

Research concerning the balancing of a plane mechanism

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Abstract: By statically balancing of the plane mechanisms and especially those functioning at high speeds is being pursued the decrease of the value of the resultant force of all inertia forces that work on the component elements, thus obtaining a significant decrease in vibrations and shocks during the functioning. On the other hand, the existence of balancing masses which ensure the balancing of the mechanism leads to increased gauge and its mass. In this paper are presented some possibilities of statically balancing a plane mechanism which is composed of three independent contours. First is analyzed the case when the mechanism is totally balanced. Then a solution is proposed for a partial balancing of the mechanism based on the balancing of the first harmonic of the inertia force developed in a piston of the mechanism. Finally, are presented some simulation results concerning the variation of the value of the resultant inertia force during a cinematic cycle when the mechanism is unbalanced and when it is partially balanced. Also, it is analyzed the variation of the motor moment when the mechanism is unbalanced and when is totally and partially balanced.

Keywords: plane mechanism, inertia forces, balancing

1. Introduction

During the operation of the mechanisms, the forces and moments of inertia that work on the component elements vary over time, which leads to the occurrence of dynamic forces in joints whose value increases as the mechanism is faster. When the values of these dynamic forces exceed certain limits, a premature wear of the joints comes out that generate vibrations, sometimes even shocks during operation. By statically balancing of the mechanisms is being pursued the decrease of the value of the resultant force of all inertia forces that work on the component elements [1, 2]. This is achieved by introducing some balancing masses mounted in the prolongation of the component elements which leads to the increase in the gauge of the mechanism and its mass. So, the problem of balancing the mechanisms has to be studied very carefully. Finding solutions of partial balancing that lead to the diminution of the values of the resultant inertia force without a significant increase in the gauge of the mechanism is highly indicated.



In this paper are presented some possibilities of statically balancing a plane mechanism. First, is analyzed the mechanism kinematics. Then a statically total balancing solution of the mechanism, when the resultant inertia force is cancelled, is presented. Also, it is proposed a solution for a partial balancing of the mechanism based on the balancing of the first harmonic of the inertia force developed in a piston of the mechanism. Finally, are presented some simulation results concerning the variation of the value of the resultant inertia force during a cinematic cycle when the mechanism is unbalanced and when it is partially balanced. Also, it is analyzed the variation of the motor moment when the mechanism is unbalanced and when is totally and partially balanced.

2. Mechanism kinematics

In figure 1 the analyzed plane mechanism is presented. With $C_i, i = \overline{1,7}$, were noted the mass centres of the component elements. The dimensions of the component elements are: $OA = 0.14$ m; $OC_1 = 0.07$ m; $AD = 0.9$ m; $AB = 0.45$ m; $BF = 0.8$ m; $BE = 0.4$ m; $FG = 0.5$ m; $FC_6 = 0.25$ m; $O_2E = 0.6$ m; $EC_5 = 0.3$ m. The coordinates of the G point as against the Oxy coordinate system are: $x_G = 0.75$ m, $y_G = 0.7$ m. The nominal angular speed of the leader element of the mechanism is $\omega_1 = 20$ rad/s. The positional and cinematic analysis of the mechanism was realized by applying the method of the projection of the closed and independent vectorial contours [3, 4].

