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Determination of the critical bending speeds of a multy-rotor shaft from the vibration signal analysis

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Abstract. The purpose of this paper is the development and validation of an impulse excitation technique to determine flexural critical speeds of a single rotor shaft and multy-rotor shaft. The experimental measurement of the vibroacoustic response is carried out by using a condenser microphone as a transducer. By the means of Modal Analysis using Finite Element Method (FEM), the natural frequencies and shape modes of one rotor and three rotor specimens are determined. The vibration responses of the specimens, in simple supported conditions, are carried out using algorithms based on Fast Fourier Transform (FFT). To validate the results of the modal parameters estimated using Finite Element Analysis (FEA) these are compared with experimental ones.

1. Introduction

Rotating machinery is seen in most industrial or domestic applications of our everyday life. During their operation rotating machines are carrying a kinetic energy of rotation and a vibratory kinetic energy. The purpose of rotor dynamics is to determine how a ratio between vibrational and rotational energy can be kept as small as possible. This can be done if the operation of the machines takes place in a speed range far away from the critical speeds.

The first reference to a limit speed over which rotating machines can not function safely was Rankine [1], which introduced the term whirling into the rotor dynamics vocabulary. The proposed model for studying the dynamics of a rotor was with two-degree system of freedom, which did not take into account the effect of Coriolis's inertial force, leading to the unlimited increase of radial deformations. This model has caused confusion amongst engineers and has delayed the development of turbo machines for nearly 50 years.

Laval was the first engineer who felt the operation of the machinery over the critical speed building a steam turbine to operate at 40 000 rpm. Dunkerly [2] and Föppl [3] have continued investigations in this area recognizing the existence of several critical speeds which in some circumstances, coincides with the natural frequencies of the rotor shaft system.

The first fundamental work in the field of rotor dynamics was that of Jeffcott [4], in which he considered a model of flexible shaft without mass and a rigid disc placed in the middle of the shaft, and which confirmed Laval and Föppl predictions. This model is named Jeffcott Laval. Important developments in rotor dynamics have been made by Stodola [5], which, among other things, has laid down some approximate methods for determining critical speeds. The graphical procedure proposed by him was widely used until the introduction of the transfer matrix method by Myklestad [6] and Prohl [7].

With the development of digital computer capacity, a series of codes have been developed to solve numerical problems in rotor dynamics. The Finite Element Method (FEM) became the strongest and most used code since the 1970s. Now, with the explosion of computer computing power, FEM techniques can be combined with model techniques to generate simulations that allow coupled behavior of flexible discs, flexible shafts, and flexible support structures into a single, massive, multidimensional model [8].

In this paper an approximate analytical / analytical determination of the critical speeds is made, for a shaft with a rotor and a three-rotor shaft. The results are compared with those obtained numerically by applying the Modal Analysis Method using the meshing technique based on the Finite Element Method. In the experimental way is developed a method for determination the critical speeds from the analysis of some vibroacoustic signals that validate the results obtained by the analytical or numerical methods.

2. Analytical analysis

Figure 1 shows a negligible mass shaft relative to the mass m of a Jeffcott rotor, placed midpoint between the rigid bearings.

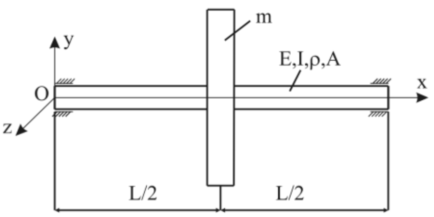

Figure 1. A Jeffcott rotor model in xoy plane

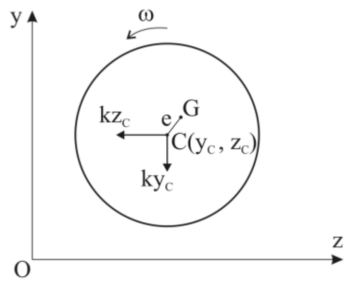

Figure 2. Free body diagram of the rotor in yoz plane

The geometric center of the disk C is located at the point (y_C, z_C) along coordinate axis defined about the bearing center line, and the disk center of mass G is located at (y_G, z_G).The vector **e** connecting the points C and G, represents the unbalance in the rotor disk. The rotating speed of the disk is given by ω. The lateral bending stiffness at the axial center of a simply supported uniform shaft is given by

