

Identification of exponent from load-deformation relation for soft materials from impact tests

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Abstract. When two bodies are brought into contact, the magnitude of occurring reaction forces increase together with the amplitude of deformations. The load-deformation dependency of two contacting bodies is described by a function having the form $F = Cx^\alpha$. An accurate illustration of this relationship assumes finding the precise coefficient C and exponent α . This representation proved to be very useful in hardness tests, in dynamic systems modelling or in considerations upon the elastic-plastic ratio concerning a Hertzian contact. The classical method for identification of the exponent consists in finding it from quasi-static tests. The drawback of the method is the fact that the accurate estimation of the exponent supposes precise identification of the instant of contact initiation. To overcome this aspect, the following observation is exploited: during an impact process, the dissipated energy is converted into heat released by internal friction in the materials and energy for plastic deformations. The paper is based on the remark that for soft materials the hysteresis curves obtained for a static case are similar to the ones obtained for medium velocities. Furthermore, utilizing the fact that for the restitution phase the load-deformation dependency is elastic, a method for finding the α exponent for compression phase is proposed. The maximum depth of the plastic deformations obtained for a series of collisions, by launching, from different heights, a steel ball in free falling on an immobile prism made of soft material, is evaluated by laser profilometry method. The condition that the area of the hysteresis loop equals the variation of kinetical energy of the ball is imposed and two tests are required for finding the exponent. Five collisions from different launching heights of the ball were taken into account. For all the possible impact-pair cases, the values of the exponent were found and close values were obtained.

1. Introduction

The impact phenomenon is frequently encountered both in everyday life and in applied engineering, and its main characteristic is the sudden variation of the velocity of the components of a system but with a less evident change of their position. Two stages can be identified during an impact phenomenon: *the compression phase* defined as the time period between the instant when the first points of the two bodies come into contact, and the instant when the normal approach between the bodies reaches maximum, with the immediate consequence that at this moment the relative velocity between the two bodies is zero. The second stage, *the restitution phase*, starts once the compression phase ends and lasts until the instant when the last two points of the two contacting bodies detach.



In daily life the impact phenomenon happens any time when two solid bodies make direct contact. The affirmation is based on the observation that avoiding impact imposes that the two bodies are brought into contact at an infinitely small speed. This aim is practically impossible to attain and therefore when two bodies are contacting, the relative velocity is non zero and decreases rapidly, the order of impact time being in the range $10^{-3} \div 10^{-4} \text{ sec}$ [1]. The sudden velocity variation has as consequence the development of appreciable forces characterized by accelerations within the domain $10^3 g \div 10^4 g$. In applied engineering the impact phenomenon occurs, for example, in gear boxes when the rotation sense of a shaft is changed, in all joints with clearance [2-4] etc. Two manners of approach are distinguished in technical literature concerning the impact phenomenon. The first methodology accepts the hypothesis of rigid bodies, so the method is uncomplicated but has the drawback that the impact is considered instantaneous and the impact force cannot be estimated. The second manner accepts the assumption of deformable bodies and permits the identification of all kinematical and dynamical parameters of the system. A requisite parameter in the study of impact phenomenon is e , the normal coefficient of restitution, defined as the ratio between \mathbf{v}'' , the normal component of the post-impact relative velocity and \mathbf{v}' , the normal component of the pre-impact relative velocity:

$$e = - \frac{\mathbf{v}'' \cdot \mathbf{n}}{\mathbf{v}' \cdot \mathbf{n}} \quad (1)$$

where \mathbf{n} is the unit vector of the normal to the contacting surfaces.