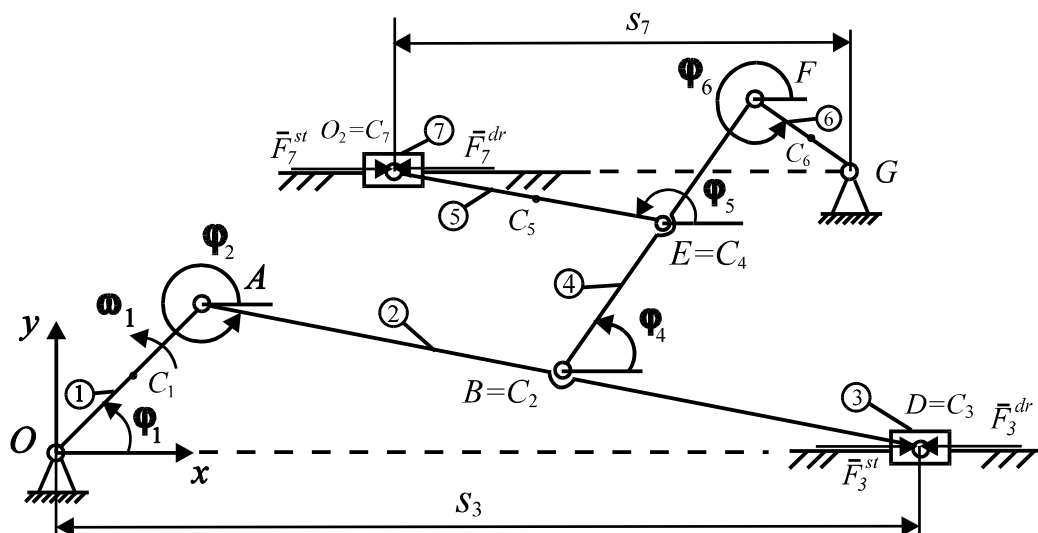


Figure 1. Plane mechanism.

The mechanism has three closed and independent contours: $O-A-D-O$, $O-A-B-F-G-O$ and $O-A-B-E-O_2-G-O$. By projecting on the x and y axes the vectorial closing equations corresponding to the three contours the following systems of equations were obtained:

$$\begin{cases} l_1 \cdot \cos \varphi_1 + l_2 \cdot \cos \varphi_2 - s_3 = 0 \\ l_1 \cdot \sin \varphi_1 + l_2 \cdot \sin \varphi_2 = 0 \end{cases} \quad (1)$$

$$\begin{cases} l_1 \cdot \cos \varphi_1 + AB \cdot \cos \varphi_2 + l_4 \cdot \cos \varphi_4 + l_6 \cdot \cos \varphi_6 - x_G = 0 \\ l_1 \cdot \sin \varphi_1 + AB \cdot \sin \varphi_2 + l_4 \cdot \sin \varphi_4 + l_6 \cdot \sin \varphi_6 - y_G = 0 \end{cases} \quad (2)$$

$$\begin{cases} l_1 \cdot \cos \varphi_1 + AB \cdot \cos \varphi_2 + BE \cdot \cos \varphi_4 + l_5 \cdot \cos \varphi_5 + s_7 - x_G = 0 \\ l_1 \cdot \sin \varphi_1 + AB \cdot \sin \varphi_2 + BE \cdot \sin \varphi_4 + l_5 \cdot \sin \varphi_5 - y_G = 0 \end{cases} \quad (3)$$

where: $l_1 = OA$; $l_2 = AD$; $l_4 = BF$; $l_5 = EO_2$; $l_6 = FG$.

By solving the equations system (1) results the angle φ_2 and the displacement s_3 , then by solving the equations system (2) results the angles φ_4 and φ_6 , and finally by solving the equations system (3) results the angle φ_5 and the displacement s_7 . The kinematics study of the mechanism was transposed into a computer program using the Maple programming environment [5] that holds a lot of packages of powerful symbolically computation functions, so that solving the three equations systems became extremely easy. Once established the angles values mentioned above, it was determined the coordinates of the various points of the mechanism, including those of the mass centres of the component elements. The projections on the x and y axes of the speeds and of the accelerations of any point on the mechanism may be calculated by deriving in relation to time the expressions of its coordinates, for example in the case of the mass centres $C_j, j = \overline{1,7}$, by using the following relations:

$$\begin{cases} (v_{C_j})_x = \dot{x}_{C_j} = \frac{dx_{C_j}}{d\varphi_1} \frac{d\varphi_1}{dt} = \omega_1 \cdot \frac{dx_{C_j}}{d\varphi_1} \\ (v_{C_j})_y = \dot{y}_{C_j} = \frac{dy_{C_j}}{d\varphi_1} \frac{d\varphi_1}{dt} = \omega_1 \cdot \frac{dy_{C_j}}{d\varphi_1} \end{cases} \quad (4)$$

$$\begin{cases} (a_{C_j})_x = (\dot{v}_{C_j})_x = \varepsilon_1 \cdot \frac{dx_{C_j}}{d\varphi_1} + \omega_1^2 \cdot \frac{d^2x_{C_j}}{d\varphi_1^2} \\ (a_{C_j})_y = (\dot{v}_{C_j})_y = \varepsilon_1 \cdot \frac{dy_{C_j}}{d\varphi_1} + \omega_1^2 \cdot \frac{d^2y_{C_j}}{d\varphi_1^2} \end{cases} \quad (5)$$

