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Periodic Heterogeneous Vehicle Routing Problem With Driver Scheduling

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Abstract. The paper develops a model for the optimal management of logistic delivery of a given commodity. The company has different type of vehicles with different capacity to deliver the commodity for customers. The problem is then called Periodic Heterogeneous Vehicle Routing Problem (PHVRP). The goal is to schedule the deliveries according to feasible combinations of delivery days and to determine the scheduling of fleet and driver and routing policies of the vehicles. The objective is to minimize the sum of the costs of all routes over the planning horizon. We propose a combined approach of heuristic algorithm and exact method to solve the problem.

Keywords: Vehicle routing problem, scheduling, combined approach

1. Introduction

The classical Vehicle Routing Problem (VRP) is particularly intended to determine an optimal routing plan for a fleet of homogeneous vehicles such that to satisfy the need of a set of customers. As the root of this logistic problem is derived from the concept of Hamiltonian graph, it is necessarily to impose conditions that each vehicle route starts and ends at the starting point, and each customer is visited once by one vehicle. Since then many researchers have been working in this area to discover new variants and new methodologies. There are a number of survey can be found in literature for VRP, such as, [1], [2], [3], [4], and books [5], [6].

In literature there are some variants of VRP which are grouped according to specific constraints. Some of the well known variants are: Capacitated VRP (CVRP), the vehicles are restricted to carry limited capacity; VRP with time windows (VRPTW), each customer is served within a defined time frame; multiple depots VRP (MDVRP), in this variant goods can be delivered to a customer from a set of depots; VRP with pick-up and delivery (VRPPD), goods not only need to brought from the depot to the customers, but also must be picked-up at a number of customers and brought back to the depot. Taking into account several days of planning for routing problems is another variant of the VRP, known as the priodic VRP (PVRP).

In PVRP, within a given time horizon, there is a set of customers needs to be visited once or several times. There would be a visiting schedules associated with each customer. A fleet of vehicles is available and each vehicle leaves the depot, serves a set of customers, when its work shift or capacity is over, returns to the depot. The problem is to minimize the total length of the routes travelled by the vehicles on the time horizon. This problem is very important in real world applications, such as, distribution for bakery companies [7], blood product distribution [8], or pick-up of raw materials for a manufacture of automobile parts [9].

A survey on PVRP and its extensions can be found in [10]. Due to the complexity of the problem most of the works present heuristic approaches, nevertheless [11] proposed an exact method. [12] addressed a combined of heuristic and exact method for solving PVRP. The extension of the VRP in which one must additionally decide on the fleet composition is known as the

Heterogeneous Vehicle Routing Problem (HVRP) is rooted in the seminal paper of [13] published in 1984 and have recently evolved into a rich research area. There have also been several classifications of the associated literature from different perspectives. [14] provided a general overview of papers with a particular focus on lower bounding techniques and heuristics. The authors also compared the performance of existing heuristics described until 2008 on benchmark instances. [15] presented a review of exact algorithms and a comparison of their computational performance on the capacitated VRP and HVRPs, while [16] reviewed several industrial aspects of combined fleet composition and routing in maritime and road-based transportation. Afshar-Nadjafi and afshar-Nadjafi (2017) propose a constructive heuristic approach for solving HVRP time windows with multi-depot and time-dependent.

Note that all the literature survey of HVRP mentioned in the previous paragraph do not involve PVRP. This paper proposes a model of HVRP in which that the customer can be visited more than once, and there would be a schedule for visiting customers.