2. Impact phenomenon modelling under the deformable bodies hypothesis

In impact modelling, besides the fundamental theorems of dynamics, a series of supplementary hypothesis must be considered for establishing the manner in which certain kinematical or dynamical parameters of the analyzed system change. The general study of the impact between two bodies of random shape is complex due to the complicate form of the equations describing the phenomena occurring during the entire process. The intricate form of differential equations makes it impossible to obtain an analytical solution and therefore numerical methods are applied in obtaining a particular solution. In order to obtain analytical solutions, many of the studies concerning the subject consider the impact between bodies having simple geometries. Timoshenko [5] applies the results from Hertzian contact theory and finds the maximum force, maximum normal approach and the impact time between two elastic spheres. Even for this very simple model, the consideration of some aspects which take it closer to an actual situation lead to a complexity difficult to surpass. Goldsmith [6] shows that in the case of impact, the kinetical energy variation is found as a work of internal friction, plastic deformation work, energy of elastic waves propagating through colliding bodies etc. Lankarani [7] proposes a nonlinear differential equation to describe the impact with internal friction between two metallic balls. To obtain the differential equation of the model, the hypothesis that the work of internal friction has the same value both for the compression phase (from contact initiation until the moment of maximum normal approach) and for the restitution phase is accepted. The hypothesis restricts the application of the model only for quasielastic bodies. Recently, Flores [8] corrected the Lankarani equation, starting from the assumption that in the phases plane, a current point moves on an elliptical trajectory, making it applicable for any type of impact. Both models, Flores and Lankarani, are based on the assumption that the impact phenomenon is not accompanied by plastic deformations. Considering the observation made by Goldsmith [9] who cites Davies, this hypothesis is hard to accept; Goldsmith shows that a $1/2 \text{ inch}$ diameter steel ball produces a plastic imprint on steel if the impact velocity is approximately $3 \text{ in/sec} \approx 0.076 \text{ m/s}$, corresponding to a free fall from a height of 0.3 mm . Johnson [10] confirms that the plastic deformation following the impact between a ball in free fall and an immobile horizontal plane requires a launching height of $\approx 1 \text{ mm}$. These experimental observations confirm the necessity of a model that considers the dissipation of impact kinetical energy as work of plastic deformation. In a recent work, Anderson [11] proposes a model for the study of impact considering that the variation of kinetical energy of the system is retrieved as work of internal

friction and work of plastic deformation, and experimentally validates the presence of the two types of work for the case of an automobile impacting a fixed obstacle. The hysteresis loop for the case of impact with internal friction and plastic deformation is presented in Figure 1.

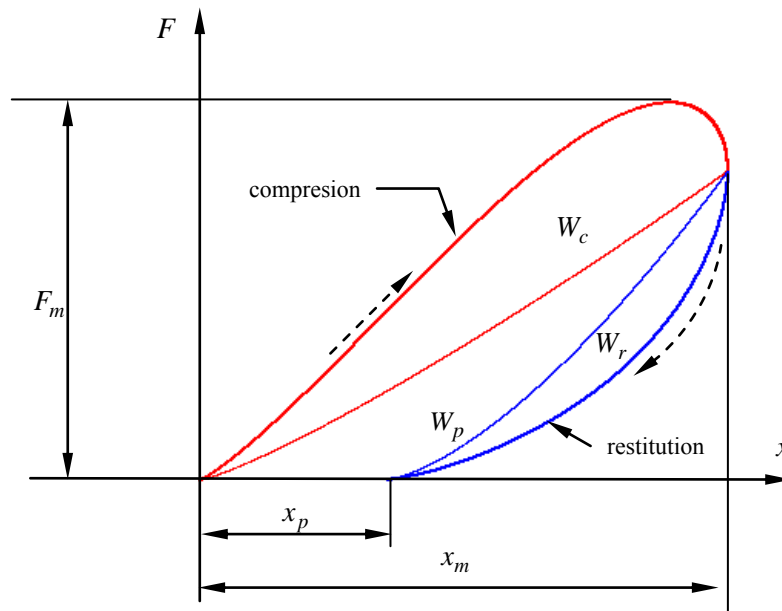


Figure 1. Hysteresis loop for the case of impact with internal friction and plastic deformation

The shape of the hysteresis curve is influenced by the ratio between the values of internal friction work $W_c + W_r$ and the plastic deformation work W_p and two limit cases can be distinguished: collision without plastic deformation, and collision only with plastic deformation. Goldsmith [6] presents the experimental hysteresis loops for several pairs of materials and identifies the cases mentioned above.

