Possibility of using the smoothed spline functions in approximation of average course of terrain inclinations caused by underground mining exploitation conducted at medium depth

J Orwat

Silesian University of Technology, Department of Mining and Geology, Akademicka street, no. 2, 44100 Gliwice, Poland

E-mail: justyna.orwat@polsl.pl

Abstract. In this paper was presented an obtainment way of the average values of terrain inclinations caused by an exploitation of the *338/2* coal bed, conducted at medium depth by four longwalls. The inclinations were measured at sections of measuring line established over the excavations, perpendicularly to their runways, after the termination of subsequent exploitation stages. The average courses of measured inclinations were calculated on the basis of average values of measured subsidence obtained as a result of an average-square approximation done by the use of smooth splines, in reference to their theoretical values calculated via the S. Knothe's and J. Bialek's formulas. The typical values of parameters of these formulas were used. Thus it was obtained for two average courses after the ending of each exploitation period. The values of standard deviations between average and measured inclinations σ_I and variability coefficients of random scattering of inclinations M_I were calculated. Then they were compared with the values appearing in the literature and based on this the possibility evaluation of use smooth splines to determination of average course of observed inclinations of mining area was conducted.

1. Introduction
The results of geodesic measurements conducted by the use of standard methods and equipment, namely lengths and angles between measuring points based on which the situational coordinates of points are calculated, and heights differences between the points based on which the altitude coordinates of these points are calculated, are distorted by some errors. These errors are mainly binding with an accuracy of measuring devices, a human factor and the variable weather conditions.

Of course the errors appear also in case of surveys made by the use of The Global Navigation Satellite Systems or even The Unmanned Aerial Vehicles (*UAV*) [1-3]. These errors come from the additional sources like satellites constellation, troposphere parameters or gradient angle of camera lens to the terrain surface.

The above errors and the factors such as: a type and a shape of observational system (particularly the distances between measuring points); a range, a multiple [4], [5], a height and a depth of hard coal excavation; the geological structure of rock mass and its fissures [6], [7], and a random factor have an influence on the values and their uncertainties of deformation indicators of mining area determine

from the results of surveys [8]. For these reasons the courses of measured deformation indicators (inter alia subsidence, inclinations) are irregular and characterized by the fluctuation (a random dispersion).

It's obvious that it should be identified the average courses of these indicators. So far the orthogonal polynomials have been mainly used to designate the average courses of measured values of subsidence, inclinations and horizontal strains. The problem with their use depends on determination of an optimal degree of a polynomial for a given indicator and it requires division of a whole profile of subsidence through into some parts [9].

Therefore in this article it has been analysed the use possibility of the smooth splines functions for designation of average courses of terrain inclinations, which were caused by an underground exploitation of hard coal bed conducted on the medium depth.

2. The courses of measured inclinations

In this section have been presented: an example of hard coal extraction from the bed located on medium depth, a description of surveys made on the points of observational line located above the exploitation field and an obtainment way of the courses of measured inclinations.

2.1. A description of made extraction

Some hard coal mine situated within borders of The Upper Silesian Coal Basin in Poland was running the exploitation of coal bed marked as *338/2* with four longwalls [10].

These longwalls were numbered as *001, 002, 005* and *007*. The depth of an extraction was amounted from 580 m in the northern part of region (longwall no. *001*) to 700 m in the southern part of coal bed (longwall no. *007*). The average length of longwalls was close to 250 m and their runs were amounted from 750 m (longwall no. *001*) to 1080 m (longwall no. *007*). The coal bed was exploited at a height of 2 m with the roof rocks fall to the post-mining emptiness.

It needs to be highlighted that an exploitation was conducted in the rock mass which was violated an earlier excavation. The over layers thickness amounted to 60 m. These layers were mainly formed in Quaternary and Triassic periods.

2.2. A description of made surveys

The deformations of mining area caused by an exploitation of four longwalls in coal bed *338/2* were observed on measuring line no. *1*. It was established perpendicularly to the runs of longwalls and consisted of 53 points. The average distance between measuring points was amounted 37 m.

The measurements of the points heights were carried out by the use of precise leveller, with an accuracy of 0.1 mm. There was used a double geometric levelling made from the middles of segments between observational points.

The sections lengths were measured using a distance-meter device, with an accuracy of 1 mm.

