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Developing a feasible neighbourhood search for solving hub location problem in a communication network

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Abstract. The hub location with single assignment is the problem of locating hubs and assigning the terminal nodes to hubs in order to minimize the cost of hub installation and the cost of routing the traffic in the network. There may also be capacity restrictions on the amount of traffic that can transit by hubs. This paper discusses how to model the polyhedral properties of the problems and develop a feasible neighbourhood search method to solve the model.

Keywords: Communication network, hub location, MILP, neighborhood search.

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1. Introduction

Hub facilities can be found in most of the transportation and communication networks. The main role of hubs for this sort of networks usually act as a central point for distribution of communication services. The hub facility will distribute most types of communications services.

Hub location problems arise in all network design, particularly when it is necessary to distribute services. Due to the nature of a location problem, the aim is to find the location of hub nodes (location point) and the allocation of service nodes to these located hub nodes. In standard location problems when the locations of the facilities are determined then logically, each customer node receives service from its nearest facility. However, for hub location problems, it is not necessary to assign each demand node to its nearest hub will give optimal solutions. Therefore the optimal allocations of demand centers to the located hubs must be determined. In term of applications rather than in telecommunication, the hub location problem can also be found in public transportation [1]. parcel delivery [2], [3], and logistic systems [4]. Interesting review for the application of hub location problem can be found in [5] and [6].

This paper considers hab location in telecommunication network. Communication networks are built and designed in order to distribute resources. With the advent of the information age there has been increased interest in the efficient design of communication networks. Network design problems occur in many field of communication, ranging from the old style analogue telephone networks to more modern applications such as wide - area and local – area computer networks, ISDN networks for multi – media transmission, and digital cellular networks for mobile phone communications. They all have a similar intent: how to carry the expected traffic flow from origin to destination at minimum cost. The most important part of network design is to find the best way to layout the components (nodes and arcs) to minimize cost while meeting performance criterion such as transmission delay, throughput or reliability.

If one imagines the communication requirement as a set of nodes representing origin and destinations, the primary problem is to join the nodes together in the most efficient manner. This is not an easy problem; for event a small number of nodes there are many thousands of ways of joining them together. Therefore this type of problem is termed NP – *complete* [7].

Given that we cannot investigate all possible ways of linking the nodes, the challenge is to devise a method for investigating a restricted number that nevertheless obtains a good solution. Further more, communication network design problems are not time critical. Meaning that, most approaches which have been designed to solve these problems based on heuristics, such as, simulated annealing, taboo search, evolutionary computing [8], [9]. [10] proposed a hybrid heuristic based on simulated annealing, tabu list, and improvement procedures for the uncapacitated hub location problem with single allocation. [11] developed a tabu search meta-heuristic for the multiple allocation p-hub median problem. [12] presented a hybrid genetic algorithm embedded with a diagonalization method for the intermodal hub-and-spoke network design problem with multiple stakeholders and multi-type containers. [13] developed two different meta-heuristic procedures that consist of two phases: a solution construction phase and a solution improvement phase based on local search for the intermodal terminal location problem. [14] provided a review for the most recent advances of solution methodologies for the hub location problem

Exact methods are guaranteed to achieve proven optimal solutions under sufficient computation time. [15] proposed a branch-and-cut algorithm for the hub location problem with single assignment. [16] developed a Benders decomposition algorithm for the uncapacitated hub location problem with multiple assignments in telecommunication and transportation systems. [17] proposed a branch-and-price algorithm for the capacitated hub location problem with single assignment, in which Lagrangean relaxation is used to obtain tight lower bounds of the restricted master problem. [18] presented an exact algorithm capable of solving large-scale instances of the uncapacitated hub location problem with multiple assignments. The algorithm applies Benders decomposition to a strong path-based formulation of the problem. [19] proposed a Benders decomposition method for the tree of hub location problems. The exact methods are efficient at solving relatively small problems. However, they are limited on thoroughly searching all possible branches and finding an optimal solution under a reasonable amount of computational time when the problem size is particularly large.