This paper is organized as follows. Section 2 describes the problem definition and the mathematical model. Section 3 presents the feasible neighbourhood heuristic search. The algorithm is described in Section 4. Finally Section 5 describes the conclusions

2. Problem Formulation

The decision of the problem is find the feasible transportation tour of a certain type of vehicle such that to minimize the overall operating costs. In order to ease the problem, it is necessary to visualize the problem using graph. Let $\Gamma = (\varsigma , A)$ be a directed graph, where $\varsigma = \{0, 1, ... , n\}$ is the vertex set and $A = \{(i, j) : i, j \in \varsigma , i \neq j\}$ is the set of arcs, representing routes between vertex. The depot vertex is indexed by 0. $\varsigma_c = \varsigma \setminus\{0\}$ is the set of customer locations. Each vertex $i \in \varsigma_c$ has a demand $q_i \geq 0$ on each day of the planning horizon, a service time $s_i \geq 0$, a time window $[e_i, l_i]$, where e_i is the earliest time service may begin and l_i is the latest time, and requires a fixed number of visits f_i to be performed according to one of the allowable visit-day patterns. The time window specifying the interval vehicles leave and return to the depot is given by $[e_0, l_0]$. Let $K = \{1,...,k\}$ be the set of available vehicle of all types. A fleet of m vehicles of each type, with capacity Q_k is based at the depot.

For each vehicle $k \in K$, let Q_k denote the capacity in weight. We assume the number of vehicles equals to the number of drivers. Denote the set of n customers (/nodes) by $N = \{1, 2, \ldots, n\}$. Denote the depot by $\{0, n+1\}$. Each vehicle starts from $\{0\}$ and terminates at $\{n+1\}$. Each customer $i \in N$ specifies a set of days to be visited . On each day, customer $i \in N$ requests service with demand of q_i^t in weight, within time window $[a_i, b_i]$. Note that, for the depot $i \in \{0, n+1\}$, we set $q_i^t = 0$. Denote the set of preferable vehicles for visiting customer i by K_i ($K_i \in K$) and the extra service time per pallet by e if a customer is not visited by a preferable vehicle.

This notations used are given as follows :

Set:

DI	The set of internal drivers,
DE	The set of external drivers,
D	The set of drivers $D = DL \cup DE$,
K	The set of vehicles,

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N The set of customers,
N_0 The set of customers and depot $N_0 = \{0, n + 1\} \cup N$,
K_i The set of preferable vehicles for customer $i \in N$,

Parameter:
M_k The available number of vehicles of type k
Q_k The weight capacity of vehicle $k \in K$,
$[a_i, b_i]$ The earliest and the latest visit time at node $i \in N_0$,
q_i The weight demand of node $i \in N_0$
$[g_l, h_l]$ The start time and the latest ending time of driver $l \in D$
α_i Pick up quantity for customer i
β_i Delivery quantity for customer i
CV^k Fixed cost for each type of vehicle $k \in K$ from depot
CVA_{ij}^k Travelling cost of vehicle $k \in K$ along the edge $(i, j) \in A$
CVN^k Cost due to the visit of a customer by non preferable vehicle $k \in K$
CDI_l^k Cost for internal driver $l \in DI$ using vehicle $k \in K$
CDE_l^k Cost for external driver $l \in DE$ using vehicle $k \in K$
CPQ_j^k Cost to pick up a number of quantity for customer $j \in N$ using vehicle $k \in K$
CDD_j^k Cost to deliver a number of quantity for customer $j \in N$ using vehicle $k \in K$

Variables:
x_{ij}^k Binary variable indicating whether vehicle $k \in K$ travels from node $i \in N_0$ to $j \in N_0$
w_i Binary variable indicating whether customer $i \in N_0$ is visited by a non-preferable vehicle
v_i^k The time at which vehicle $k \in K$ starts service at node $i \in N_0$
y_l^k Binary variable indicating whether vehicle $k \in K$ is assigned to internal driver $l \in DI$
z_l^k Binary variable indicating whether vehicle $k \in K$ is assigned to external driver $l \in DE$
θ_j^k Number of pick up demand of customer j served by vehicle $k \in K$
σ_j^k Number of delivery demands of customer j served by vehicle $k \in K$

