

## Software-in-the-loop simulation of a quadcopter portion for hybrid aircraft control

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**Abstract.** In this paper, we present the design of the software-in-the-loop simulation framework for a quadcopter that is incorporated in our hybrid aircraft. The hybrid aircraft comprises a quad-copter and a fixed wing with one forward thrust rotor. We need to develop a split control system that utilizes a typical quadcopter controller to control four motors/propellers and a supervisor controller to control a forward thrust rotor. The supervisor controller shall take feedback signals from the quadcopter and will command the fifth rotor for stabilizing the hybrid aircraft and resolves problems like thrust saturation. The simulation simulates the control algorithm and verifies the quadcopter's behavior using MATLAB and Simulink together. Achieving these results, we come to know how our hybrid controller will be implemented, what results to expect once the forward thrust rotor is attached to the quadcopter. The software-in-the-loop simulation of a quadcopter is one of the most effective methods for verifying overall control performance and safety of the hybrid aircraft before actual hardware implementation and flight test.

### 1. Introduction

Hybrid aircrafts are unique type of aircrafts that combine the hover capability of helicopters or quadcopters along with the speed and range of airplanes. Computational tools presently are a prime source of saving efforts, money and time. Artificial stability can be given to aircraft via such control systems. Stability and maneuverability have an inverse effect to each other. In this article we present the modular control of a quadcopter portion of a hybrid aircraft. The aircraft comprises a standard quadcopter, a fixed wing and one forward thrust rotor. A mathematical model is used for modeling the simulations of quadcopter control part of the whole system. Figure 1 shows a prototype model for the hybrid aircraft. The purpose for designing the quadcopter controller is to obtain Thrust Percentage values in relation to the motor speed of each motor of the quadcopter. The basic idea is to have a supervisor controller, which takes the motion command, and the necessary data from a standard quadcopter controller and in result controls the forward thrust rotor. The reason for having a control for a forward thrust motor is to deal with problems like thrust saturation, and a better stability for the hybrid aircraft. Initially the mathematical model for the whole system is developed.





**Figure 1:** Hybrid Aircraft comprising a quad-copter, fixed wing and a forward thrust rotor

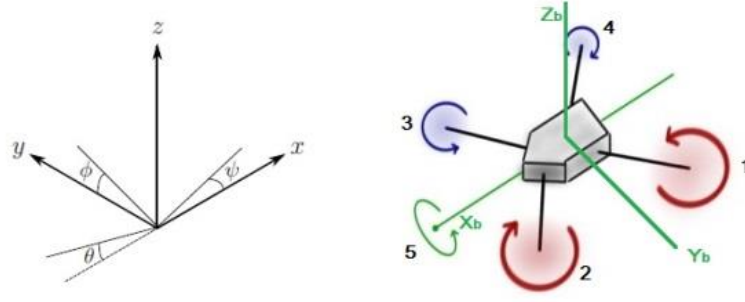
For the successful progress of our research, we carried out a detailed literature review of the previous works related to our project. A detailed summary of the literature review is discussed in section 1.1.

### *1.1. Literature Review*

One type of very basic UAVs (Unmanned Air Vehicles) is quadcopters. Starting with dynamics of UAVs, the research study conducted on Robotics and Automation by Marcus, Heinrich and Paolo [1] mainly explains the concept of a quadrotor UAV with tilting propellers. Another similar research article by Pounds and Paul from Australian National University [2] has defined the progress towards realizing practical quadrotor robot helicopters and, in particular, the conceptual design of Australian National University's 'X-4 Flyer' platform. Two challenges faced by the 'X-4' were generating sufficient thrust and managing unstable dynamic behavior. They have addressed these issues with a rotor design technique for maximizing thrust and the application of a novel rotor mast configuration. Relating to kinematics of UAVs, the research article presented by Dongming, Cai, Jorge and Lakmal from Institute Khalifa University UAE [3] considered the work on attitude control of quadrotor UAVs. They applied an intuitive kinematic representation, called the rotation vector. Based on the property that the rotation vector rate is equivalent to the body angle velocity when the rotation is small, a simple and intuitive attitude reference is proposed. A proportional-derivative (PD) law was used by integrating the new attitude reference for the attitude control of quad-rotor UAVs. Along with considerable knowledge of kinematics and dynamics of quadrotor vehicles, we proceeded further with the work on control method and control theory. The work by Shakev, Andon, Kaynak and Shiev on stabilization of quadrotor using feedback controllers [4] showed that five different types of nonlinear feedback laws with saturation elements, previously proposed for global control of systems with multiple integrators, are applied and tested to control the quad-rotor rotorcraft's roll and pitch angles. Moving forward with the control theory, the research by Adriano on SIL (Software In Loop) simulation using MATLAB (Simulink) and X-plane [5] introduced a new method to simulate a guidance algorithm running on Simulink that controls a fixed wing unmanned aircraft model running on the flight simulator X-Plane, which simulates the vehicle dynamics, sensors, and actuators.