The angular speeds and accelerations $\omega_j, \varepsilon_j, j = 2,4,5,6$, can be determined in the same way by deriving in relation to time the expressions of the angles $\varphi_j(\varphi_1), j = 2,4,5,6$:

$$\begin{cases} \omega_j = \dot{\varphi}_j = \frac{d\varphi_j}{d\varphi_1} \frac{d\varphi_1}{dt} = \omega_1 \cdot \frac{d\varphi_j}{d\varphi_1} \\ \varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 \cdot \frac{d\varphi_j}{d\varphi_1} + \omega_1^2 \cdot \frac{d^2\varphi_j}{d\varphi_1^2} \end{cases} \quad (6)$$

3. Static balancing of the mechanism

It is presented a solution of statically total balancing of the mechanism by using the method of static concentration of the mass of the component elements [1]. In figure 2 is presented the model of the mechanism with concentrated masses. Because it was considered the mass centres $C_j, j = 1,2,4,5,6$, at the middle of the elements, it results that the concentrated masses m_{1O} and m_{1A} are equal to half of the mass of element 1, m_{2A} and m_{2D} are equal to half of the mass of element 2, m_{4B} and m_{4F} are equal to half of the mass of element 4, m_{5E} and m_{5O_2} are equal to half of the mass of element 5 and m_{6F} and m_{6G} are equal to half of the mass of element 6.

First it is added the balancing mass m_{E1} in the prolongation of the element 2 so that:

$$m_{E1} \cdot r_{E1} = m_D \cdot l_2 + m_B \cdot AB \quad (7)$$

where: $m_D = m_3 + m_{2D}$ and $m_B = m_{4B}$, m_3 being the mass of the piston 3.

From equation (7) it is calculated the balancing mass m_{E1} and the length of its arm r_{E1} , by imposing the value of one of the two unknowns. Neglecting the mass of the arm of length r_{E1} , it results that in point A will be now concentrated a mass whose value is $M_A = m_{E1} + m_A + m_D$, where $m_A = m_{1A} + m_{2A}$.

It is added the balancing mass m_{E2} in the prolongation of the crank l so that:

$$m_{E2} \cdot r_{E2} = M_A \cdot l_1 \quad (8)$$

The balancing mass m_{E2} and the length of its arm r_{E2} are calculated in the same way, by imposing the value of one of them. Neglecting the mass of the arm of length r_{E2} , it results that in point O is now concentrated a mass whose value is $M_O = m_{E2} + M_A + m_O$, where $m_O = m_{1O}$. Then, it is added the balancing mass m_{E3} in the prolongation of the element 5 so that:

$$m_{E3} \cdot r_{E3} = m_{O2} \cdot l_5 \quad (9)$$

where $m_{O2} = m_{5O2} + m_7$, m_7 being the mass of the piston 7. After setting the values for m_{E3} and r_{E3} by imposing the value of one of them in equation (9), it results (neglecting the mass of the arm of length r_{E3}) that in point E is concentrated a mass whose value is $M_E = m_{E3} + m_{O2} + m_E$, where $m_E = m_{5E}$.

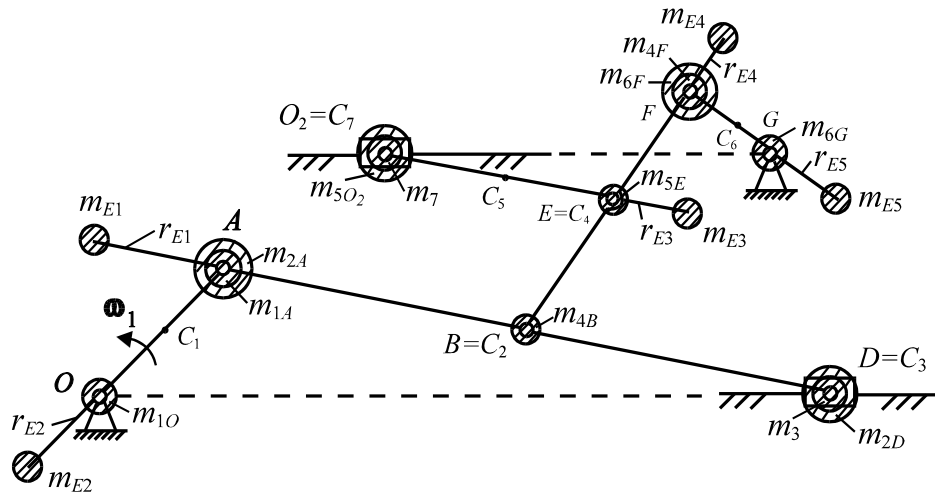


Figure 2. Solution of statically total balancing of the mechanism.

