

Operational limit conditions of the spur gears in lubricated modes

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Abstract: The calculation of the gear teeth resistance, shows the using of a certain number of coefficients determined experimentally and which are accepted by the various international standards. However, this kind of calculation determines the gears by excess material and does not support the tribological parameters of operation. We propose in this work the support of these parameters, to determine the limit operation conditions of the spur gears, using the equivalent geometry. This is represented by two cylinders, which geometrically models of the contact between two teeth of a gear and whose lubrication is generally in mixed lubrication mode. The concept of Mc cool is used to determine the distribution of the load and the friction force, which are distributed in liquid (elastohydrodynamic) and solid domains and interact with each other. The phenomenon of interaction between the two domains is used, to predict the tribological limit conditions of operation. The proposed model is based on the resolution of elastohydrodynamic equations for the determination of load and friction as well as the deduction of mixed friction by tracing the Stribeck curve. This is calculated by the model of the decomposition of the patterns profile of rough surfaces in contacts. The results of non-dimensional calculations allow us to deduce the boundary conditions and can be adapted for any type of gear pair defined according to pre-established operating conditions.

Keywords: —: lubrication, gears, mixed lubrication, Stribeck curve, limit condition

1. Introduction

The design of gears includes calculations of resistance of materials, the geometrical aspect, manufacture and the checking. Among all these parameters, it is essential to know with precision the stresses being in the tooth of gears to prevent certain risks of rupture. Consequently, several theoretical and experimental methods were developed, starting from the end of the 19th century. O. Corina[1] indexed in its thesis the various methods of calculating of resistance. This reveals that the various standards AGMA and ISO that those take into account the parameters of fatigue and flexural strength. AGMA 218.01 names is frequently used in North America. and normalizes it ISO 6336-1: 2006 are used in Europe; In the two standards of calculation we notice the existence of several experimental parameters for the assumption of responsibility of the tribological parameters[2][3]. into this work we introduce the tribological parameters, for the determination of the limiting conditions of operation of the spur gears, by using the equivalent geometry and the theory of mixed lubrication. our approach is based on the basic equations of the material resistance with which we combine the various equations of elastohydrodynamic lubrication to have adequate limiting conditions of operation.



2. Static resistance of a gear teeth

We consider the resistance of the tooth of a spur gear height H , width L and thickness E as indicated by figure 1. It is supposed that the effort F is applied at the end of the tooth which represents the most unfavourable case and which only the component tangential T produces a bending of the tooth, that we regard as a fixed beam.

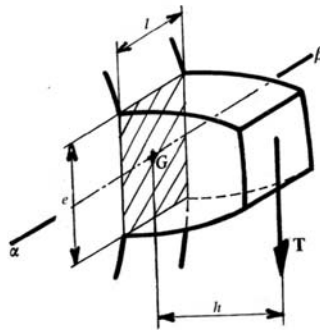


Figure1: gear teeth resistance

The equation of the maximum bending moment is written:

$$Mf_{\max} = T.h \quad (1a)$$

The tangential stress on diameter according to the load is written:

$$T = W \cos \alpha \quad (1b)$$

The equation of the bending moment becomes:

$$Mf_{\max} = W.h \cos \alpha \quad (2)$$

The flexural strength condition of the tooth is written:

$$\frac{I}{v} \geq \frac{Mf_{\max}}{\sigma_{\max}} \quad (3a)$$

who is written:

$$Mf_{\max} \leq \left(\frac{I}{v} \right) \sigma_{\max} \quad (3b)$$

the quadratic moment of a square surface is written:

$$\frac{I}{v} = \frac{l.e^2}{6} \quad (4)$$

The condition of Hertzian pressure resistance is written:

$$\sigma_{\max} = 0.3 p_0 \quad (5a)$$

With:

$$p_0 \leq 0.6 HB \quad (5b)$$

The equation (5a) becomes:

$$\sigma_{\max} \leq 0.18 HB \quad (5c)$$

Thus the equation of the moment (3b), becomes

$$Mf_{\max} \leq 0.03 (HB) l e^2 \quad (6)$$

By replacing the equation (2) in the equation (6), the equation of the load is written:

$$W \leq \frac{0.03 (HB) l e^2}{h \cos \alpha} \quad (7)$$

3. Variation of the load on teeth in lubricated contacts

We modelize geometrically the contact between two teeth as being the contact between two cylinders or two half rolls as indicated by figure 2 and their lubrications is generally in mode of mixed lubrication, and the concept of Mc cool is often used (figure 3) to determine the burden-sharing in the contact [3].