$$k = \frac{48EI}{L^3}, \tag{1}$$

where E is the elastic modulus of the shaft, L is the length between the bearings, and I is the geometric inertia moment of cross area section. For a uniform cylindrical shaft with diameter d, the equation for the area moment of inertia is

$$I = \frac{\pi d^4}{64}. \tag{2}$$

The dynamic equations for the rotor are derived by applying Newton’s law of motion to the rotor disk. With the assumption that the shaft is massless, the forces acting on the disk are the inertial force, the stiffness and damping forces generated by the lateral deformation of the shaft. The lateral equations of motion in the y- and z-axes, in terms of the disk geometric center as shown in Figure 2 are found to be

$$m\ddot{y}_C + c\dot{y}_C + ky_C = m\omega^2 e\cos\omega t \tag{3}$$

$$m\ddot{z}_C + c\dot{z}_C + kz_C = m\omega^2 e \sin\omega t\,, \tag{4}$$

where c is the corresponding damping.

The undamped free vibration analysis deals with the rotor vibration in the case of negligible unbalance eccentricity and damping. The equations of motion are

$$m\ddot{y}_C + ky_C = 0 \tag{5}$$

$$m\ddot{z}_C + kz_C = 0\,. \tag{6}$$

The above equations have the same undamped characteristic equation

$$ms^2 + k = 0 \tag{7}$$

Solving the above equation it is obtain the following solution:

$$s_{1,2} = \pm i\sqrt{\frac{k}{m}} = \pm i\omega_n\,, \tag{8}$$

where ω_n is the undamped natural frequency of the single rotor with the massless shaft. The undamped critical speed of the system is defined as

$$n_{cr} = \frac{30}{\pi}\sqrt{\frac{48EI}{mL^3}}\,[\text{rpm}] \tag{9}$$

Because in the experimental determinations the sample has the mass of the shaft comparable to that of the disc, it will also take into account the mass of the shaft through the density ρ, the length L and the area of the section A.

Adopting Euler Bernoulli classical theory of simple supported beam the equation of motion for a uniform shaft with a rotor disk (Figure 1) can be written as [9]:

$$EI\frac{\partial^4 V(x,t)}{\partial x^4} + \rho A\frac{\partial^2 V(x,t)}{\partial t^2} + m\delta(x - L/2)\frac{\partial^2 V(x,t)}{\partial t^2} = 0 \tag{10}$$

where δ is Dirac function.

Using Ritz method [10], a series approximate analytical solution of Eq. 10 is assumed to be:

$$V(x,t) = \sum_{i=1}^{n}\Phi_i(x)q(t)_i = \{\Phi\}^T\{q\} \tag{11}$$

where $\Phi_i(x)$ are the mode shape functions assumed to satisfy geometric boundary conditions of the shaft, and $q_i(t)$ are the generalized coordinates.

The total kinetic energy of shaft whit a rotor is given by:

$$E_c = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial V(x,t)}{\partial t}\right)^2 dx + \frac{1}{2}m\left(\frac{\partial V(x,t)}{\partial t}\right)^2_{x=L/2} = \frac{1}{2}\{\dot{q}\}^T\left([M_1]+[M_2]\right)\{\dot{q}\} = \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\} \tag{12}$$

where the matrix $[M_1]$ and $[M_2]$ are:

$$[M_1] = \rho A\int_0^L \{\Phi\}\{\Phi\}^T dx; [M_2] = m\left[\{\Phi\}\{\Phi\}^T\right]_{x=L/2} \tag{13}$$

The total potential energy of the system is given by:

$$E_p = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 V(x,t)}{\partial^2 x}\right)^2 dx = \frac{1}{2}\{q\}^T[K]\{q\} \tag{14}$$

where the stiffness matrix [K] is:

$$[K] = EI\int_0^L \frac{\partial^2}{\partial x^2}\{\Phi\}\frac{\partial^2}{\partial x^2}\{\Phi\}^T dx \tag{15}$$

It should be noted that the transverse displacement V (x, t) can take place both in the xoy plane and in the xoz plane. In the following, the two displacement functions can be written:

$$Y(x,t) = \sum_{i=1}^{n} Y_i(x)q_i(t); Z(x,t) = \sum_{i=1}^{n} Z_i(x)q_i(t) \tag{16}$$

where both the $Y_i(x,t)$ and $Z_i(x,t)$ functions are admissible functions. The number n of the chosen admissible functions determines the number of eigenfrequencies and the eigenvectors.

