

Structuring students' analogical reasoning in solving algebra problem

S Lailiyah¹, T Nusantara², C Sa'dijah², E B Irawan², Kusaeri¹, and A H Asyhar³

¹Faculty of Education and Teacher Training, Islamic State University of Sunan Ampel, Jl. Ahmad Yani 117, Surabaya, East Java, Indonesia

²Faculty of Mathematics and Science, State University of Malang, Jl. Semarang 5, Malang, East Java, Indonesia

³Faculty of Science and Technology, Islamic State University of Sunan Ampel, Jl. Ahmad Yani 117, Surabaya, East Java, Indonesia

lailiyah@uinsby.ac.id

Abstract. The average achievement of Indonesian students' mathematics skills according to Benchmark International Trends in Mathematics and Science Study (TIMSS) is ranked at the 38th out of 42 countries and according to the survey result in Program for International Student Assessment (PISA) is ranked at the 64th out of 65 countries. The low mathematics skill of Indonesian student has become an important reason to research more deeply about reasoning and algebra in mathematics. Analogical reasoning is a very important component in mathematics because it is the key to creativity and it can make the learning process in the classroom become effective. The major part of the analogical reasoning is about structuring including the processes of inferencing and decision-making happens. Those processes involve base domain and target domain. Methodologically, the subjects of this research were 42 students from class XII. The sources of data were derived from the results of thinks aloud, the transcribed interviews, and the videos taken while the subject working on the instruments and interviews. The collected data were analyzed using qualitative techniques. The result of this study described the structuring characteristics of students' analogical reasoning in solving algebra problems from all the research subjects.

1. Introduction

The report on assessment framework TIMSS 2011 stated that the average achievement of participants was 386 which is meant Indonesia is in the low level [1]. Average achievement of Indonesian participants in TIMSS 2011 was decreased from the average achievement in TIMSS 2007 that is 397, the TIMSS 2011 framework was like the TIMSS 2007 framework. This finding showed that the mathematics skills of Indonesian students ranked 38th of 42 countries. The research finding of The Programme for International Student Assessment (PISA) 2009 said that the mathematics skills of Indonesian students are at the 61st level from 65 countries with average score 371. Students' skill was seemed low especially in finding algorithms, interpreting the data, and using the steps in solving the problems. The research finding of TIMSS 2011 and PISA 2009 indicates that Indonesian students already have their mathematics skill, but that skill is not sufficient to solve regular problems about



manipulating mathematical form and choose the accomplishment strategies and to solve non-regular problem such as mathematical problems as well.

The low skill of Indonesian students based on the TIMSS 2011 and PISA 2009 report become an important reason to do a deeper research about reasoning in mathematics. The low achievement of Indonesian students on the TIMSS 2011 needs specific study related to the content domain and cognitive domain. Mullis stated that the average percentage of the correct answer on the content domain and cognitive domain is 22% of Indonesian participants who are able to solve algebra problems and 17% of Indonesian participants who are able to solve the reasoning problem. Based on these results, it is needed to do an in-depth study of reasoning and algebra [2].

Learning mathematical concepts need to have special skills. Reasoning is a skill that is used both for teaching and learning [3]. This reasoning must have harmony to real life. Reasoning and proving become major goals in mathematics and learning process as one of the skills that must be mastered by students in learning mathematics [4]. The curriculum of NCTM that was adapted to Indonesian curriculum, especially for the curriculum 2006, states that teaching and learning mathematics at secondary schools aims to make the students are able to master the concepts, do reasoning, solve the problems, communicate their ideas and have the manner to respect the usefulness of mathematics in life [5]. Based on that curriculum, it can be said that reasoning is a very important skill in learning mathematics.

In mathematics, reasoning is divided into three types. They are deductive reasoning, analogical reasoning and inductive reasoning [3]. Analogical reasoning is reasoning skill that made to better learning in mathematical context. The advantages of analogical reasoning, if it is applied to mathematics teaching, are able to: increase students' creativities, develop students' reasoning abilities and motivation, increase their problem-solving abilities, make students remember the mathematics concepts in a long-term memory, associate the abstract concepts of mathematics to the students' real world, and give other examples from mathematics analogy examples. Analogical reasoning is a very important component in mathematics learning because it is the key to creativity and it can make the teaching-learning process in the classroom become effective [6]. Many cognitive processes involve analogy-making in one way or another: perceiving a stone as a human face, solving a problem in a way similar to another problem previously solved, arguing in court for a case based on its common structure with another case, understanding metaphors, communicating emotions, learning, translating poetry from one language to another [7].