It is added the balancing mass m_{E4} in the prolongation of the element 4 so that:

$$m_{E4} \cdot r_{E4} = M_E \cdot EF \quad (10)$$

equation from which results the value for m_{E4} and r_{E4} , by imposing the value of one of them. So, in point F is now concentrated a mass whose value is $M_F = M_E + m_F + m_{E4}$, where $m_F = m_{4F} + m_{6F}$.

Finally, it is added the balancing mass m_{E5} in the prolongation of the rocker 6 so that:

$$m_{E5} \cdot r_{E5} = M_F \cdot l_6 \quad (11)$$

wherefrom results the values of the balancing mass m_{E5} and the length of its arm r_{E5} , by imposing the value of one of them in equation (11). Neglecting the mass of the arm of length r_{E5} , it results that in point G is now concentrated a mass whose value is $M_G = m_{E5} + M_F + m_G$, where $m_G = m_{6G}$.

It can be observed that after adding the balancing masses the entire mass of the mechanism has been concentrated in the points O and G . So, the mass centre of the mechanism occupies a fix position between O and G during operation, and in this way the resultant force of all inertia forces that work on the component elements is cancelled. On the other hand it is obvious that introducing the balancing masses leads to the increase in the gauge of the mechanism and its mass. That is why it is necessary

finding solutions of partial balancing that lead to the diminution of the values of the resultant inertia force without a significant increase in the gauge of the mechanism.

Further, it is presented in figure 3 such a partial balancing solution in the case of the analyzed mechanism. In this case, first it is added the balancing mass m_E in the prolongation of the crank l so that:

$$m_E \cdot r_E = m_A \cdot l_1 \quad (12)$$

The balancing mass m_E and the length of its arm r_E are calculated by imposing the value of one of them in equation (12).

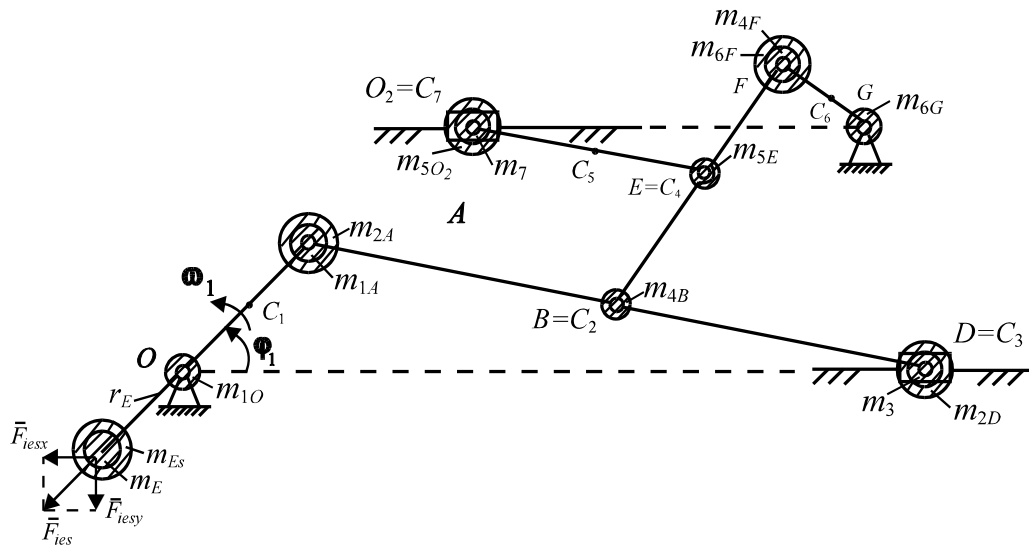


Figure 3. Solution of partially balancing of the mechanism.