The mathematical formulation for this problem is presented as follows:

$$\begin{aligned}\text{Min} \sum_{k\in K}\sum_{j\in N_0} CV^k x_{oj}^k + \sum_{k\in K}\sum_{(i,j)\in A} CVA_{ij}^k x_{ij}^k + \sum_{i\in N_0}\sum_{k\in K} CVN^k w_i^k + \sum_{l\in DI}\sum_{k\in K} CDI_l^k y_l^k + \\ \sum_{l\in DE}\sum_{k\in K} CDE_l^k z_l^k + \sum_{j\in N}\sum_{k\in K} CPQ_j^k \theta_j^k + \sum_{j\in N}\sum_{k\in K} CDD_j^k \sigma_j^k\end{aligned} \tag{1}$$

Subject to

$$\sum_{j \in N_0} x_{0j}^{k} = 1, \qquad \forall k \in K \tag{2}$$

$$\sum_{k \in K} \sum_{j \in N} x_{ij}^{k} = 1, \qquad \forall i \in N_0 \tag{3}$$

Constraints (2) and (3)) are to ensure that exactly one vehicle of each type enters and departs from every customer and from the central depot.and comes back to the depot.

$$\sum_{i \in N} x_{ij}^{k} - \sum_{i \in N} x_{ji}^{k} = 1; \qquad \forall j \in N_0, \forall \mathrm{k} \in \mathrm{K} \tag{4}$$

A flow conservation equation is necessarily needed to maintain the continuity of each vehicle route on each period of time. This equation is presented in Constraints (4).

$$\sum_{j \in N_0} x_{0j}^{k} \le M_k, \qquad \forall k \in K \tag{5}$$

Constraint (5) represents that each customer is served only by the available and active vehicle of type m.

$$\sum_{j \in N_0} x_{1j}^{k} \le 1; \qquad \forall k \in K \tag{6}$$

$$\sum_{i \in N, i > 1} x_{i1}^{k} \le 1; \qquad \forall k \in K \tag{7}$$

Constraints (6) and (7) state the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively.

$$\sum_{i \in N} q_i \sum_{j \in N_0} x_{ij}^{k} \le Q_k; \qquad \forall k \in K_m \tag{8}$$

Constraints (8) ensure that each delivery does not exceed the capacity of each type of vehicle.

$$\sum_{k \in K} \sum_{j \in N_0} x_{ijk}^{t} = 1 \qquad \forall\ i \in N, t \in T_i \tag{9}$$

$$\sum_{k \in K \setminus K_i} \sum_{j \in N_0} x_{ijk}^{t} = w_i^{t} \qquad \forall\ i \in N, t \in T_i \tag{10}$$

$$\sum_{i \in N} \sum_{j \in N_0} q_i^{t} x_{ijk}^{t} \le Q_k \qquad \forall\ k \in K, t \in T \tag{11}$$

$$b_i \ge v_{ik}^{t} \ge a_i \qquad \forall i \in N,\ k \in K, t \in T_i \tag{12}$$

$$v_{0k}^t \geq \sum_{l \in D} (g_l^t \cdot y_{lk}^t) \qquad \forall k \in K, t \in T \tag{13}$$

$$v_{n+1,k}^t \leq \sum_{l \in D} (h_l^t \cdot y_{ik}^t) \qquad \forall k \in K, t \in T \tag{14}$$

$$\sum_{k \in K} \theta_{jk}^t = \alpha_j^t \qquad \forall j \in N, t \in T \tag{15}$$

$$\sum_{k \in K} \sigma_{jk}^t = \beta_j^t \qquad \forall j \in N, t \in T \tag{16}$$

$$x_{ijk}^t, w_i^t, z_{ik}^t, y_{lk}^t \in \{0,1\} \qquad \forall i, j \in N_0, l \in D, \; k \in K, t \in T \tag{17}$$

$$v_{ik}^t, u_{ik}^t, r_l^t, s_l^t \geq 0 \qquad \forall i, j \in N_0, l \in D, \; k \in K, t \in T \tag{18}$$

$$\theta_{jk}^t, \sigma_{jk}^t \in \{0,1,2,...\} \qquad \forall j \in N, k \in K, t \in T \tag{19}$$

Constraints (11) state that each customer must be visited by one vehicle on each of its delivery days. Constraints (12) define whether each customer is visited by a preferable vehicle. Constraints (13-14) guarantee that the vehicle capacities are respected in both weight and volume.