In many applications however, it is necessary to add redundancy into the network to ensure reliability; if one link fails it would still be possible to connect from origin to destination. We have used a combination of linear Programming (LP) and heuristic based on neighbournood search approach to tackle this problem; and these approaches are discussed in Sections 4 and 5.

2. The Network Design Problem

The design problem is to satisfy all traffic requirements at minimum cost. For a tree network synthesis problem involving n nodes, there are n^{n-2} possible tree structures, e.g. one hundred million possibilities for a network as small as 10 nodes. The information required to formulate the problem is the traffic demand between each origin and destination (O – D) pair, and the linier cost function for carrying traffic on each (possible) link (i,j) between nodes i and j.

In order to model the problem, we define first the following notation.

Sets and Indices

N Set of nodes,
T Set of time periods.
M_k Set of modules with different types available for a hub located at node $k \in N$

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Traffic communication flow

f_{ij}^t Flow from node $i \in N$ that will go to node $j \in N$ at time period $t \in T$

TFE_i^t Total flow emerged from node $i \in N$ at time period $t \in T$

TFD_j^t Total flow for node $j \in N$ at time period $t \in T$

Capacities

C_{mk} Capacity of module type $m \in M_k$ available for node $k \in N$

Costs

α_k^t Setup cost to locate a hub at node $k \in N$ in period $t \in T$

β_{kj}^t Costs to operate a hub li9nk between hubs $k \in N$ and $j \in N$ in period $t \in T$

δ_{mk}^t Cost for installing a module of type $m \in M_k$ at hub $k \in N$ in period $t \in T$

γ_k^t Cost to operate per unit of flow for hub $k \in N$ in period $t \in T$

σ_{ij}^t Cost for sending flow from node $i \in N$ to node $j \in N$ in period $t \in T$

Decision variables

x_{ij}^t Binary variable indicating whether there is a link between node $i \in N$ to hub $j \in N$ in period $t \in T$

z_{kj}^t Binary variable indicating whether a hub link between hubs $k \in N$ and $j \in N$ in period $t \in T$

v_{km}^t Binary variable indicating whether a module of type $m \in M_k$ is installed at hub $k \in N$ in period $t \in T$

y_{ijk}^t Amount of communication flow from node $i \in N$ routed from hub $k \in N$ to hub $j \in N$ in period $t \in T$.

3. The Model

The objective of the communication network problem such that to find the hub location is to minimize the overall costs. From reality it can be understood that the costs which could be involved are as follows: cost to link a node to a hub, cost to install a module, cost to locate a hub and cost relating to the amount of communication flow.. Mathematically the objective can be expressed as follows.

$$
\begin{aligned}
Minimize & \sum_{t \in T} \sum_{i \in N} \alpha_t^k (x_{ii}^t - x_{ii}^{t-1}) + \sum_{t \in T} \sum_{k \in N} \sum_{j \in N, j>k} \beta_{kj}^t z_{kj}^t + \sum_{t \in T} \sum_{k \in N} \sum_{m \in M_k} \delta_{mk}^t v_{mk}^t \\
& + \sum_{t \in T} \sum_{k \in N} \gamma_k^t \Big(\sum_{i \in N} TFE_i^t x_{ik}^t + \sum_{i \in N} \sum_{j \in N} y_{ijk}^t\Big) + \sum_{t \in T} \sum_{i \in N} \sum_{j \in N} \sigma_{ij}^t x_{ij}^t \\
& + \sum_{t \in T} \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sigma_{ij}^t y_{ijk}^t + \sum_{t \in T} \sum_{i \in N} \sum_{j \in N} \sigma_{ij}^t TFD_j^t x_{ij}^t
\end{aligned} \tag{1}
$$

There are constraints imposed for the decision variables should be considered. The restrictions are formulated in the following equations.