Then the balancing mass m_E is supplemented with the mass m_{Es} so that the horizontal component of the inertia force developed by it (\bar{F}_{iesx}), whose value is $m_{Es} \cdot r_E \cdot \omega_1^2 \cdot \cos \varphi_1$, balances the first harmonic of the inertia force developed by the mass m_D : $\bar{F}_{iD} = -m_D \cdot \bar{a}_D$, whose expression is [1, 6]: $m_D \cdot l_1 \cdot \omega_1^2 \cdot \cos \varphi_1$, \bar{a}_D being the acceleration of point D . From this equality it results: $m_{Es} = m_D \cdot l_1 / r_E$. It can be observed that this partial balancing solution leads to an insignificant increase in the gauge of the mechanism. However, the inertia forces developed by the other concentrated masses of the model of the analyzed mechanism remain unbalanced, as well as the higher harmonics of the inertia force \bar{F}_{iD} and the projection on the y axis of the inertia force \bar{F}_{ies} .

4. Simulation results

It was considered that the elements of the mechanism are made of aluminium, the elements 1, 2, 4, 5 and 6 having circular cross section with a radius of 0.015 m. The mass of each of the pistons 3 and 7 is 1.5 kg.

In the case of statically total balancing of the mechanism we have imposed the lengths of the arms of the balancing masses in this way: $r_{E1} = 0.8 \cdot l_2$; $r_{E2} = l_1$; $r_{E3} = 0.8 \cdot l_5$; $r_{E4} = 0.8 \cdot l_4$ and $r_{E5} = 1.2 \cdot l_6$. So, the balancing masses have resulted with the following values: $m_{E1} = 2.948$ kg; $m_{E2} = 6.3$ kg; $m_{E3} = 2.59$ kg; $m_{E4} = 4.226$ kg and $m_{E5} = 9.555$ kg.

In the case of the partial balancing solution by imposing $r_E = l_1$, it has been obtained $m_E + m_{Es} = 3.351$ kg.

In figure 4 it is presented the variation on a cinematic cycle of the total inertia force F_{itot} in the case when the mechanism is unbalanced (curve 1) and when it is partially balanced (curve 2). The total inertia force F_{itot} has been calculated by considering its projections on the x and y axes with the relation: $F_{itot} = (F_{itotx}^2 + F_{itoty}^2)^{1/2}$, where:

$$\begin{cases} F_{itotx} = -\sum_{j=1}^7 m_j \cdot (a_{C_j})_x \\ F_{itoty} = -\sum_{j=1}^7 m_j \cdot (a_{C_j})_y \end{cases} \quad (13)$$

when the mechanism is unbalanced and:

$$\begin{cases} F_{itotx} = -\sum_{j=1}^7 m_j \cdot (a_{C_j})_x - (m_E + m_{Es}) \cdot (a_E)_x \\ F_{itoty} = -\sum_{j=1}^7 m_j \cdot (a_{C_j})_y - (m_E + m_{Es}) \cdot (a_E)_y \end{cases} \quad (14)$$

when the mechanism is partially balanced. In relations (13) and (14) with m_j was noted the mass of the element j and with $(a_E)_x$ and $(a_E)_y$ were noted the projections on the x and y axes of the acceleration of the point where is concentrated the balancing mass $m_E + m_{Es}$.

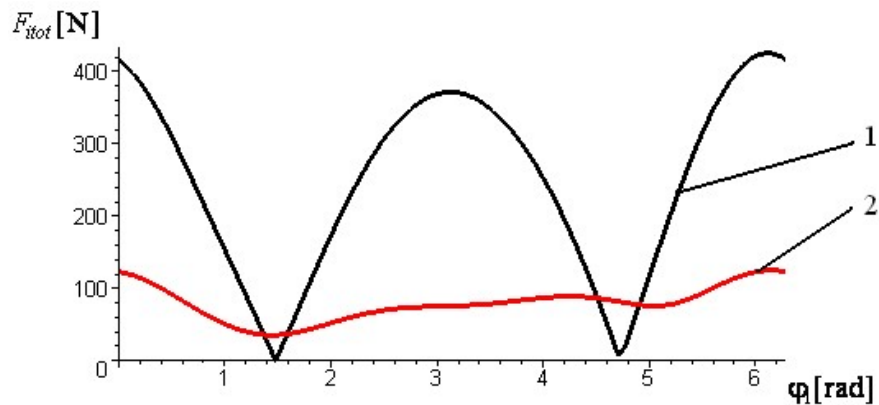


Figure 4. The variation on a cinematic cycle of the total inertia force F_{itot} in the case when the mechanism is unbalanced (curve 1) and when it is partially balanced (curve 2).