## **2. Mathematical Model**

A mathematical model of the aircraft was identified and constructed. It is then implemented in the MATLAB Simulink environment, to be tested as a simulation. After verifying the correctness of the model, the control algorithms are crafted to control a simulated aircraft. The mathematical model of the hybrid aircraft has been derived from the quadcopter dynamics and control [6-7], and the thrust vector in forward direction (x-axis) has been added from the fifth propeller. Figure 2 shows the inertial and body axes of the hybrid aircraft along with the propeller numbering.



**Figure 2.** Inertial and body axes

Before delving into the physics of quadcopter motion, the kinematics in the body and inertial frames are formalized. We define the position and velocity of the quadcopter in the inertial frame as  $\mathbf{x} = (x, y, z)^T$  and  $\dot{\mathbf{x}} = (\dot{x}, \dot{y}, \dot{z})^T$ , respectively. Similarly, we define the roll, pitch, and yaw angles in the body frame as  $\theta = (\phi, \theta, \psi)^T$  with corresponding angular velocities equal to  $\dot{\theta} = (\dot{\phi}, \dot{\theta}, \dot{\psi})^T$ . However, note that the angular velocity vector is  $\boldsymbol{\omega} \neq \dot{\theta}$ . The angular velocity is a vector pointing along the axis of rotation, while  $\dot{\theta}$  is just the time derivative of yaw, pitch, and roll. In order to convert these angular velocities into the angular velocity vector, we can use the following relation:

$$\boldsymbol{\omega} = \begin{Bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{Bmatrix} \dot{\theta} \quad (1)$$

where  $\boldsymbol{\omega}$  is the angular velocity vector in the body frame.

We can relate the body and inertial frame by a rotation matrix  $\mathbf{R}$  which goes from the body frame to the inertial frame. This matrix is derived by using the Euler angle conventions and successively “undoing” the yaw, pitch, and roll.

$$\mathbf{R}_B^I = \begin{Bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\psi\sin\theta + \cos\phi\cos\psi & \cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{Bmatrix} \quad (2)$$

In order to properly model the dynamics of the system, we need an understanding of the physical properties that govern it. We will begin with a description of the motors being used for our hybrid aircraft, and then use energy considerations to derive the forces and thrusts that these motors produce on the entire quadcopter. All motors on the hybrid aircraft are identical, so we can analyze a single one without loss of generality. Note that adjacent propellers, however, are oriented opposite to each other; if a propeller is spinning “clockwise,” then the two adjacent ones will be spinning “counter-clockwise,” so that torques are balanced if all propellers are spinning at the same rate.

Considering that our major component of solution is thrust, we already know that brushless motors are used for all hybrid copter applications. Hence, solving for the thrust magnitude  $T$ , we obtain that thrust is proportional to the square of angular velocity of the motor:

$$\mathbf{T} = \left( \frac{K_v K_T \sqrt{2\rho A}}{K_T} \boldsymbol{\omega} \right)^2 = k\boldsymbol{\omega}^2 \quad (3)$$

where  $k$  is some appropriately dimensioned constant,  $K_T$  is the torque proportionality constant,  $K_v$  is a proportionality constant (indicating back-EMF generated per RPM),  $\rho$  is the density of the surrounding air and  $A$  is the area swept out by the rotor. Summing over all the motors, we find that the total thrust on the hybrid aircraft (in the body frame) is given by

$$\mathbf{T}_B = \sum_{i=1}^4 T_i = k \begin{bmatrix} T_X \\ 0 \\ \sum \boldsymbol{\omega}_i^2 \end{bmatrix} \quad T_X = k\boldsymbol{\omega}_x^2 \quad (4)$$

Since the system of the hybrid aircraft consists of nonlinear coupled dynamics, we need to define it in the form of  $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ .

### 2.1. Equations of Motion

In the inertial frame, the acceleration of the hybrid aircraft is due to thrust, gravity, and linear friction. We can obtain the thrust vector in the inertial frame by using our rotation matrix  $\mathbf{R}$  to map the thrust vector from the body frame to the inertial frame. Thus, the linear motion can be summarized as

$$[m\ddot{\mathbf{x}}]^I = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \mathbf{R}_B^I \mathbf{T}_B + \mathbf{F}_D \quad (5)$$

where  $\mathbf{x}$  is the position of the aircraft,  $g$  is the acceleration due to gravity,  $\mathbf{F}_D$  is the drag force, and  $\mathbf{T}_B$  is the thrust vector in the body frame. We derive the rotational equations of motion from Euler's equations for rigid body dynamics. Expressed in vector form, Euler's equations are written as

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega}) = \boldsymbol{\tau} \quad (6)$$

where  $\boldsymbol{\omega}$  is the angular velocity vector,  $\mathbf{I}$  is the inertia matrix, and  $\boldsymbol{\tau}$  is a vector of external torques. We can rewrite this as

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \mathbf{I}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega})) \quad (7)$$

We can model our hybrid aircraft as two thin uniform rods crossed at the origin with a point mass (motor) at the end of each. With this in mind, it is clear that the symmetries result in a diagonal inertia matrix of the form

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (8)$$

Therefore, we obtain our final result for the body frame rotational equations of motion

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \tau_\phi I_{xx}^{-1} \\ \tau_\theta I_{yy}^{-1} \\ \tau_\psi I_{zz}^{-1} \end{bmatrix} - \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}} \omega_y \omega_z \\ \frac{I_{zz}-I_{xx}}{I_{yy}} \omega_x \omega_z \\ \frac{I_{xx}-I_{yy}}{I_{zz}} \omega_x \omega_y \end{bmatrix} \quad (9)$$

Considering the Newton-Euler equations, the hybrid aircraft is considered as a rigid body. In the body frame, the force required for the acceleration of mass and the centrifugal force are described as follows:

$$m\mathbf{v}_B + \mathbf{v} \times (m\mathbf{v}_B) = \mathbf{R}^T \mathbf{G} + \mathbf{T}_B \quad (10)$$

where  $\mathbf{R}^T = \mathbf{R}^{-1}$  and  $\mathbf{T}_B$  = Thrust of the body.

In the inertial frame the centrifugal force is nullified. Thus, only the gravitational force and direction of thrust are contributing in the acceleration of the hybrid aircraft.

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + [\mathbf{R}] \frac{\mathbf{T}}{m} \quad (11)$$

This becomes:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{m} \begin{bmatrix} T_x(\cos\theta\cos\psi) + T_z(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ T_x(\cos\theta\sin\psi) + T_z(\cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi) \\ T_x(-\sin\theta) + T_z(\cos\phi\cos\theta) \end{bmatrix} \quad (12)$$

In the body frame, the angular acceleration of inertia  $\mathbf{I}\dot{\boldsymbol{\nu}}$ , the centripetal forces  $\mathbf{v} \times I\mathbf{v}$  and the gyroscopic forces  $\boldsymbol{\Gamma}$  are equal to the external torque  $\boldsymbol{\tau}$

$$\mathbf{I}\dot{\boldsymbol{\nu}} + \mathbf{v} \times I\mathbf{v} + \boldsymbol{\Gamma} = \boldsymbol{\tau} \quad (13)$$

Therefore,

$$\dot{\mathbf{v}} = \mathbf{I}^{-1} \left( - \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \mathbf{r} \end{bmatrix} \times \begin{bmatrix} I_{xx} \mathbf{p} \\ I_{yy} \mathbf{q} \\ I_{zz} \mathbf{r} \end{bmatrix} - I_r \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \mathbf{r} \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{w}_\Gamma + \boldsymbol{\tau} \right) \quad (14)$$

