of malaria disease can be investigated through a mathematical model. To control the disease spread in a population, vaccine can be administered to population. We administered the vaccination variable to the recovered population and applied optimal control theory to minimize the amount of vaccine being administered and the cost. The addition of penalties to the objective function gives better value to the outcome. In this case the system does not only produces smaller amount of the vaccination but also gives the optimal solution for the system. The results of the simulations suggest that control strategy through the optimal control system with penalty is more effective than the one without penalty.

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