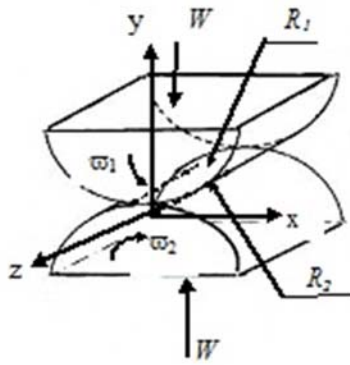


Figure 2. Geometrical model

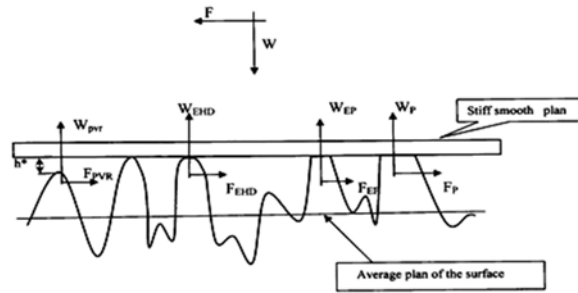


Figure 3. Mc Cool Model

We note in this model the influence of the asperities in the movement of a body compared to the other and causes the load (W) to be distributed in a liquid lift (W_L) and the other solid (W_S)

The asperities the least far away from the smooth plan, become deformed geometrically in an elastic range to form the elastohydrodynamic field (EHD), whose resultant is W_{EHD} . The sum of these efforts contributes to the lift of viscous origin, dependant amongst other things the speed of slip and on the lubricant. This field generates a liquid friction (f_L), which in the case of tightens in its majority the gears towards an elastohydrodynamic friction (f_{EHD}), whose limiting value does not exceed 0.02[4], and is written:

$$f_{EHD} = \frac{F_{EHD}}{W_{EHD}} \quad (8)$$

The asperities intercepted by the plan smoothes are subjected to a crushing, which can be weak of elastic type (W_E), moderately crushed of elastoplastic type (W_{EP}) and lastly of plastic type (W_P) for the asperities the most requested. The sum of these efforts contributes to a lift of the solid type independent speed.

The solid lift is written:

$$W_S = W_E + W_{EP} + W_P \quad (9)$$

It results a solid friction force which is written:

$$F_S = F_E + F_{EP} + F_P \quad (10)$$

The coefficient of total solid friction is written:

$$f_S = \frac{F_S}{W_S} \quad (11)$$

According to the authors [3] the coefficient f_S vary between 0.12 to 0.09

The total load, results from the sum of the elastohydrodynamic and solid load, and is written:

$$W = W_{EHD} + W_S \quad (12)$$

The total friction force, results from the sum of the elastohydrodynamic and solid force, and is written:

$$F = F_{EHD} + F_S \quad (13)$$

It results a global coefficient of friction which is written:

$$f = \frac{F}{W} = \frac{F_{EHD} + F_S}{W_{EHD} + W_S} \quad (14a)$$

Who is written

$$f = \frac{f_{EHD}W_{EHD} + f_S W_S}{W_{EHD} + W_S} \quad (14b)$$

By replacing the equation (14b) in the equation (12), the solid load (W_s) is written:

$$W_S = W_{EHD} \frac{(f - f_{EHD})}{(f_S - f)} \quad (15)$$

The equation of the total load (12) becomes:

$$W = W_{EHD} \left(1 + \frac{(f - f_{EHD})}{(f_S - f)} \right) \quad (16a)$$

Thus:

$$\frac{W}{W_{EHD}} = \left(1 + \frac{(f - f_{EHD})}{(f_S - f)} \right) \quad (16b)$$