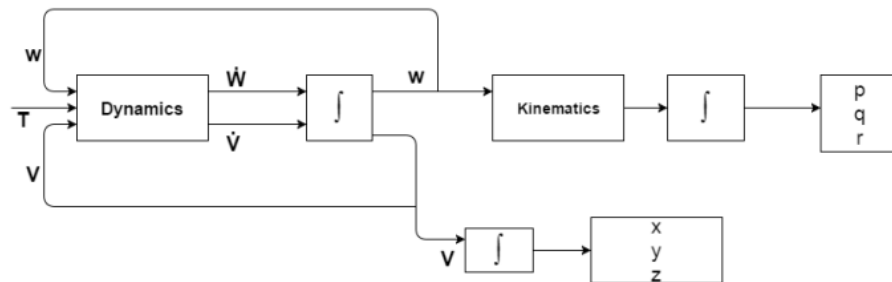
$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \\ \dot{\mathbf{r}} \end{bmatrix} = \begin{bmatrix} \frac{(I_{yy}-I_{zz})}{I_{xx}} \mathbf{q}\mathbf{r} \\ \frac{(I_{zz}-I_{xx})}{I_{yy}} \mathbf{p}\mathbf{r} \\ \frac{(I_{xx}-I_{yy})}{I_{zz}} \mathbf{p}\mathbf{q} \end{bmatrix} - I_r \begin{bmatrix} \mathbf{q} \\ -\mathbf{p} \\ 0 \end{bmatrix} \mathbf{w}_\Gamma + \begin{bmatrix} -\frac{\tau_\phi}{I_{xx}} \\ \frac{\tau_\theta}{I_{yy}} \\ \frac{\tau_\psi}{I_{zz}} \end{bmatrix} \quad (15)$$

where  $\mathbf{w}_\Gamma = \boldsymbol{\omega}_1 - \boldsymbol{\omega}_2 + \boldsymbol{\omega}_3 - \boldsymbol{\omega}_4$ .

With the dynamics and kinematics of the system solved, we now need to create a control scheme which will give the solution to our system, in terms of position and angular velocity. The next section contains the control schematic which is the solution to our system, and the simulation is required to give us output in terms of thrust of the rotor, motor velocity, position and attitude of the UAV.

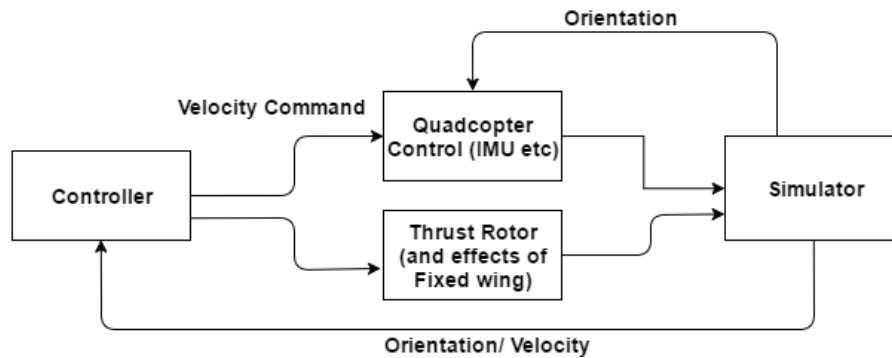
### 3. Control Structure

The hybrid aircraft works on the schematic shown in figure 3, a control structure for extracting the position, velocity and attitude of hybrid aircraft such that the thrust is fed into the dynamics block along with the feedback of angular velocity and velocity in x, y, z axis of the hybrid aircraft. The dynamics are solved, integrated and fed into feedback system. The integrated data is fed into kinematics block for the kinematics to be solved so that we can get the positional velocity ( $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$ ). The data from the dynamics block is doubly integrated so that we can have the position of the hybrid aircraft.



**Figure 3.** Control schematic of Hybrid Aircraft

Furthermore, figure 4 explains the control structure in more detail. Our idea is to use the previously designed quadcopter control and simulations for our split control. We shall check in the results if the quadcopter control system works well for our control system. We will begin with the simulation of quadcopter to maintain its height and cover some distance in x-axis. Regarding the split control structure, our input for the controller will be the velocity command (thrust for motors), this command will be processed using the dynamics and state equations. The output will be the angular velocities, roll pitch and yaw angles and the position of the quadcopter. The results will let us know what changes to expect when we implement our forward thrust rotor.

**Figure 4.** Split control structure

### 3.1. Parameters

Initial conditions are established for the programmed simulations. Table 1 shows the parameters for the initial conditions for which the simulation is accomplished.