Applying the theorem of conservation of mechanical energy

$$E_c + E_p = \text{const} \tag{17}$$

and after derivation in respect to time, the differential equation that governs the movement of the system in matrix form is obtained:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\}. \tag{18}$$

Assuming a modal solution

$$\{q\} = \{\mu\}e^{i\omega t}, \tag{19}$$

yields the eigenvalue problem

$$\left[-\omega^2[M] + [K]\right]\{\mu\} = \{0\}, \tag{20}$$

from which the characteristic equation, the approximate natural frequencies, eigenvectors and approximate eigenfunctions are obtained.

Given that the first vibration mode of the rotor shaft system is interested, it is necessary to choose a single admissible function for determining the critical speed. Consequently, the two functions which approximate the displacements in oxy plane and oxz plane are chosen such:

$$Y(x,t) = Z(x,t) = \sin\frac{\pi x}{L}\cos\omega t, \tag{21}$$

where ω is the fundamental circular frequency and the function sin πx/L is admissible function which the geometric boundary condition are exactly satisfied for simple supported shaft.

Applying this method to a shaft with a single rotor as in Figure 1, from the mechanical energy conservation theorem, for the admissible function given by equation (21), is obtained fundamental circular frequency

$$\omega^2 = \frac{\int_0^L EI\left(\frac{\pi}{L}\right)^4 \sin^2\frac{\pi x}{L}dx}{\int_0^L \rho A\sin^2\frac{\pi x}{L}dx + m\left(\sin\frac{\pi}{L}\frac{L}{2}\right)^2}, \tag{22}$$

which corresponds to the critical bending speed

$$n_{cr} = \frac{30}{\pi}\omega = \frac{30}{L}\pi\sqrt{\frac{EI}{(\rho AL + 2m)L}} \tag{23}$$

For the same rotor considering the shaft without mass, using the approximate analytical solution given by equation (21), the critical bending speed is

$$n_{cr} = \frac{30}{\pi}\omega = \frac{30}{L}\pi\sqrt{\frac{EI}{2mL}} \tag{24}$$

For a three-rotor shaft placed as in Figure 3, choosing the same admissible function as for a single rotor shaft, i.e. $\Phi(x) = \sin\pi x/L$, following the same procedure, the fundamental frequency of the bending vibrations is obtained

$$\omega^2 = \frac{\int_0^L EI\left(\Phi''(x)\right)^2 dx}{\int_0^L \rho A\Phi^2(x)dx + \sum_{i=1}^{n} m_i\Phi^2(x_i)}, \tag{25}$$

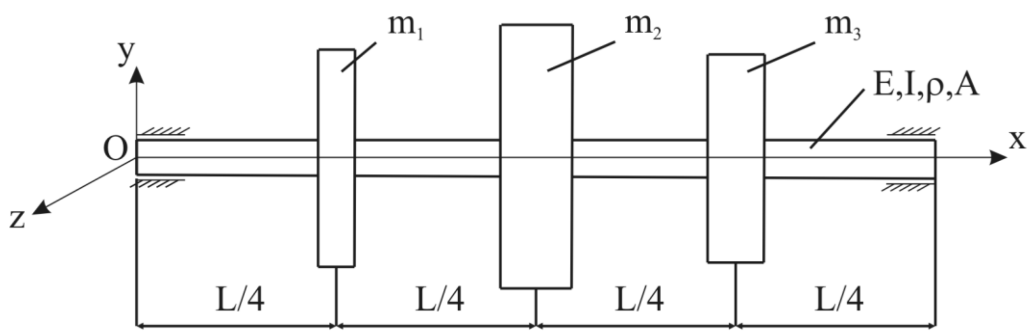

Figure 3. Three-rotor shaft

which corresponds to the critical bending speed

$$n_{cr} = \frac{30}{\pi}\omega = \frac{30}{L}\pi\sqrt{\frac{EI}{(\rho AL + m_1 + 2m_2 + m_3)L}} \tag{26}$$

3. Numerical analysis

In this paragraph are given basic steps of the numerical modal analysis simulation. The modal analysis is carried out on the steel simple supported shaft with single rotor and three-rotor simple supported shaft. The simple supported shaft is designed in the graphical environment of the ANSYS [11]. Mode shapes and natural frequencies are computed in programs ANSYS with numerical formulation of the direct solver including the block Lanczos method.