3. Theoretical considerations

A model for impact considering only plastic deformations was proposed by Lankarani [12]. More explicitly, he accepts the hypothesis that the variation of kinetical energy of a system consisting of two colliding balls is recovered as work of plastic deformation.

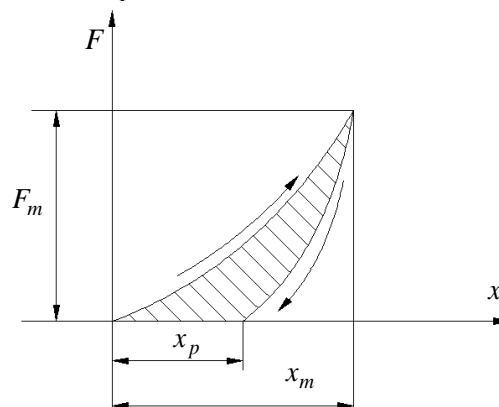


Figure 2. Impact force variation for Lankarani model

Noting by x the normal approach between the two balls and by x_m and x_p the values of maximum approach and permanent plastic deformation, respectively, Lankarani proposes for the interaction force represented in Figure 2 the expression:

$$F = \begin{cases} Cx^{3/2}, & 0 \leq x \leq x_m \\ C' \left(\frac{x - x_p}{x_m - x_p} \right)^{3/2}, & x_p \leq x \leq x_m \end{cases} \quad (2)$$

Assuming the geometrical and elastic characteristics of the balls are known, the initial relative velocity v' and the coefficient of restitution e , the expressions for maximum approach x_m and permanent deformation x_p can be found with the proposed model. It is obvious that for small impact velocities, the shape of hysteresis loops should be identical to the ones obtained for quasistatic contact. The hysteresis loops traced for both static and dynamic regime, presented by Goldsmith [6], confirm this hypothesis. In a recent work, Alaci [13] finds for the impact between a ball in free fall and a fixed horizontal surface, both made of steel, the coefficient of restitution and the depth of impact imprint and shows that the values of the residual imprint depth are approximately twice greater than the ones given by the Lankarani model. An analysis of the experimental results presented by Goldsmith, Figure 2, reveals that when one of the bodies is made of a soft material, the curves of impact force variation present a similar shape to the curves given by Lankarani's model. However, when both bodies are made of hard materials, the shape of the hysteresis curves changes and resembles the curves presented by Anderson [11]. In the curves given by Anderson the effect of internal friction cannot be neglected any longer. A first solution for finding the exponent from the force-deformation dependence consists in using the force-deformation curve obtained for quasistatic compression. The weak point of the method, as shown in [14], [15] is the poor accuracy in finding the instant when the contact initiation takes place. To avoid this drawback, a method is proposed for finding the law of impact force variation using hysteresis curves in the case of impact at low speeds.

4. New theoretical model

The proposed theoretical model is appropriate for the case of collision of soft bodies, characterised by reduced values of the coefficient of restitution and neglecting the effects of internal friction. The law of variation for the impact force is based on the remarks made by Goldsmith [9] and Tabor [16].

