IOP Conf. Series: Materials Science and Engineering **296** (2018) 012046 doi:10.1088/1757-899X/296/1/012046

Line integral on engineering mathematics

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Abstract. Definite integral is a basic material in studying mathematics. At the level of calculus, calculating of definite integral is based on fundamental theorem of calculus, related to anti-derivative, as the inverse operation of derivative. At the higher level such as engineering mathematics, the definite integral is used as one of the calculating tools of line integral. the purpose of this is to identify if there is a question related to line integral, we can use definite integral as one of the calculating experience. The conclusion of this research says that the teaching experience in introducing the relation between both integrals through the engineer way of thinking can motivate and improve students in understanding the material.

1. Introduction

The line integral is one of the topics in engineering mathematics courses for electrical engineering department during the last three years in Bandung Institute of Technology. Other departments learn the line integral in advanced calculus. This study describes how to teach the line integral in engineering mathematics course [1].

Students learn line integral after studying definite integral in calculus [2,3] (first year). As we know, a definite integral is usually introduced from the large of area under a curve. Since the calculation using limit of the Riemann sum is not easy, the students are introduced to a tool called Fundamental Theorem of Calculus. We can see the theorem in references such as Varberg et. al. [4]. Therefore, the definite integral is calculated based on anti-derivative. It is compulsory for the first-year students being able to calculate the definite integral for simple functions.

Based on the knowledge in the first-year calculus, the students in electrical engineering department should study line integral as the continuation of the definite integral. What is it and how is it calculated? From mathematical approach, it can be introduced a similar process in definite integral: discretize, Riemann sum and limit, see for example in Kreysig [5] and Wiryanto [6]. But it is not interesting for engineering students. They require a physical explanation to motivate their study. It presents in this paper.

2. Line Integral

In explaining a line integral, there is prerequisite that is usually given in the previous section, i.e. line curves presented in vector function, scalar, and vector fields. Basically, there are two types of line integral written as

$$\int_C f(x,y)ds \text{ and } \int_C \bar{F}(x,y).ds.$$

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Here, we discuss the integral along a curve C in a plane. f is scalar field and $\overline{F}$ is vector field [7]. In this paper, we focus only for the first type of integral.

Mathematically, the notation of the line integral is a number that is the result of the limit of Riemann sum [8,9]. The process to construct the Riemann sum and then calculate the limit are impractical, and are useful in analytical work. Practically, we can calculate the Riemann sum using the following formula:

$$\int_C f(x,y) = \int_a^b f\big(x(t),y(t)\big)s'(t)\,dt. \tag{1}$$

for $C{:}\,\boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j},\;\; a \le t \le b$ and relation $[s'(t)]^2 = \boldsymbol{r}'(t)\cdot\boldsymbol{r}'(t)$. The relation (1) indicates that we can write the line integral into definite integral [10]. In practice, we can describe the line integral as the area of a wall built above a curve C, and the height of the wall is various, following $z = f(x,y)$. In real life, it can be told as the same height of the wall, but the contour of the ground is up and down, so that the total height is $z = f(x,y)$. So, how to calculate the area of the wall? Engineers will answer by pulling the wall into straight (see Fig.1.), and then it is calculated such as we know in Calculus. We could formulate in (1) mathematically.

In case $f(x,y) = 1$, (1) becomes

$$\int ds = \int_a^b s'(t)dt \tag{2}$$

From the above explanation of wall area, (2) is the formulae to calculate the area of a wall built along a curve C with homogenous height 1. But, we can also explain the left-hand side of (2) as the length of a wire with shape following C, and the right-hand side is the way that we measure the wire by straightening. No-one measures a wire in curve form.

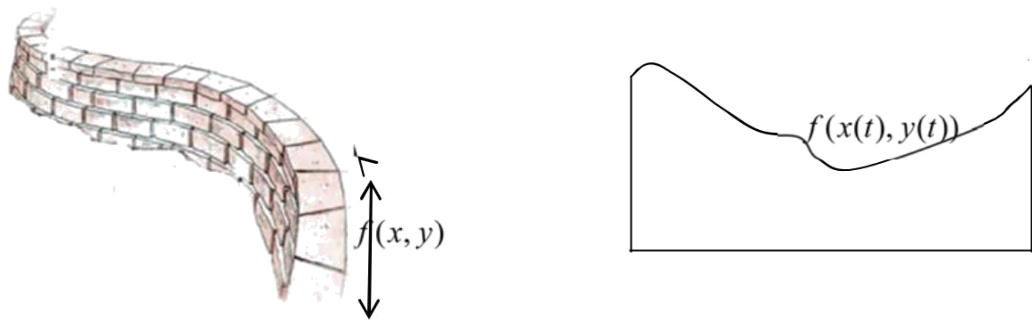

Figure1. Curved wall (left) with un-flat bottom and after the wall is straightened (right), but the bottom is made flat (source: https://www.google.com/search?q=curve+wall&client=firefox-b-ab&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjA4cap4OHTAhXHM48KHQotA0oQ_AUICigB&biw=845&bih=491)

Figure 2. A wire entwines a cylinder (source: https://www.google.co.id/search?q=entwines&source=lnms&tbm=isch&sa=X&ved=0ahUKEwj-vrKa2OHTAhUQTo8KHcmxBykQ_AUIBigB&biw=1267&bih=736#tbm=isch&q=kumparan&imgrc =9wfGI-_dF5HIaM)

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From the idea of wire length, we can extend the problem into C -curve in 3-D. If we have a horizontal cylinder with radius *R*, and we want to entwine *n* rounds of a wire having radius *r* from left to right, see Fig.2. How length of wire does we need? The first step is to construct the curve *C* as

$$C: \boldsymbol{r}(t) = R\cos t\ \boldsymbol{i} + R\sin t\,\boldsymbol{j} + t\,\boldsymbol{k}, \qquad \text{for } 0 \le t \le 2n\pi.$$

In one circle, each 2*p* of *t*, the wire must be shifted 2*r*, so that the wire does not pile up each other. The next step is to calculate using (2), i.e.

$$\boldsymbol{r}'(t) = -R\sin t\,\boldsymbol{i} + R\cos t\,\boldsymbol{j} + \frac{r}{\pi}\boldsymbol{k}$$

$$s'(t) = \sqrt{R^2 + \frac{r^2}{\pi^2}}$$

and the length is

$$\int_C f(x,y)ds = \int_0^{2n\pi} \sqrt{R^2 + \frac{r^2}{\pi^2}}dt = 2n\pi\sqrt{R^2 + \frac{r^2}{\pi^2}}$$

the similar problem could be calculated the length of a rope rolled such as shown in Figure 3. To do that, we straight the rope into a line. The difficult thing was how to formulate the vector function representing the rolled rope.

Figure 3. The length of a rope in that form measured by integral. (source: https://www.google.com/search?q=rope&client=firefox-b-ab&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiGopbK5OHTAhVMQo8KHQtbADgQ_AUICig B&biw=845&bih=491)

3. Conclusion

The definite integral is used as one of the calculating tools of line integral. The relation between both integrals through the engineer way of thinking, so that can motivate and improve students in understanding material.

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Acknowledgment
The research of this paper was supported by P3MI Research Grant ITB 2017 and PUPT DIKTI 2017. Therefore, the author acknowledged.