Figure 4 shows a significant decrease of the total inertia force in the case of partial balancing of the mechanism under conditions of very small increase of its gauge.

Also, it was analyzed the variation of the motor moment when the mechanism is unbalanced and when is totally and partially balanced.

The variation on a cinematic cycle of the motor moment M_m may be obtained by expressing the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component elements of the mechanism [1, 7]. So, when the mechanism is unbalanced the motor moment may be calculated with the following relation:

$$\bar{M}_m \cdot \bar{\omega}_1 + \sum_{j=1}^7 \bar{G}_j \cdot \bar{v}_{C_j} + \sum_{j=1}^7 (\bar{F}_{ij} \cdot \bar{v}_{C_j} + \bar{M}_{ij} \cdot \bar{\omega}_j) + \bar{F}_3 \cdot \bar{v}_D + \bar{F}_7 \cdot \bar{v}_{O_2} = 0 \quad (15)$$

where: $G_j = m_j \cdot g, j = \overline{1,7}$, are the weight forces corresponding to the component elements ($g = 9.81 \text{ m/s}^2$ is the gravitational acceleration); $\bar{F}_{ij} = -m_j \cdot \bar{a}_{C_j}, j = \overline{1,7}$, are the inertia forces; $\bar{M}_{ij} = -I_{C_j} \cdot \bar{\varepsilon}_j, j \in \{1,2,4,5,6\}$, are the inertia moments, where $I_{C_j}, j \in \{1,2,4,5,6\}$, are the inertia mass moments of the elements; $\bar{F}_j, j = 3,7$, are the technological forces equal to $\bar{F}_j^{dr}, j = 3,7$, when the j piston moves to the right and equal to $\bar{F}_j^{st}, j = 3,7$, when the movement is to the left (figure 1); \bar{v}_D and \bar{v}_{O_2} are the speeds of the points D and O_2 , respectively.

The simulations were performed considering $F_3^{dr} = 1500 \text{ N}$; $F_3^{st} = 800 \text{ N}$; $F_7^{dr} = 800 \text{ N}$ and $F_7^{st} = 1500 \text{ N}$.

When the mechanism is statically total balancing the relation (15) becomes:

$$\bar{M}_m \cdot \bar{\omega}_1 + \sum_{j=1}^7 \bar{G}_j \cdot \bar{v}_{C_j} + \sum_{j=1}^5 \bar{G}_{Ej} \cdot \bar{v}_{Ej} + \sum_{j=1}^7 (\bar{F}_{ij} \cdot \bar{v}_{C_j} + \bar{M}_{ij} \cdot \bar{\omega}_j) + \sum_{j=1}^5 \bar{F}_{iEj} \cdot \bar{v}_{Ej} + \bar{F}_3 \cdot \bar{v}_D + \bar{F}_7 \cdot \bar{v}_{O_2} = 0 \quad (16)$$

where: $G_{Ej} = m_{Ej} \cdot g, j = \overline{1,5}$, are the weight forces corresponding to the balancing masses; $\bar{F}_{iEj} = -m_{Ej} \cdot \bar{a}_{Ej}, j = \overline{1,5}$, are the inertia forces corresponding to the balancing masses; $\bar{v}_{Ej}, \bar{a}_{Ej}, j = \overline{1,5}$, are the speeds and the accelerations, respectively of the points where the balancing masses are concentrated.