The points heights and segments lengths were determined in the same time, after the termination of each exploitation stage. Therefore it was obtained the four observation cycles which demonstrate the impacts coming from an exploitation of longwalls *001*; *001* and *002*; *001, 002* and *005*; *001, 002, 005* and *007*.

2.3. The obtainment way of measured inclinations courses

On the basis of measurements of points heights before and after the termination of subsequent exploitation stages in coal bed *338/2*, there were designated the subsidence of all points from the formula (1) [11].

$$W_i^{k+1} = H_i^{k+1} - H_i^k,\qquad(1)$$

where: H – height of point; W – subsidence; i – number of point; k – number of observational cycle.

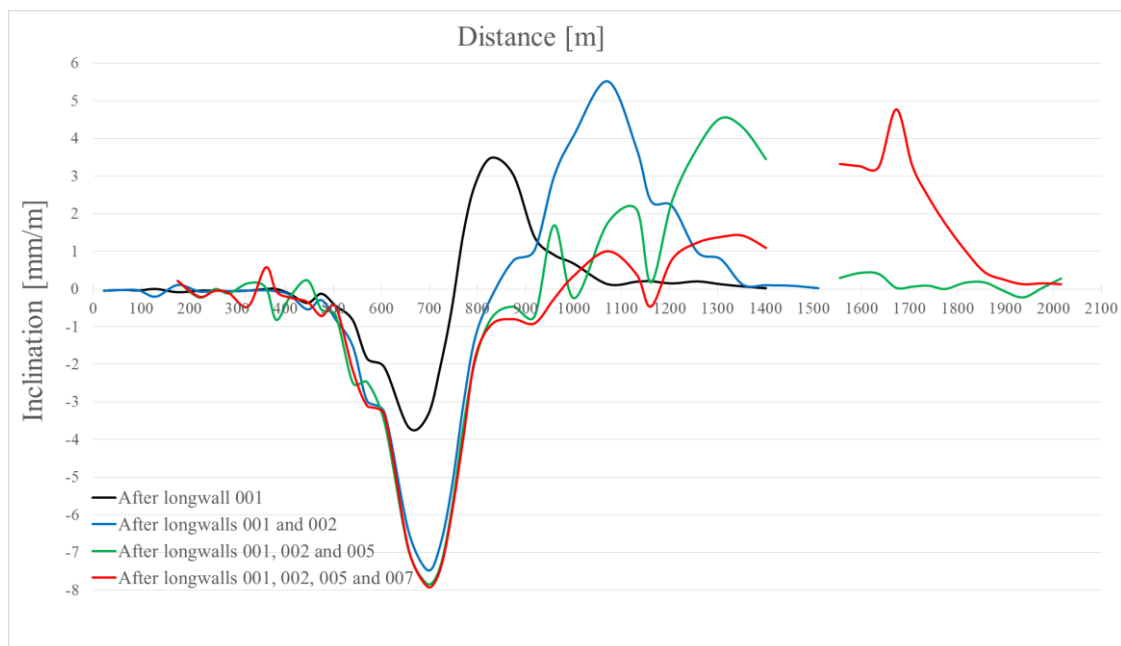The courses of measured subsidence were shown at the Figure 1.
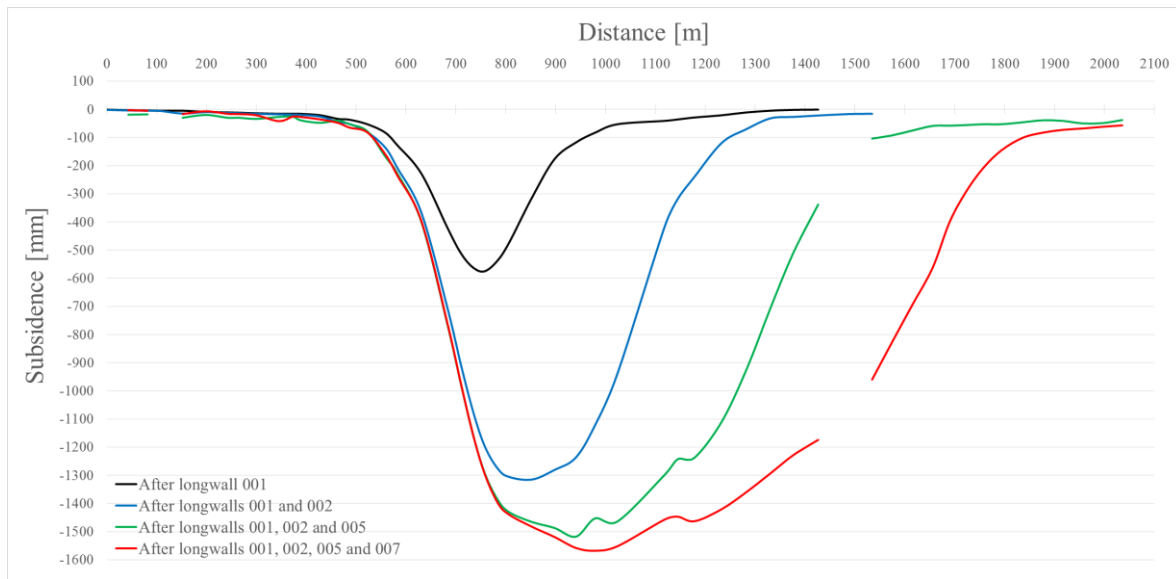
Figure 1. The courses of subsidence measured after termination of the subsequent exploitation stages in coal bed *338/2*