Analogical reasoning plays a significant role in problem-solving [8]. The ability to use base problem (source domain or base domain) that has an identical structure in solving the target problem (target domain) can increase the performance of problem-solving. The definition of analogical reasoning in this study is a thinking process of inferring by using the same framework between the base problem and the target problem. Ruppert said that the analogical reasoning components include 4 things, they are the structure of encoding and inferring, mapping, applying, and verifying [9]. The component of analogical reasoning that will be applied in this study is stated by Ruppert.

Structuring is one of the important components in analogical reasoning because the rule of problem-solving in analogical reasoning from base domain to target domain is based on the structure-mapping. Gentner said that structure-mapping was first introduced to clarify the importance of structural similarity between the source domain and target domain or the relation between objects for each domain [10]. The purpose of structuring work-mapping is to catch the psychology process while doing the analogical mapping and tell the way of someone in using the analogy to make a conclusion.

Algebra is one of four cognitive domains from TIMSS that has the highest percentage that is 30%. The question number 6 is an algebra question in TIMSS 2011 about the quantity that full fills two differences which are presented using the balance of weights. The analogical problem consisted of the sixth question which is about the balance problem between first weights and second weights. The average percentage of the correct answer for the question number 6 for TIMSS is 18%. The percentage of students who answered the question correctly is low because the students could not identify the relation happened on the base domain (on those two weights balance) so that students were not able to

do the structuring in analogical reasoning. Therefore, it needs further discussion about structuring algebra problems.

2. Method

This study is a qualitative research and aims to discover the structuring of students' analogical reasoning in solving an algebra problem. This research was held at 3 Senior High Schools in Surabaya. The students who had been chosen were decided based on the following consideration: the students have good command in Mathematics, they are recommended by teachers, and they are communicative. In analyzing the result, this study used constant ratio between two subjects chosen from each group.

The data were collected from the result of algebra measuring instruments. The source data in this study were taken from the result of thinks aloud, transcribed interview, and video that was recorded while the subjects doing the worksheet and during the interview. The algebra measuring instrument in this study was presented in Figure 1 below.

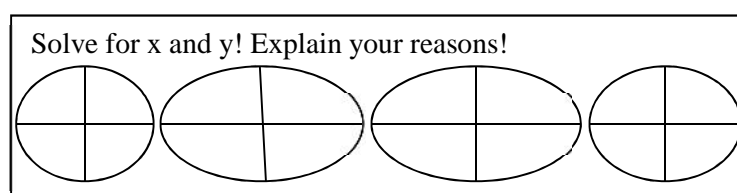


Figure 1. Algebra Measuring Instrument in This Study

The data had been collected by giving the questions to a student to be completed or solved. In the problem-solving process, students were asked to say loudly what they are thinking. The researcher then recorded the verbal expression from them and took a note of their expression including the unique things done by students when they solve the problem. When a student finished doing that question, the data collection techniques mentioned above were also applied for the other students till the researcher got the expected subject. This kind of data collection technique is called think out loud or think aloud.

The collected data were analyzed using qualitative data analysis. One of the models applied is interactive analysis technique. In this study, the arrangement of units is based on the problem that would be investigated; structuring, mapping, applying and verifying. And then, the students' spoken or written answers were categorized and coded.