**Table 1.** Parameters for the control algorithm and simulations

Parameters	Value	Units
$g$	9.81	$\text{m/s}^2$
$m$	4.468	kg
$l$	0.225	m
$k$	$2.980 \cdot 10^{-6}$	
$b$	$1.140 \cdot 10^{-7}$	
$I_M$	$3.357 \cdot 10^{-5}$	$\text{kg m}^2$
$I_{xx}$	$4.856 \cdot 10^{-3}$	$\text{kg m}^2$
$I_{yy}$	$4.856 \cdot 10^{-3}$	$\text{kg m}^2$
$I_{zz}$	$8.801 \cdot 10^{-3}$	$\text{kg m}^2$
$A_x$	0.35	kg/s
$A_y$	0.25	kg/s
$A_z$	0.25	kg/s
$T_{\max}$	20	N
$H$	10	m

$T_{\max}$  = Maximum thrust of each rotor

$H$  = Altitude

The control algorithm was designed in MATLAB. The Simulink-based supervisory controller runs the kinematic and dynamic models of quad-copter based on the initial conditions of the model. The initial conditions are aligned with the parameters in table 1. The quadcopter will start from the height of 10 meters and maintain its height while following a path of 275 meters in the direction of the x-axis. We expect a constant positive pitch and stable behavior of the quadcopter in the conditions applied.

### 3.2 Control Algorithms

The designed control algorithms are based on the equations explained in Section 2 (Mathematical Modeling) and the reference was established in figure 2.

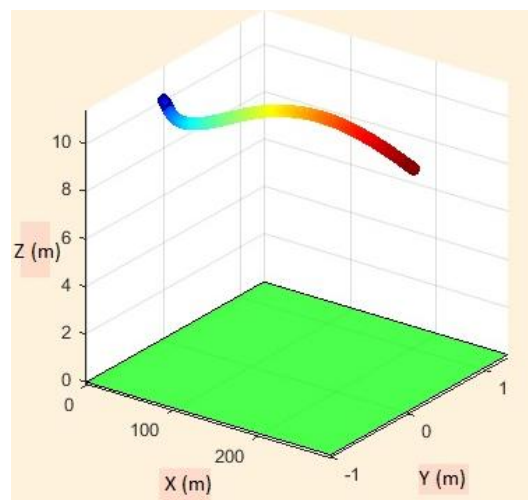
An algorithm is established for altitude commands in which step commands are given to the quadcopter system. These commands are processed via the quadcopter dynamic controller. The quadcopter control includes the command and control of altitude along with the dynamics. The simulation is run and the results are obtained. The altitude controller takes the altitude commands, processes the data and applies correction using the designed PID controller. The roll, pitch and yaw corrections are applied in real time to the quadcopter for constant altitude and attitude improvement. Figure 6 shows the basic control structure, it is the input-output summary of the designed controller. We command the thrust, which is converted into

RPM. The quadcopter dynamic equations account for the commanded input and the disturbances. Hence the output results are given in the form of figure 7.

#### 4. Simulation Results and Analysis

The results shown are for the quadcopter maintaining its height of 10 meters, the run time of simulation is 10 seconds. The quadcopter initially starts from 10 meters, maintains the altitude of 10 meters throughout and covers the length in x-direction of 275 meters. Initially there is a constant throttle command for every motor, to hover at the height of 10 meters, therefore each motor has a considerable motor speed.

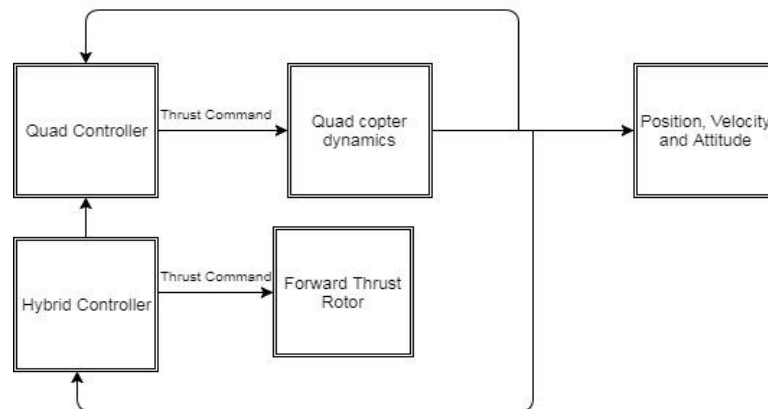
The results help us understand the behavior of the quadcopter as shown in figure 7. The quadcopter undergoes the simulation conditions displaying stable behavior. The angular velocities and rotational angles change minutely, as the quadcopter maneuvers itself with stability at the height of 10 meters.