Based on the subsidence calculated in two neighboring points, it can be determined an inclination of segment between these points from the following formula [11]:

$$T_{i,i+1}^{k+1} = \frac{W_{i+1}^{k+1} - W_i^{k+1}}{l_{i,i+1}}, \tag{2}$$

where: T – inclination; W – subsidence; i – number of point; k – number of observational cycle; l – measured length of segment.

The courses of measured inclinations were shown at the Figure 2.

Figure 2. The courses of inclinations measured after termination of the subsequent exploitation stages in coal bed *338/2*

3. The average courses of measured inclinations

In this section have been shown how to obtain the average values of measured inclinations by the use of smooth spline functions and the basic rules of work of *smooth.spline* function from the *R* computer programme.
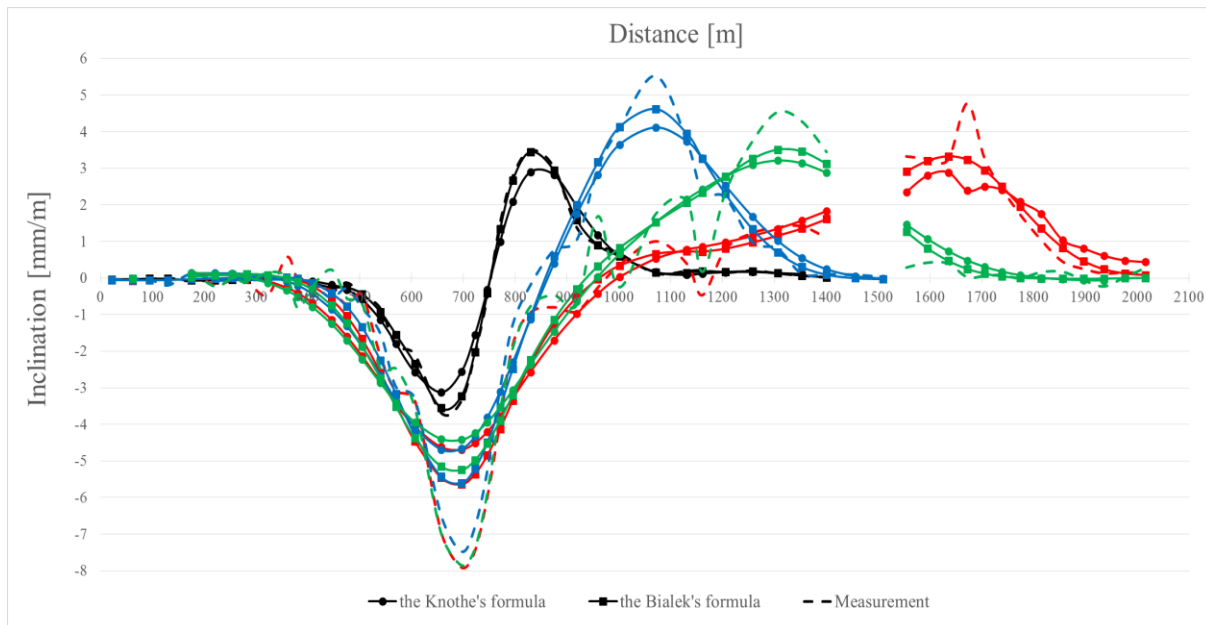
3.1. Calculation of average values of measured inclinations

The average values of inclinations measured after the termination of exploitation in subsequent longwalls were calculated basing on the formula (3) [12]:

$$T_{i,i+1}^{aver} = \frac{W_{i+1}^{approx} - W_i^{approx}}{l_{i,i+1}}, \qquad (3)$$

where: T^{aver} – average value of inclination; W^{approx} – approximated value of subsidence; i – number of point; l – measured length of segment.