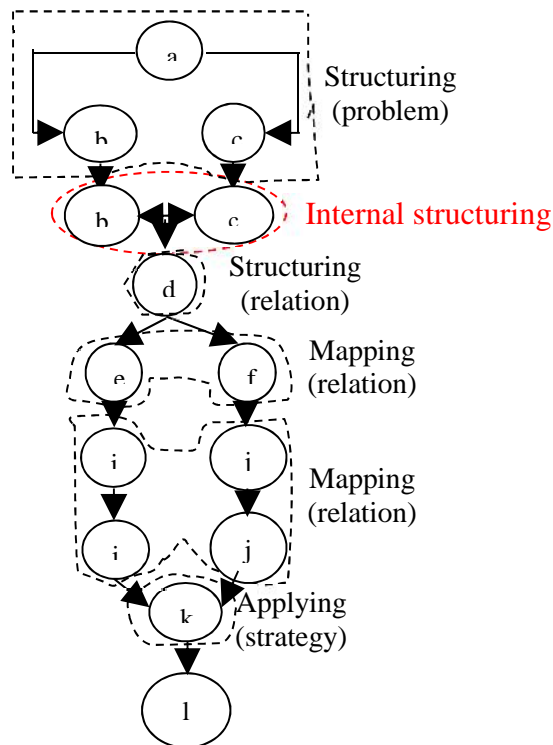
3. Discussion

Structuring in analogical reasoning divided into two types; internal structuring and connective external structuring [11]. As for the results of this study are described as follow:

3.1. Internal Structuring

Internal structuring is an analogical reasoning that is applied by identifying every mathematics object in every base domain. The subjects included in internal structuring group are subject 1 (S1) and subject 2 (S2). The analogical reasoning of S1 and S2 have the same characteristics, that is subjects identifying relation item in each of base domains and subjects mapping the characteristic code form base domain to target domain.

In the process of reasoning, S1 and S2 can identify the base problem by linking parts only to the base domain (first circle and fourth circle) so that characteristic codes are formed. S1 and S2 can map the characteristic code to the target domain. S1 and S2 can apply the characteristic code to the target domain so that completion is obtained. Although S1 and S2 can make the connection of the source characteristic source code, map, and apply the characteristic code to the target problem, these two subjects do not check the correctness of the answer. The truth of both subjects' answers was clarified at the time of the interview. As for the occurrence of internal structuring on analogy reasoning is presented in Figure 2.



Description:

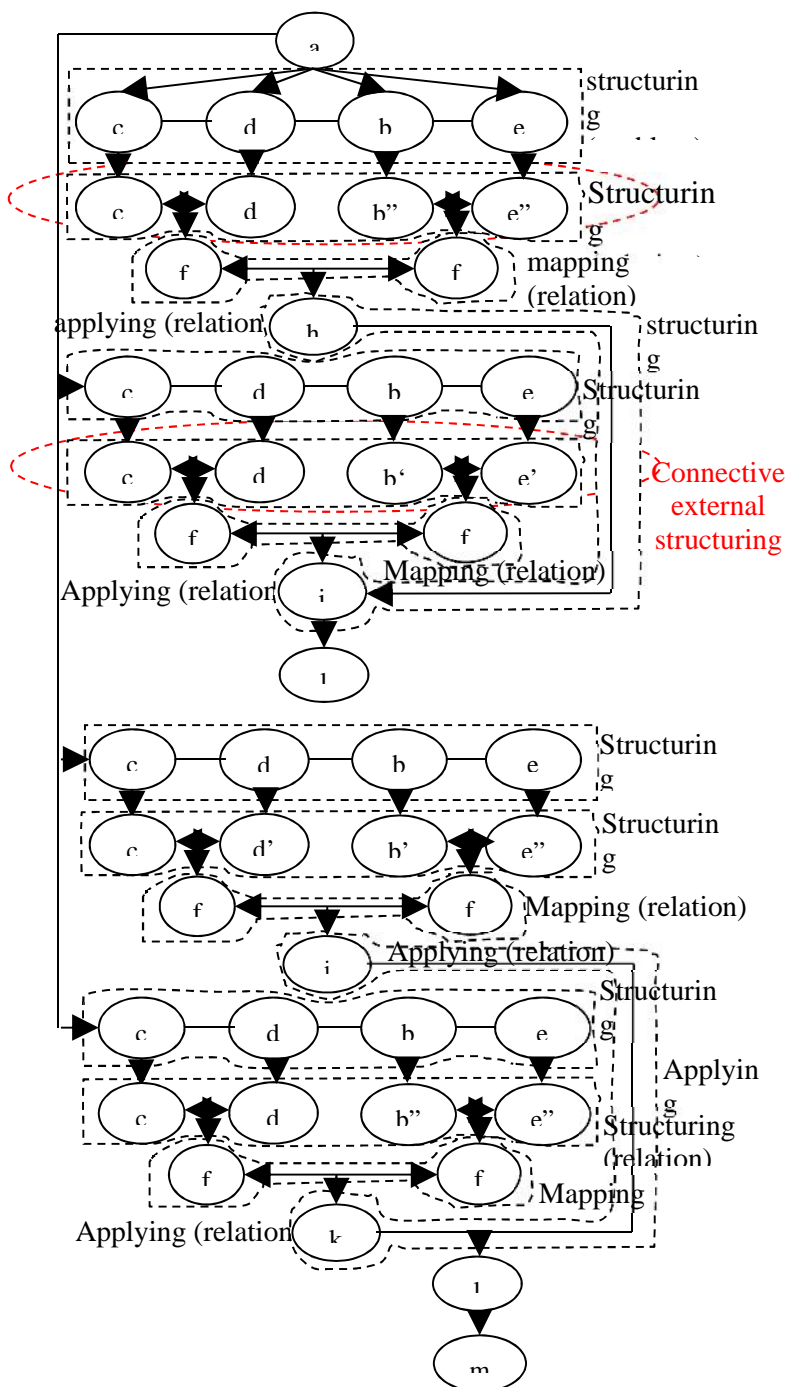
- a = determine the value of x and y based on 4 images in solving math problems.
- b = identification circle 1.
- b'' = create characteristic code starting from quadrant I, II, III and IV in circle 1.
- c = Identification circle 4.
- c'' = create characteristic code starting from quadrant I, II, III and IV in circle 4.
- d = draw a conclusion from the characteristic code on L I and L IV.
- f = identification circle 3.
- i = summation or subtraction from quadrant I, II, III, and IV in circle 2.
- i' = the first two linear equations are formed.
- k = summation or subtraction from quadrant I, II, III, and IV in circle 3.
- l = the second two linear equations are formed.