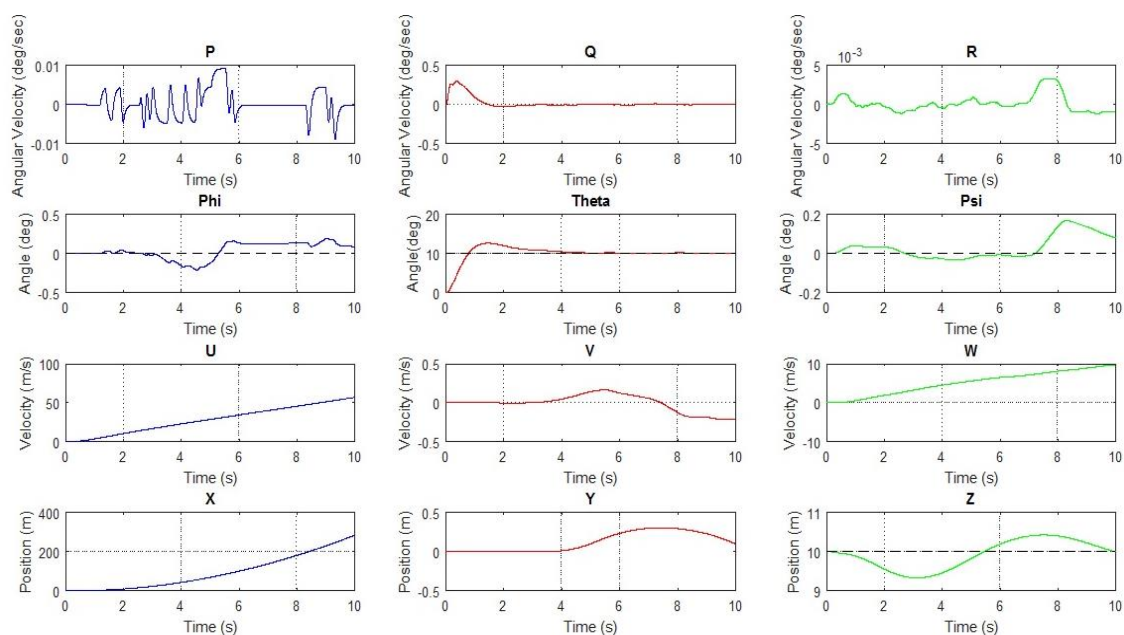
**Figure 5.** Path of the quadcopter

Figure 5 shows the path of the quadcopter in the simulation.

While maintaining the altitude of 10 meters, the quadcopter continues to move with a constant acceleration of  $5.17 \text{ m/s}^2$ . It completes the path of 275 meters in x-direction while maintaining the height of 10 meters in z-axis. There is hardly any change in position in y-axis as expected, since it is not commanded to maneuver itself in y-axis. Between 4-6 seconds it experiences a disturbance in y-direction but stabilizes itself well enough. The information retrieved from the results is very beneficial for us. We get to know the behavior of the quadcopter under a specific set of conditions. We get to know the position, velocity and attitude of our aircraft as shown in figure 7. The attitude and velocity of our aircraft along with the rotational angles of roll, pitch and yaw motions can now be determined for any maneuver. Also, as per our discussion in section 3, we now know that, for forward motion at the height of 10 meters, there is a positive pitch angle of 10 degrees, which shows that our simulation is in line with the command given as well as theory. There is a forward pitch, since the quadcopter is moving forward while maintaining the altitude. When we implement our fifth rotor, we know that there should be no positive pitch, in fact the pitch angle should be zero, since the forward thrust rotor will cater for the thrust required in the direction of x-axis.



**Figure 6.** Input-output structure for the complete design of simulation model



**Figure 7.** Position, velocity and attitude of the quadcopter

## 5. Conclusion

The results obtained are in line with the theory. Also, the results are satisfactory as expected according to the literature review.

Considering all the collected data, we are now able to set up the input and feed back to our controller from the simulation results for our forward thrust rotor and thereby our developed control structure can now be further expanded to incorporate a fixed wing and a forward thrust rotor. As a further expansion of the project, a fully developed split control can then be made by incorporating this designed control structure to introduce a totally unique form of hybrid aircraft and its control in the industry. This fully developed hybrid control structure shall be able to provide solutions for problems such as thrust saturation.

## Acknowledgment

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