The average courses of measured inclinations calculated in relation to the Knothe's (circular signs) and Bialek's (square signs) formulas were shown at the Figure 3.

Figure 3. The average courses of measured inclinations obtained in reference to their theoretical values calculated by the use of the Knothe's and Bialek's formulas

For comparison of measured inclinations with their average values, there were calculated:
− r_T – correlation coefficient;
− σ_T – standard deviation of inclinations;
− P_T – percentages of the extreme measured values of inclinations which correspond the extreme average values of measured inclinations.

There were also calculated the values of variability coefficient of random dispersion of inclinations M_T as a standard deviation of inclinations σ_T divided by the module of average value of measured extreme inclination.

Table 1. The correlation between measured inclinations and their average values

Parameter	Reference	
	The Knothe's formula	The Bialek's formula
After longwall 001		
r_T	0.9815	0.9978
σ_T [mm/m]	0.31	0.10
M_T [%]	10.07	2.77
P_{Tmax} [%]	83.09	98.85
P_{Tmin} [%]	84.78	96.47
After longwalls 001 and 002		
r_T	0.9502	0.9738
σ_T [mm/m]	0.89	0.63
M_T [%]	19.04	11.31
P_{Tmax} [%]	74.64	83.70
P_{Tmin} [%]	62.70	74.87
After longwalls 001, 002 and 005		
r_T	0.9069	0.9398
σ_T [mm/m]	1.15	0.92
M_T [%]	25.98	17.51
P_{Tmax} [%]	70.70	77.31
P_{Tmin} [%]	56.31	66.75
After longwalls 001, 002, 005 and 007		
r_T	0.9245	0.9618
σ_T [mm/m]	1.09	0.76
M_T [%]	23.25	13.51
P_{Tmax} [%]	60.59	69.81
P_{Tmin} [%]	59.22	71.21

3.2. Approximation of average values of measured subsidence
The average values of measured subsidence were obtained as a result of least – square approximation conducted by the use of smooth spline in the *R* programme. There was used the *smooth.spline* function for that.

To the basic arguments of this function belong:

− *df* – the number of the degree of freedom assuming values from *0* to *n,* where *n* is the number of measuring points;
− *spar* − smoothing parameter of an approximating function assuming values from *0* to *1,* where value *0* denotes a total lack of smoothing and then an approximating function assumes the form of an approximated function;
− *λ* – penalized criterion (*PC*);
− *cv* – cross validation, when *cv = truth*, then we deal with *leave-one-out,* if *cv = false*, then we deal with *general cross validation* (*GCV*);
− *nknots* – number of knots, if *all.knots = false*, the value is smaller than the number of measuring points.

The least − square approximation was done in reference to the theoretical values of subsidence. There were calculated via the Knothe's formula [13] with the standard values of its parameters ($a = 0.8$; $tg\beta = 2.0$; $A_{obr} = 0$) and via the Bialek's formula with the typical values of its parameters ($a = 0.8$; $tg\beta = 2.3$; $A_{obr} = 0.15$). The detailed information about the reasons of such procedure were shown in the works [12], [16].

The parameters values of approximation of subsidence measured after the ending of particular exploitation stages have been juxtaposed in the Table 2.

Table 2. The parameters values of made approximation

Parameter	Reference of approximation	
	The Knothe's formula	The Bialek's formula
After longwall 001		
df	11.88	23.07
spar	0.47	0.32
λ	3.67×10^{-5}	2.02×10^{-6}
PC [mm^2]	5841	225
After longwalls 001 and 002		
df	7.24	10.54
spar	0.63	0.55
λ	3.90×10^{-4}	7.73×10^{-5}
PC [mm^2]	135434	35184
After longwalls 001, 002 and 005		
df	7.40	10.29
spar	0.68	0.61
λ	4.39×10^{-4}	1.06×10^{-4}
PC [mm^2]	254332	107365
After longwalls 001, 002, 005 and 007		
df	8.23	11.50
spar	0.66	0.58
λ	2.81×10^{-4}	6.46×10^{-5}
PC [mm^2]	205941	66549