$$\sum_{j\in N} x_{ij}^{t} = 1, \ \ \forall i \in N, \forall t \in T \tag{2}$$

$$x_{ij}^{t} \le x_{jj}^{t}, \ \ \forall i \in N, \forall t \in T \tag{3}$$

$$x_{jj}^{t} \le x_{jj}^{t-1}, \ \ \forall j \in N, \forall t \in T \tag{4}$$

$$\sum_{m\in M_k} v_{km}^{t} \le x_{kk}^{t}, \ \forall k \in N, \forall t \in T \tag{5}$$

$$\sum_{i\in N} TFE_i^t \, x_{ik}^{t} + \sum_{j\in N} \sum_{i\in N} y_{ijk}^{t} \le \sum_{m\in M_k} C_{km} \, v_{km}^{t}, \ \forall k \in N, \forall t \in T \tag{6}$$

$$\sum_{j\in N, j\ne k} y_{kji}^{t} - \sum_{j\in N, j\ne k} y_{jki}^{t} = TFD_i^t x_{ik}^{t} - \sum_{j\in N,} f_{ij}^{t} \, x_{jk}^{t}, \ \ \forall i,k \in N, \forall t \in T \tag{7}$$

$$z_{kj}^{t} \le x_{kk}^{t}, \ \ \forall k,j \in N, k < j, \forall t \in T \tag{8}$$

$$z_{kj}^{t} \le x_{jj}^{t}, \ \ \forall k,j \in N, k < j, \forall t \in T \tag{9}$$

$$y_{ijk}^{t} + y_{ikj}^{t} \le TFE_i^t z_{kj}^{t}, \ \ \forall i,j,k \in N, k < j, \forall t \in T \tag{10}$$

$$x_{ij}^{t}, z_{kj}^{t}, v_{km}^{t} \in \{0,1\}, \ \forall i,j,k \in N, \forall m \in M_k, \forall t \in T \tag{11}$$

$$x_{ii}^{0} = 0, \forall i \in N \tag{12}$$

$$y_{ijk}^{t} \ge 0, \forall i,j,k \in N, \forall t \in T \tag{13}$$

It can be observed that the model is in the form of large scale mixed integer linear programms.

4. A Suboptimal Search

In integer programming the reduced gradient vector, which is normally used to detect an optimality condition, is not available, even though the problems are convex. Therefore it is necessary to impose a certain condition for the local testing search procedure in order to assure that we have obtained the "best" suboptimal integer feasible solution.

A quantity test to replace the pricing test for optimality in the integer programming problem was proposed by Scarf [18]. The test is conducted by a search through the neighbours of a proposed feasible point to see whether a nearby point is also feasible and yields an improvement to the objective function. However, the result obtained is just a feasible suboptimal solution.

Let $[x_k]$ be the integer feasible point which satisfies the neighborhood conditions $[x_k] + 1 \in N([x_k])$. We could then say if $[x_k] + 1 \in N([x_k])$ implies that the point $[x_k] + 1$ is either infeasible or yields an inferior value to the objective function obtained with respect to $[x_k]$. In this case $[x_k]$ is said to be an "sub-optimal" integer feasible solution to the integer programming problem.

5. The Algorithm

After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let

$$x = [x] + f, \quad 0 \leq f \leq 1$$

be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable x and f is the fractional component.

Stage 1.

Step 1. Obtain $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Find $v_{i^*}^T = e_{i^*}^T B^{-1}$

Step 3. Calculate the maximum movement of nonbasic variable $\sigma_{ij} = v_{i^*}^T a_j$

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB.

If none go to next i^*.

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Do integer lines search to improve the integer feasible solution

6. Conclusions

The approach described in this paper has been demonstrated to be efficient on a limited range of moderately sized problem. However, the success of these investigations lends hope that it will prove to provide a valuable tool for communication network design. An advantage of the approach described here over other heuristic approaches is the ability to terminate the procedure early and still be assured of a reasonably good solution which still satisfies feasibility requirement.

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