Let us write: $\bar{W} = \frac{W}{W_{EHD}}$ Thus the equation (16b) is written:

$$\bar{W} = \left(1 + \frac{(f - f_{EHD})}{(f_S - f)} \right) \quad (17)$$

Same manner let us divide the equation (7) by W_{EHD} , it becomes

$$\frac{W}{W_{EHD}} \leq \frac{0.03(HB)le^2}{W_{EHD}h \cos \alpha} \quad (18a)$$

Who is written too

$$\bar{W} \leq \frac{0.03(HB)le^2}{W_{EHD}h \cos \alpha} \quad (18b)$$

Let us introduce the dimensional parameter K , who is written:

$$K = \frac{0.03(HB)le^2}{h \cos \alpha}$$

The equation (18b) becomes:

$$\bar{W} \leq K \left(\frac{1}{W_{EHD}} \right) \quad (19)$$

For a module of cutting, a material and a geometry of the tooth which is fixed, the parameter K is constant. It is noticed that the inequation (19) depends on the load in the elastohydrodynamic field. The latter is determined by the resolution of the equation given by Belarifi et al[3].

$$H_e \widehat{W}_{EHD}^{0.1} - 0.63 \widehat{W}_{EHD} - H_0 = 0 \quad (20a)$$

Dowson equation film thickness [3] are used at the center for linear contact

$$\hat{H}_c = 3.06 \hat{U}^{0.69} \hat{G}^{0.56} \hat{W}_{EHD}^{-0.1} \quad (20b)$$

We introduce:

$$\hat{H}_c = H_e \hat{W}_{EHD}^{-0.1} \quad (20b)$$

With:

$$\hat{H}_c = \left(\frac{h_c}{R_e} \right); \hat{U} = \left(\frac{\eta_0 U}{E^* R_e} \right); \hat{G} = (\alpha E^*); \hat{W}_{EHD} = \left(\frac{W_{EHD}}{E^* R_e} \right); R_e = \frac{R_1 R_2}{R_1 + R_2}; E^* = \frac{E_1 E_2}{E_1 + E_2}$$

These equations are valid, as long as load W_{EHD} max; remain lower than the elastoplastic mode ($W_{EHDMax} < W_{Ep}$), and is written:

$$\hat{W}_{EHD}^{lim} = 2.18 \left(\frac{R_{pe}}{E^*} \right)^2 \quad (21)$$

Knowing that:

$$HB \leq 2.77 R_{pe} \quad (22a)$$

Finally the elastohydrodynamic limiting load is written:

$$\hat{W}_{EHD}^{lim} = 16.73 \left(\frac{HB}{E^*} \right)^2 \quad (22b)$$

4. Results and discussions

The various computational results indicated by figures 4 and 5, were established for the various characteristics indicated to table 1, for a thickness of film in the center $h_0 = 10^{-6}$ m, solid friction ($f_s = 0.12$) [3], elastohydrodynamic friction ($f_{EHD} \leq 0.02$), slip velocity ($V_g 0.5 \text{ ms}^{-1}$) and hertzian pressures variable.

Table 1. Characteristics of material and lubricant

geometry of the solid		Young Modulus	Lubricant		
R_e	l		η	α	τ_0
$4.7 \cdot 10^{-3} \text{ m}$	$6 \cdot 10^{-3} \text{ m}$	231 GPa	$\frac{0.324 \text{ Pa}}{\text{s}}$	$19 \cdot 10^{-9} \text{ Pa}^{-1}$	2.5 MPa