The average courses of measured subsidence, designated in reference to the theoretical values of subsidence which were calculated by the use of the Knothe's (circular signs) and Bialek's (square signs) formulas with typical values of their parameters, were shown at the Figure 4.
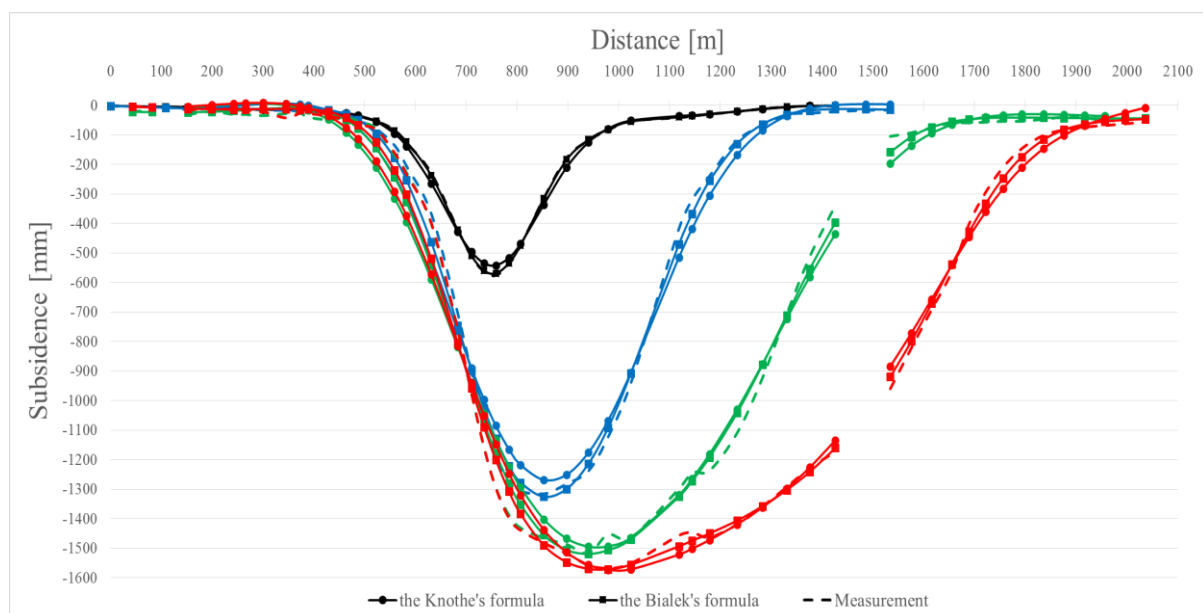
Figure 4. The approximated courses of subsidence measured after termination of the subsequent exploitation stages in coal bed *338/2*

The correlation between the measured and approximated values of subsidence has been shown by the usage of correlation coefficient r_W, standard deviation of subsidence σ_W and percentage P_W of the extreme measured value of subsidence which corresponds the extreme average value of measured subsidence. It was also calculated the value of variability coefficient of random dispersion of subsidence M_W as a standard deviation divided by the module of extreme average measured value.

Table 3. The correlation between measured and approximated subsidence

Parameter	Reference of approximation	
	The Knothe's formula	The Bialek's formula
After longwall 001		
r_W	0.9978	0.9999
σ_W [mm]	13.11	2.47
M_W [%]	2.42	0.43
P_W [%]	94.25	98.95
After longwalls 001 and 002		
r_W	0.9935	0.9982
σ_W [mm]	58.93	30.04
M_W [%]	4.64	2.27
P_W [%]	96.48	100.81
After longwalls 001, 002 and 005		
r_W	0.9923	0.9967
σ_W [mm]	73.56	46.81
M_W [%]	4.92	3.08
P_W [%]	98.47	100.06
After longwalls 001, 002, 005 and 007		
r_W	0.9944	0.9982
σ_W [mm]	64.83	36.85
M_W [%]	4.12	2.35
P_W [%]	100.28	100.18

4. Discussion of the results

The data included in the Table 2 indicate that the values of smoothing parameter *spar* of function approximating the measured courses of subsidence grow generally with the increase of exploitation range: from 0.47 after the ending of exploitation in first longwall to 0.68 after the termination of excavation in three longwalls (in case of approximation made in reference to the Knothe's formula), and from 0.32 after the first stage of exploitation to 0.61 after the third exploitation stage (in case of approximation made in reference to the Bialek's formula). It means that the approximation function reflects the measurement data in a lesser degree, when the exploitation proceeds. Above statement is confirmed by the values of penalized criterion (a residual sum of squares), which also grow together with the number of exploitation stage.