In order to determine the critical speed of the disc is required measurements of the fundamental frequency in two planes: Oxy and Oxz. Mesh of the shaft and rotor is generated automatically by ANSYS, while is used the spatial element SOLID187. The element is defined by 10 nodes while each node has three degrees of freedom. The SOLID187 has a quadratic shifting behaviour and is suitable for modelling of the finite element irregular mesh. The maximum size of the element is 5 mm. The mesh is created of 3200 elements and of 22459 nodes.

To validate the rotor shape modes of vibration, in correlation with experimental resonance frequencies test, it is necessary to do a modal analysis. For an n-degree of freedom system, by Finite Element Method the motion equation of sample can be expressed similarly as equation (18), where {q} is the vector of the displacement, and [M] and [K] are mass, and stiffness matrices. For free vibration of the shaft and rotor, the characteristic equation is:

$$\left|-\omega^2[M]+[K]\right|=0 \tag{27}$$

To solve eigenvalues problem, given by equation (14), the Block Lanczos method is use because it is efficient to extract of large number of modes in most models. It is used in complex models with mixture of solids/shells/beams etc. It offers an efficient extraction of modes in a frequency range.

The first mode shapes of the single simple supported rotor shaft in Oxy plane and in Oxz plane are shown in Figure 4 and Figure 5.

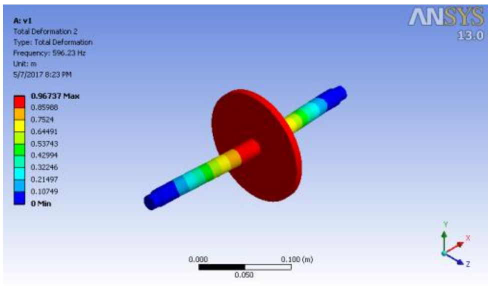

Figure 4. First bending mode in Oxy plane for single simple supported rotor shaft

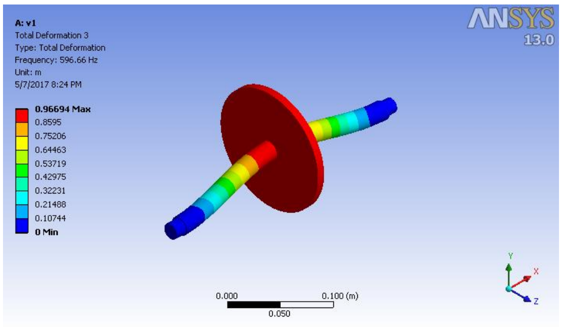

Figure 5. First bending mode in Oxz plane for single simple supported rotor shaft

The three-rotor simple supported shaft is design in the same graphical environment of the ANSYS with the same spatial element solid 187.The first mode shapes of the three-rotor simple supported shaft in Oxy plane and in Oxz plane are shown in Figure 6 and Figure 7.

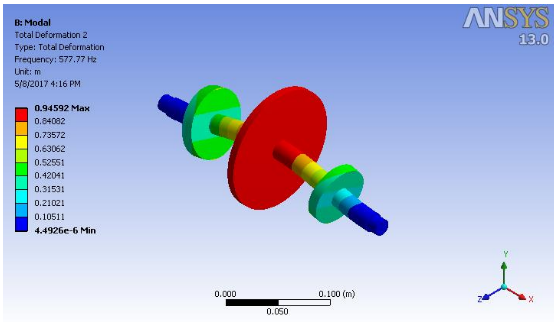

Figure 6. First bending mode in Oxy plane for three-rotor with simple supported shaft