Thus, the impact force variation is according to the relation:

$$F = \begin{cases} Cx^\alpha, & 0 \leq x \leq x_m \\ C' \left(\frac{x - x_p}{x_m - x_p} \right)^\beta, & x_p \leq x \leq x_m \end{cases} \quad (3)$$

For the compression phase, the variation of the impact force obeys the Meyer's law and the α exponent:

$$1 \leq \alpha \leq 3/2 \quad (4)$$

takes the value $\alpha = 3/2$ for the case of perfect elastic materials and $\alpha = 1$ for the impact between perfect plastic materials. The assumption that during the restitution phase the plastic deformations do not occur any longer implies that:

$$\beta = 3/2 \quad (5)$$

Applying the theorem of kinetic energy for the compression phase, it results:

$$-\frac{1}{2} m_{red} v'^2 = -\int_0^{x_m} F dx = -\frac{C x_m^{\alpha+1}}{\alpha+1} \quad (6)$$

For the restitution phase, the work energy theorem leads to:

$$\frac{1}{2}m_{red}(e \cdot v')^2 = -\int_{x_m}^{x_p} C' \left(\frac{x - x_p}{x_m - x_p} \right)^\beta dx = \frac{C'(x_m - x_p)}{\beta + 1} \quad (7)$$

where m_{red} is the reduced mass of the system:

$$m_{red} = \frac{m_1 m_2}{m_1 + m_2} \quad (8)$$

The condition that the maximum force should be the same for both compression and restitution phase must be added to the equations (6) and (7):

$$C x_m^\alpha = C' \quad (9)$$

The relations (6), (7) and (9) are the available equations for an impact test. The values of the coefficients C and C' , the exponent α and the maximum normal approach x_m should be found in order to make possible the use of the relation (3). To evaluate the unknown parameters from relation (3), experimental tests were made using a bearing ball, 12.7mm diameter, as projectile, and a disc made of unhardened tool steel as target. Five launchings were made and the coefficient of restitution e was established by measuring the time elapsed between the first and the second impact of the ball with the target. The depth of the plastic imprint was measured using a laser scanner, Figure 3, and the profile of the imprint was approximated with a parabolic function as shown in Figure 3b. The results of the five tests are presented in Table 1.

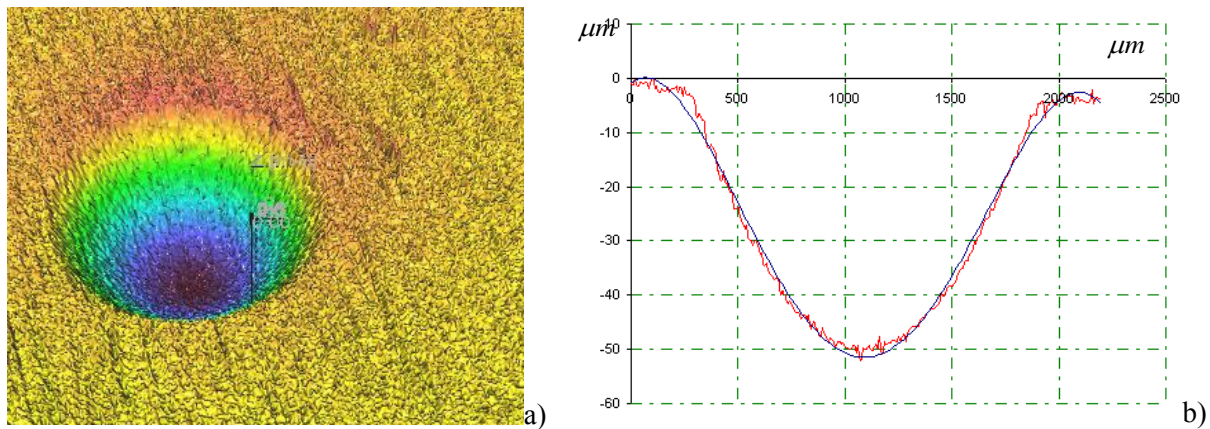


Figure 3. Evaluation of the depth of the plastic imprint

Table 1. Experimental impact parameters

Test no.	v' [m/sec]	e [–]	x_p [μm]
1.	2.334	0.530	23.5
2.	2.940	0.512	29.7
3.	4.089	0.479	41.6
4.	4.504	0.467	45.97
5.	4.993	0.453	51.03

It is assumed that for each impact test the loading is made on the same curve and the constants C and α have to be established. The unloading happens on different curves having the same exponent