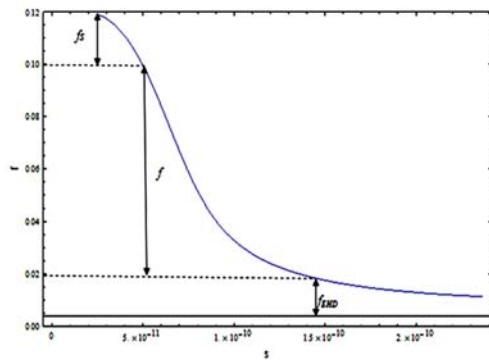


Figure 4. Stribeck curve

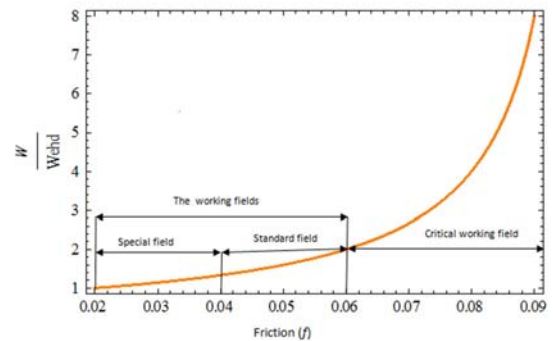


Figure 5. Operational Limit condition

The curve of Stribeck (figure 4), is plotted according to the model of Robbe-Valloir & al [5][6], indicates the variation of the coefficient of friction (f) according to the parameter of Somerfield ($S = \frac{\eta V}{p}$), the values of frictions are introduced into the equation (17), to determine the adimensional

limits loads operation, indicated by figure 5. We notice on this curve two distinct area.

The first area is known as working area, it is included for varying friction coefficients from 0.02 to 0.06 for adimensional loads varying between values 1 to 2. We distinguish in this area two parts.

- A special field : requires specific conditions to guarantee friction lower than 0.04, which requires very good surface qualities (shaving), we notice that we work in minimal values of the limiting adimensional load what requires an abundant lubrication and a high speed without reaching the phenomenon of resonance.

- A Standard field: we can work under reasonable conditions where frictions varies between 0.04 to the 0.06. The adimensional load can reach the maximum value of 2. Our various calculations showed that this part can be used for all the types of gears.

The second area represents the critical field of operation, where frictions vary from 0.06 to 0.09 in this area the load believes in an exponential way. It is noticed that tends some mixed lubrication towards a boundary lubrication.

5. Conclusion

The model proposed, defined the various ranges operation of gears by giving the limiting adimensional loads according to lubricated frictions. Our various work on tribometer pin- disc confirms that we are in mixed lubrication but great difficulties persist in to guarantee good conditions of operation, those is noticed by the Stribeck curve, which shows low numbers of Somerfield which due to strong the hertzian pressures applied. These low values will tend to bring closer the couple to gears towards boundary lubrication what is to be avoided. Thus the knowledge of the limiting loads defined in this work will make it possible to avoid the phenomena of rupture or seizing, because we will manage to define the optimal operating ranges beforehand.

References

1. C. Oancea, Analyse des dents d'engrenages droits par la méthode des potentiels complexes," Thèse de Doctorat. université de Laval (Québec-Canada), 1997.
2. G. Henriot, Engrenages : détermination des charges sur les dentures et calculs de résistance," Technique de l'ingénieur Transm. puissance mécanique engrenages liens souples, vol. base docum, no. ref. article : bm5623, 2002.
3. F. Belarifi, J. Blouet, G. Inglebert, and A. Benamar, Confrontation between a mixed lubrication model and an experimental survey on the behaviour to the friction of spur gear teeth, *Mechanics & Industry*, vol.7, 527–536, 2006
4. F. Belarifi and G. Inglebert, Numerical modelling of the hydrodynamic behaviour of linear contacts, *Journal of the Balkan Tribological Association*, vol. 21, no. 3, 630-639, 2015.
5. F. Robbe-Valloire, B. Paffoni, and R. Progri, Load transmission by elastic, elasto-plastic or fully plastic deformation of rough interface asperities, *Mechanics of Materials*, vol. 33, no. 11, 617–633, 2001.
6. F. Robbe-Valloire, R. Progri, B. Paffoni, and R. Gras, Theoretical prediction and experimental results for mixed lubrication between parallel surfaces, *Boundary and Mixed Lubrication Science and Applications Proceedings of the 28th Leeds-Lyon Symposium on Tribology*, Vol. 40, 129–137, 2002.