Figure 2. Internal Structuring in Analogy Reasoning

3.2. Connective External Structuring

Connective external structuring is an analogical reasoning done by identifying every mathematics object in corresponding parts of the source domain and target domain. The subjects included in internal structuring group are Subject 3 (S3) and Subject 4 (S4). The analogical reasoning of S3 and S4 have the same characteristics, that is the subject identify the parts corresponding to the base domain and target domain on connective (connecting the part corresponding among all base domain and target domain with finding identical relationship) and the subject maps the relationship/characteristic code partially to the other corresponding parts.

In the process of reasoning, S3 and S4 have recognized the problem of solving the mathematical problem; S3 and S4 identify the corresponding sides between base domain and target domain. The relationships that are (identical) in the corresponding parts of all circles are then mapped to the corresponding parts of the other on all circles. Based on this S3 and S4 have known the relationship between problems. S3 and S4 have also known the appropriate strategy that will be used in solving the mathematical problem that is mapping the relationship to another part between the base domain and target problem and solved with the appropriate strategy. After getting the values of x and y, S3 and S4 verify the answers. Based on the processes occurring in S3 and S4, there are some common characteristics of the occurrence of connective external. As for the occurrence of connective external structuring on analogy reasoning is presented in Figure 3.



Description:

a = determine the value of x and y based on 4 images in solving math problems.

b = identification circle 1.

b' = numbers quadrant I in circle 1.

b'' = numbers of quadrant II in circle 1.
b''' = numbers of quadrant III in circle 1.

b'''' = numbers of quadrant IV in circle 1.

c = identification circle 2.

c' = numbers of quadrant I in circle 2.

c'' = numbers of quadrant II in circle 2.
c''' = numbers of quadrant III in circle 2.

c'''' = numbers of quadrant IV in circle 2.

d = identification circle 3.

d' = numbers of quadrant I in circle 3.

d'' = numbers of quadrant II in circle 3.
d''' = numbers of quadrant III in circle 3.

d'''' = numbers of quadrant IV in circle 3.

e = identification circle 4.

e' = numbers of quadrant I in circle 4.

e'' = numbers of quadrant II in circle 4.
e''' = numbers of quadrant III in circle 4.

e'''' = numbers of quadrant IV in circle 4.

f = summation of numbers in circle 1 and circle 4 equal summation of numbers in circle 2 and circle 3.

h = the first two linear equations.

i = the second two linear equations.

j = the third two linear equations.

k = the fourth two linear equations.

l = solving two linear equations.

m = finish.