$\beta = 3/2$ but with different values of the coefficient C' and depth x_{max} . Writing the equations for all $k = 1 \div 5$ collisions, the following is obtained:

$$\begin{cases} \frac{C x_{m_k}^{\alpha+1}}{\alpha+1} = \frac{1}{2} m_{red} v_k'^2 \\ \frac{C'_k (x_{m_k} - x_{p_k})}{\beta+1} = \frac{1}{2} m_{red} (e_k \cdot v_k')^2 \\ C x_{m_k}^\alpha = C'_k \end{cases} \quad (10)$$

The system has $3k$ equations and $2k+2$ (C, α, x_{m_k}, C'_k) unknowns. For $k > 2$ the system becomes incompatible and requires finding a solution that verifies the system with minimum error. This is a difficult task considering the form of the equations and the great number of unknowns. Therefore, another method for finding the approximate solution of the system is proposed here. For $k = 2$ the system is compatible. Writing the system of equations for $k = 2$ and eliminating at each step the unknowns, finally a single transcendental equation is obtained, with unknown α :

$$\frac{\alpha+1}{\beta+1} + \frac{x_{p_1}}{x_{p_2}} \left(e_2^2 - \frac{\alpha+1}{\beta+1} \right) \left(\frac{v_2'}{v_1'} \right)^{\frac{2}{\alpha+1}} - e_1^2 = 0 \quad (11)$$

There is no difficulty in solving this equation. Once the α parameter found, the other parameters are evaluated using the relations:

$$\begin{aligned} x_{m_2} &= x_{p_2} \frac{1}{1 - \frac{\beta+1}{\alpha+1} e_2^2}; \quad x_{m_1} = x_{m_2} \left(\frac{v_1'}{v_2'} \right)^{\frac{2}{\alpha+1}}; \\ C &= \frac{m_{red} v_2'^2}{2} \frac{\alpha+1}{x_{m_2}^{\alpha+1}}; \quad C'_1 = C x_{m_1}^\alpha; \quad C'_1 = C x_{m_1}^\alpha. \end{aligned} \quad (12)$$

Combining the five tests in all possible cases, a number of 10 values results for parameter α , given in Table 2. With these values, a row of points was generated (x_j, F_j) for each case with x_j taking values up to the maximum value of maximum approach found for a pair of collisions.

Table 2. Values of α parameter

1.234	1.275	1.286	1.299	1.303	1.315	1.327	1.355	1.367	1.379
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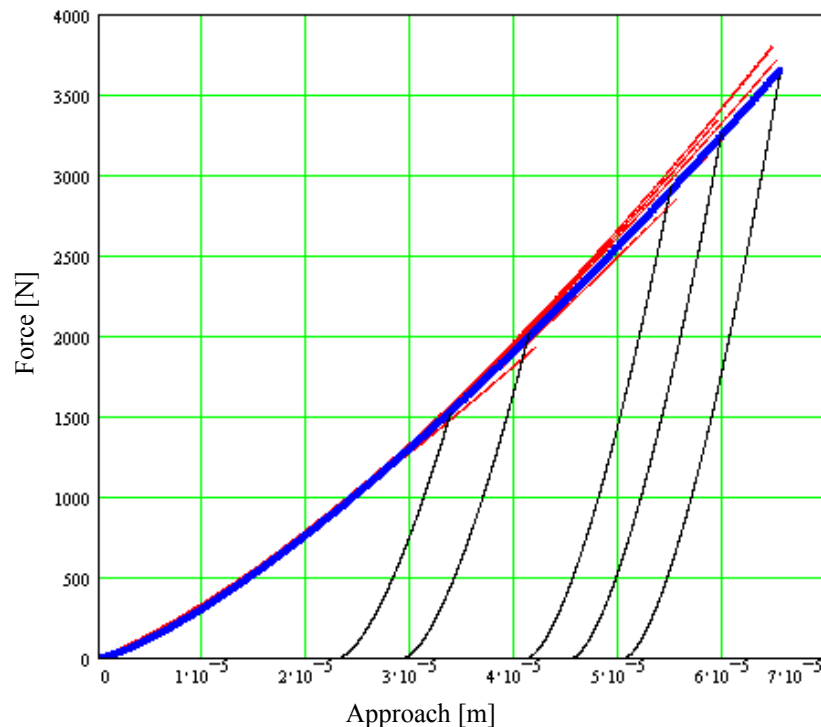
All these points are interpolated using a function having the form Cx^α , and the final values for C and α are established. It is noticed that the value found for the α exponent by interpolation, $\alpha = 1.313$ is actually the same as the mean value of the values from Table 2, $\alpha = 1.314$. With C and α now found, the maximum approach values are found using relation (6).