When the mechanism is partially balancing the calculus relation of the motor moment becomes:

$$\bar{M}_m \cdot \bar{\omega}_1 + \sum_{j=1}^7 \bar{G}_j \cdot \bar{v}_{C_j} + \bar{G}_E \cdot \bar{v}_E + \sum_{j=1}^7 (\bar{F}_{ij} \cdot \bar{v}_{C_j} + \bar{M}_{ij} \cdot \bar{\omega}_j) + \bar{F}_{iE} \cdot \bar{v}_E + \bar{F}_3 \cdot \bar{v}_D + \bar{F}_7 \cdot \bar{v}_{O_2} = 0 \quad (17)$$

where: $G_E = (m_E + m_{Es}) \cdot g$; $\bar{F}_{iE} = -(m_E + m_{Es}) \cdot \bar{a}_E$; \bar{v}_E, \bar{a}_E , are the speed and the acceleration, respectively of the point where the balancing mass $m_E + m_{Es}$ is concentrated.

Figure 5 shows the variation on a cinematic cycle of the motor moment M_m when the mechanism is unbalanced (curve 1) and when is totally balanced (curve 2) and partially balanced (curve 3).

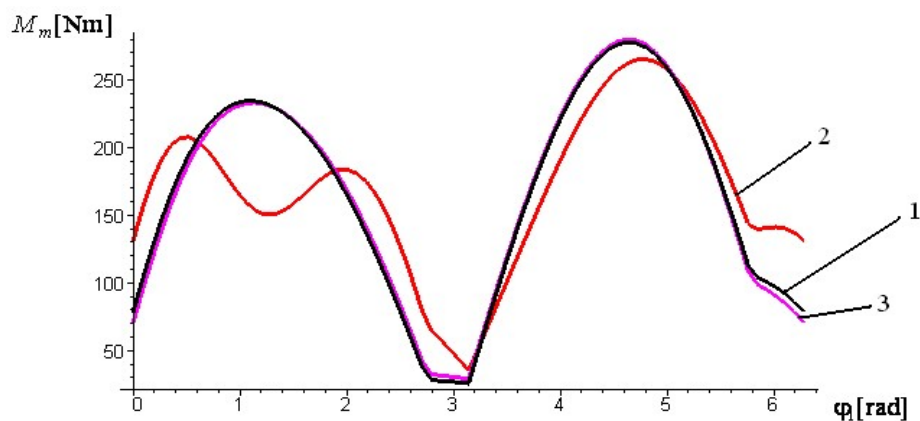


Figure 5. The variation on a cinematic cycle of the motor moment: mechanism unbalanced (curve 1); mechanism totally balanced (curve 2); mechanism partially balanced (curve 3).

Figure 5 shows that by partially balancing the mechanism the variation of the motor moment practically does not change, while in the case of totally balancing the extreme values of the motor moment have slightly decreases.

5. Conclusions

In this paper were presented some possibilities of statically balancing a plane mechanism. After presenting a statically total balancing solution of the mechanism it was proposed a solution for a partial balancing based on the balancing of the first harmonic of the inertia force developed in a piston of the mechanism. The simulations results showed a significant decrease of the total inertia force in the case of partial balancing of the mechanism under conditions of very small increase of its gauge. Also, it was analyzed the variation of the motor moment when the mechanism is unbalanced and when is totally and partially balanced. The simulations results showed in the case of totally balancing that the extreme values of the motor moment have slightly decreases compared to cases where the mechanism is unbalanced or partially balanced.

References

1. Bădoiu D., The Dynamic Analysis of Mechanisms and Machines (in Romanian), Didactical and Pedagogical Publishing House, Bucuresti, 2003.
2. Shigley J.E. and Uicker Jr.J.J., Theory of Machines and Mechanisms, McGraw-Hill, 1981.
3. Bădoiu D., Structural and Cinematic Analysis of the Mechanisms (in Romanian), Technical Publishing House, Bucuresti, 2001.
4. Bădoiu D., Petrescu M.G., Toma G., On the analysis and synthesis of the quadrilateral mechanism, Journal of the Balkan Tribological Association, vol. 20, 161-168, 2014.
5. Monagan M.B., Geddes K.O., Heal K.M., Labahn G., Vorkoetter S.M., McCarron J. and DeMarco P., Maple Introductory Programming Guide, Maplesoft, a division of Waterloo Maple Inc., 2005.
6. Manolescu N., Kovacs Fr. and Oranescu A., Mechanisms and Machines Theory (in Romanian), Didactical and Pedagogical Publishing House, Bucuresti, 1972.
7. Bădoiu D., Toma G., On a dynamic optimization problem of the quadrilateral mechanism, Journal of the Balkan Tribological Association, vol. 22, 250-260, 2016.