Figure 3. Connective External Structuring in Analogy Reasoning

4. Conclusion

Internal structuring is a structuring that occurred only in the base problem. The characteristics code from mathematics problem is built internally in the base problem. The first step of doing internal structuring is by identifying the mathematics object consisted only in the base problem to get the characteristics codes. Therefore, the second step of this internal structuring is finding the identical characteristics code from each problem and inferring. The third step of this structuring is mapping the relation of the

conclusion from base problem to target problem completely. The mapping result is applied to the target problem and then the subject chose the right strategy to solve that problem. This internal structuring is in line with the model of representative analogical reasoning stated by Holyoak, complete structure-mapping by Gabora and Saab, representative chart of analogical reasoning structure-mapping from Gentner, and general relation structure of analogical reasoning [10, 12-14].

In this connective external structuring, the characteristics codes are created among parts that are related to the base problem and target problem. The first step to do this connective external structuring is identifying the mathematical objects in the related parts between base problem and target problem so that the characteristics code is created. The second step in this connective external structuring is finding the identical characteristics codes in the corresponding parts between base problem and target problem and making the conclusion. The third step in this connective external structuring is mapping the conclusion of those correlations into other parts that are corresponding to the base domain and partial target domain. The result of that mapping was applied in others parts that are correlated with the base problem and target problem and choosing the right strategy for solving the problem.

Based on the result of this study, there are some suggestions that are: (1) in this study, there are some subjects and when they do reasoning there was also reflection. Therefore, the relation between analogical reasoning and reflection need to be analyzed. (2) In this study, students' creativities in creating characteristics code are needed for extensive external structuring. Therefore, the relation between external structuring and creativity need to be discussed. (3) this study only analyzes the structuring in students' analogical reasoning to solve the mathematics problem, but structuring and mapping are two analogical reasoning that has a correlation to each other. Because of that, mapping and analogical reasoning in solving mathematic problem needs to be analyzed.

References

- [1] OECD 2010 *Student Performance in Reading, Mathematics, and Science* **1**
<http://dx.doi.org/10.1787/9789264091450-en>
- [2] Mullis I V S, Martin M O, Foy P and Arora A 2012 *TIMSS 2011 International Result in Mathematics* (Boston: TIMSS & PIRLS International Study Center)
- [3] Mofidi S A and Parvaneh A 2012 *Indian Journal of Science and Technology* **5** (6) pp 2916—22
- [4] NCTM 2000 *Principles and Standards for Schools Mathematics*. (United States of America: NCTM)
- [5] BSNP 2006 *Standar Isi Untuk Satuan Dasar dan Menengah* (Jakarta: BSNP)
- [6] Gust H and Ka4i-Uwe K 2006 *Proc. Cognitive Science Society in cooperation with the 5th International Conference of Cognitive Science in the Asian-Pacific region (CogSci/ICCS)* (Germany: Lawrence Erlbaum) pp 417–1422
- [7] Kokinov B and R M French 2003 *Encyclopedia of Cognitive Science* Vol. 1 pp.113—8
- [8] English L D 2004 *Mathematical and Analogical Reasoning of Young Learners* (New Jersey: Lawrence Erlbaum)
- [9] Ruppert M 2013 *Proc. Eighth Congress of European Research in Mathematics Education (CERME 8)* (Turkey: Manavgat-Side, Antalya)
- [10] Gentner D 1983 *Journal of Cognitive Science* **7** pp 155-70
- [11] Lailiyah S 2017 *Jurnal Pendidikan Sains* **5** (2) pp 38-45
- [12] Holyoak K J 2012 *The Oxford Handbook of Thinking and Reasoning* (New York: Oxford University Press) pp 234-59
- [13] Gentner D and L Smith 2012 *Encyclopedia of Human Behavior* (2nd Ed) pp 130-36
- [14] Gabora L and Adam S 2011 *Proc. of the Annual Meeting of The Cognitive Science Society* (Houston TX: Cognitive Science Society) pp 3506-11