From the data juxtaposed in the Table 3 it can be seen that the values of correlation coefficient between the measured and approximated subsidence r_W decrease with the increase of exploitation range. This coefficient has reached the maximal values after first stage of extraction (0.9978 for the Knothe's formula and 0.9999 in case of the Bialek's formula). The smallest values it has reached after three longwalls (0.9923 in case of the Knothe's formula and 0.9967 for the Bialek's formula). However, its values are still high and don't come down below 0.99.

The values of subsidence standard deviation σ_W rise together with the development of exploitation in the time. It should be emphasized that for the Bialek's formula they are almost two times smaller than in the case of the Knothe's formula.

Taking into account the values of correlation coefficient of subsidence r_W and standard deviation of subsidence σ_W it can be said that the reproduction accuracy of subsidence measured courses by the approximated courses goes down with an increase of exploitation range. It's related to a random factor because its participation in the measured values of subsidence grow with the excavation development, what is confirmed by the increasing values of variability coefficient of subsidence random dispersion M_W. Its values are also two times smaller for the Bialek's formula (like in case of standard deviation).

It's surprising that the percentages of the maximal measured values of subsidence which correspond the maximal average values of measured subsidence P_W are higher for more complicated exploitation.

The correlation coefficients of inclinations r_T included in the Table 1 decrease after each exploitation stage, like in case of subsidence. Their values amount from 0.9815 to 0.9069 (for the Knothe's formula) and from 0.9978 to 0.9398 (for the Bialek's formula). As it can be seen, these values are smaller than the values of subsidence correlation coefficient r_W.

The standard deviation of inclinations σ_T increases with the exploitation development, from 0.31 mm/m after first longwall to 1.15 mm/m after three longwalls (the Knothe's formula) and from 0.10 mm/m after first longwall to 0.92 mm/m after three longwalls (the Bialek's formula). However, the differences between its values reached for the Knothe's and Bialek's formulas aren't so visible as for subsidence.

The values of variability coefficient of inclinations random dispersion M_T are higher for the Knothe's formula than for the Bialek's formula. They are almost always bigger than the values occurring in the literature. The maximal value 26 % after the third excavation stage is almost three times higher than 9.3 % identified by Kowalski in the work [8].

In case of inclinations the P_T values go down when an exploitation is developing.

5. Conclusions
It's possible to obtain the average courses of terrain inclinations measured after the termination of exploitation conducted at medium depth by the use of smoothed splines, but taking into account some conditions.

An approximating function (a smoothed spline here) should be parameterized by the use of shape parameter (*spar* here), which is responsible for fit of approximating function to the measured data.

It's recommended to do an approximation of measured courses of subsidence first and on this basis can be calculated the average courses of measured inclinations. In this way it's also possible to obtain the average courses of measured curvatures of mining area [17-19].

It should be an average – square approximation because this type of approximation ensures the greatest degree of participation elimination of random factor in the measured values.

The minimization of residual sum of squares should be referred to the theoretical courses because this criterion guarantees an obtainment of average measured courses similar to the model courses.

An accuracy of reproduction of measured courses by the approximating function is dependent on an adopted model of deformation forecasting [11], [13-15].

The reference to the Bialek's formula gives the better results than in case of the Knothe's formula. It's related to that the Bialek's formula is a three – parametric model which takes into account an existence of exploitation rim. The A_{obr} parameter is responsible for this. The Knothe's formula is only a two – parametric model [20].

Deviation of average measured courses from the measurements results is greater for the inclinations than for subsidence. Its values are increasing together with the development of exploitation in the time.

The values of variability coefficient of inclinations random dispersion M_T exceed its values occurring in the literature. It's caused by the standard values of models parameters adopted to the calculation of theoretical courses of subsidence.

To improve the results, the parameters values determined directly from the results of geodesic measurements should be used. As can be seen, the values of M_T coefficient are dependent from the parameters values of adopted model [21].

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