Constraints (15-16) define the elapsed driving time. More specifically, for the vehicle (k) travelling from customer i to j on day t, the elapsed driving time at j equals the elapsed driving time at i plus the driving time from i to j (i.e., $u_{jk}^t \geq u_{ik}^t + c_{ij}$) if the vehicle does not take a break at customer i (i.e., $z_{ik}^t = 0$); Otherwise, if the vehicle takes a break at customer i (i.e., $z_{ik}^t = 1$), the elapsed driving time at j will be constrained by (10) which make sure it is greater than or equal to the travel time between i and j (i.e., $u_{jk}^t \geq c_{ij}$). Constraints (17) guarantee that the elapsed driving time never exceeds an upper limit F by imposing a break at customer i (i.e., $z_{ik}^t = 1$) if driving from customer i to its successor results in a elapsed driving time greater than F.

Constraints (18) determine the time to start the service at each customer. If j is visited immediately after i, the time v_{jk}^t to start the service at j should be greater than or equal to the service starting time v_{ik}^t at i plus its service duration d_i^t, the extra service time $e \cdot p_i^t$ if i is visited by an inappropriate vehicle (i.e., $w_j^t = 1$), the travel time between the two customers cij , and the break time G if the driver takes a break after serving I (i.e., $z_{ik}^t = 1$). Constraints (19) make sure the services start within the customers' time window.

Constraints (20-21) ensure that the starting time and ending time of each route must lie between the start working time and latest ending time of the assigned driver. Constraints (22) calculate the total travel time for each internal driver. Constraints (23) define the working duration for each driver on every workday, which equals the time the driver returns to the depot minus the time he/she starts work. Constraints (24) make sure that the internal drivers work for no more than a maximum weekly working duration, referred to as 37 week-hour constraints. Constraints (25 – 26) define the pick up and delivery for each customer. Constraints (27-29) define the binary and positive variables used in this formulation.

3. Proposed Method

Constraints (13) – (15) express the nature of the variables used in the model.
Let

$$x = [x] + f, \quad 0 \leq f \leq 1$$

be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable x and f is the fractional component.

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Stage 1.

Step 1. Get row *i** the smallest integer infeasibility, such that $\delta_{i*} = \min\{f_i, 1 - f_i\}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i*}^T = e_{i*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i*}^T \alpha_j$

With *j* corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic *j* at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic *j* (if available). Eventually the column *j** is to be increased form LB or decreased from UB. If none go to next *i**.

Step 4.

Solve $B\alpha_{j*} = \alpha_{j*}$ for α_{j*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic *j** from its bounds.

Step 6. Exchange basis

Step 7. If row *i** = {∅} go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly lovalized neighbourhood search to verify local optimality.

4. Conclusions

This paper was intended to develop efficient technique for solving one of the most economic importance problems in optimizing transportation and distribution systems. The aim of this paper was to develop a model of Periodic vehicle Routing with Time Windows, Fleet and Driver Scheduling, Pick-up and Delivery Problem This problem has additional constraint which is the limitation in the number of vehicles. The proposed algorithm employs nearest neighbor heuristic algorithm for solving the model. This algorithm offers appropriate solutions in a very small amount of time.

References

[1] O. Bräysy, & Gendreau, M., 2005. *Vehicle Routing Problem with Times Windows, Part II: Metaheuristics*. Transportation Science, **39(1),** pp. 104-118.