The values of coefficient C'_k can be estimated either from relation (7) or from (9), using the maximum approach values. In Table 3 there are presented the values of maximum approach x_m and the coefficient C' found via both manners.

Table 3. Calculated maximum approach x_m and coefficient C'

k	1	2	3	4	5
x_m [μm]	34.08	41.61	55.34	60.16	65.77
C' , (eq. 6)	1516	2009	2935	3257	3622
C' , (eq. 8)	1547	2010	2924	3263	3668

The correctness of the methodology used in finding the C coefficient is validated by the practically equal values of the C' coefficient found in two different manners. The hysteresis loops were represented on the same graph for the five cases considered in Table 3, as presented in Figure 4. The curves have an identical appearance to the ones presented by Goldsmith for the impact between a steel ball with $12.7mm$ diameter and the frontal face of a cylinder made of unhardened steel. The difference between the plots in Figure 4 and the loops given by Goldsmith are the values of maximum forces and maximum plastic deformations, differences explained by the values of initial impact velocity v' which were smaller in the test of present work, the launching being made via free fall, while for Goldsmith's experiments special launching devices were employed.

**Figure 4.** Generated points, the interpolation curve for compression and the restitution curves

5. Conclusions

The paper presents methodology for establishing the exponent from the force-deformation expression used for the compression phase of an impact between two bodies made of soft materials. The method is proposed as alternative to the classical method that assumes a quasistatic compression test for a nonconforming contact. The disadvantage of the classical method consists in the errors introduced due to imprecision in finding the instant of contact initiation. The proposed method is based on the remark that, for a system consisting from two metallic bodies that collide, the variation of kinetical energy of the system during impact is retrieved as work of internal friction and work of plastic deformation. A random collision should be placed between two limit models: a first model, for which the kinetical energy variation is converted into work of internal friction, and the second, for which the entire

kinetical energy variation is transformed into work of plastic deformation. The second limit case is applicable when one of the impacting bodies is made of a soft material.

Admitting the hypothesis of complete transformation of kinetical energy variation into plastic deformation work, a model of impact is proposed for which the compression is obtained following a power law curve with exponent to be found, and the restitution obeys a power law curve with the exponent $3/2$ that is characteristic to elastic domain. In order to find the exponent related to the compression phase, two impact tests between the bodies to be tested are required. The values of coefficient of restitution and the depth of plastic imprint must be evaluated for the two collisions. The coefficient of restitution is found via an acoustic method and the dept of plastic imprint is measured using laser profilometry. For experiments, a bearing ball made of hardened steel, with diameter of 12.7mm, falls free on the flat surface of an immobile prism made of mild steel. Five launchings of the ball were made from different heights. The values of the exponent and the coefficient from the impact force relation for restitution phase were found for all the combinations of two possible experiments. With these values, the hysteresis curves were represented on the same graph and the curves obtained have similar shapes to the experimental curves from literature.

The proposed method has the advantage of rapidity, and promptitude may be further improved if, for finding the depth of plastic imprint, the laser profilometry is replaced, and the maximum diameter of plastic imprint is measured, the depth resulting from the supposition that the residual imprint is a spherical cap.

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