[2] Cordeau, J. F., Laporte, G., Savelsbergh, M.W.F., & Vigo, D., 2007. *Vehivle Routing.* In Barnhart, C. and Laporte, G., edotirs, *Transportation*, handbooks in Operation Research and Management Science, pp. 367-428. North-Holland, Amsterdam.

[3] Golden, B. L., Assad, A. A., & Wasil, E. A., 2002. *Routing Vehicles in the real world: Applications in the solid waste, beverage, food, dairy, and newspaper industries.* In P. Toth and D. Vigo, editors, *The Vehicle Routing Problem*, pp. 245-286. SIAM, Philadelphia, PA.

[4] Laporte G., & Sement, F., 2002. *Classical Heuristics for the Vehicle Routing Problem.* In Toth, P. and Vigo, D., editors, *The vehicle Routing Problem*, SIAM Monographs on Discrete Matheamtics and Applications, pp. 109-128. SIAM, Philadelphia, PA.

[5] Golden, B. L., Raghavan, S., &. Wasil, E. A., 2008. *The vehicle routing problem: latest advances and new challenges.* Springer.

[6] Toth, P., & Vigo, D., 2002. *The Vehicle Routing Problem Society for Industrial and Applied Matheamtics.* Philadelpiha, PA.

[7] Pacheco, J., Alvarez, A., García, I., & Angel-Bello, F., 2011. *Optimizing vehicle routes in a bakeri company allowing flexibility in delivery dates. Journal of Operationsl Research Society* http://dx.doi.org/10.1057/jors.2011.51, 2011.

[8] Hemmelmayr, V., Doerner, K., Hartl, R., & Savelsbergh, M., 2009. *Delivery strategis for blood products supplies. OR Spectrum* **31 (4)** pp. 707-725.

[9] Alegre, J., Laguna, M., & Pacheco, J., 2007. *Optimizing the periodic pick-up for raw materials for a manufacturer of auto parts.* European Journal of Operational Research **179**, pp. 736-746, 2007.

[10] Francis, P., Smilowitz, K., & Tzur, M., 2008. *The period vehicle routing problem and its extensions, in : The Vehicle Routing Problem: Latest Advances and New Challenges.* Vol. 43, Springer, 2008 (Chapter).

[11] Baldacci, R., BArtolini, E., Mingozzi, A., & Valletta, A., 2011. *An Exact algorithm for the period routing problem.* Operations research **59 (1)**, pp. 228-241.

[12] Cacchiani, V., Hemmelmayr, V. C., & Tricoire, F., 2012. *A set-covering based heuristic algorithm for the periodic vehicle routing problem.* Discrete Applied Matehametics , doi:10.1016/j.dam.2012.08.032, 2012.

[13] Golden, B., Assad, A., Levy, L., & Gheysens, F. G., 1984. *The fleet size and mix vehicle* routing problem. Computers and Operations Research, **11,** pp. 49–66.

[14] Baldacci, R., Battarra, M., & Vigo, D., 2008. *Routing a heterogeneous fleet of vehicles.* In B. L. Golden, S.Raghavan, E. A. Wasil, eds. The Vehicle Routing Problem: Latest Advances and New Challenges. pp.1–25. Springer, New York.

[15] Baldacci, R., Toth, P., & Vigo, D. 2010a. Exact algorithms for routing problems under vehicle capacity constraints.Annals of Operations Research 175, pp. 213–245

[16] Hoff, A., Andersson, H., Christiansen,M., Hasle, G., & Løkketangen, A., 2010. *Industrial aspects and literaturesurvey: fleet composition and routing.* Computers & Operations Research 37, pp. 2041–2061.

[17] Afshar-Nadjafi B and Afshar-Nadjafi A. 2017. A constructive heuristic for time-dependent multi depot vehicle routing problem with time windows and heterogeneous fleet. *Journal of King Saud University – Engineering Sciences* 29, 29-34.