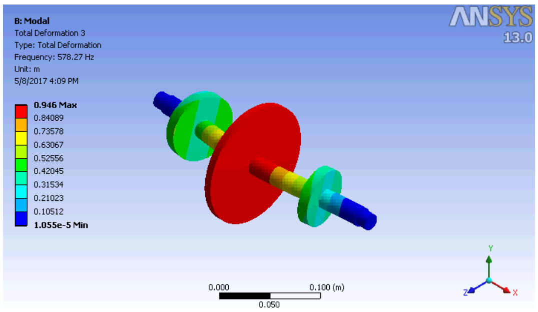

Figure 7. First bending mode in Oxz plane for three-rotor with simple supported shaft

4. Experimental analysis

The experimental stand for non-contact measurement of free vibrations of the sample having a rotor with simple supported shaft is shown in Figure 8, and it is composed by: the shaft with disk 1, which is the mechanical structure to be analyzed; impulsive mini hammer 2; brackets to support the sample 3; elastic threads for support of the structure in boundary conditions with the free ends 4; the acoustic sensor one condenser microphone 5, and the computer that has embedded and data acquisition board 6. The same stand is also used for the sample having three-rotor and simple supported shaft, as in the Figure 10. This stand or its variants used especially in the case of simulation of free- free boundary conditions was designed and used for the papers: [12], [13].

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On the stands in Figures 8 and 9 vibration measurements were performed for two samples, for a single simple supported rotor shaft and for three-rotor with simple supported shaft. The vibroacoustic signal is picked up by a good quality condenser microphone. Then the signal is transmitted as an electrical voltage to a signal acquisition board, which is actually the sound card of the computer, and it is amplified by a microphone amplifier located on this board. Acquisition board is analog digital signal from the microphone, with the possibility of adjusting the sampling frequency. The signals were discretized with a sampling frequency of 44.1 kHz, thus having the possibility of spectral analysis, according to the Shannon theorem, up to a frequency of 22 kHz, i.e. in the audible spectrum.

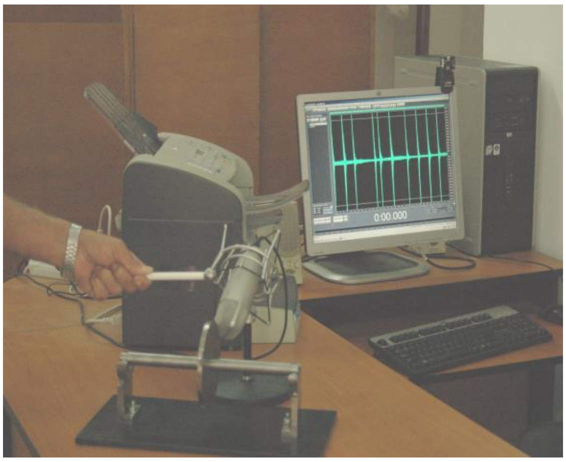

Figure 8. The experimental stand for single simple supported rotor shaft

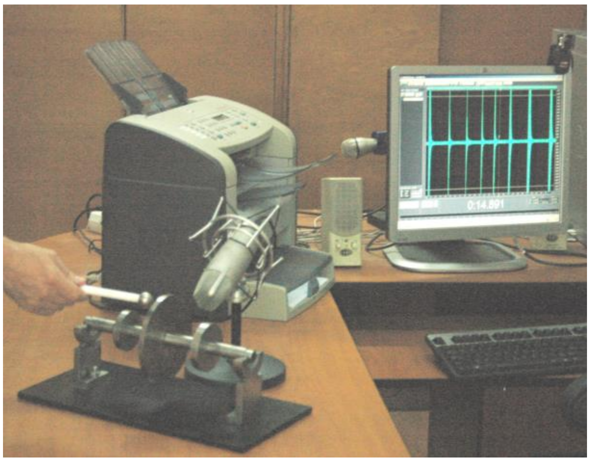

Figure 9. The experimental stand for three-rotor with simple supported shaft

The discreet signals were purchased through the Professional CoolEdit 2 [14], software as files with the wav extension. The MATLAB [15] program was used for analysis of discreet signals stored in wap files. The Figure 10 shows the recorded signal from experimental sample of a single-rotor with simple supported shaft. For each sample, several signals were recorded for spectral analysis using Fast Fourier Transform (FFT).

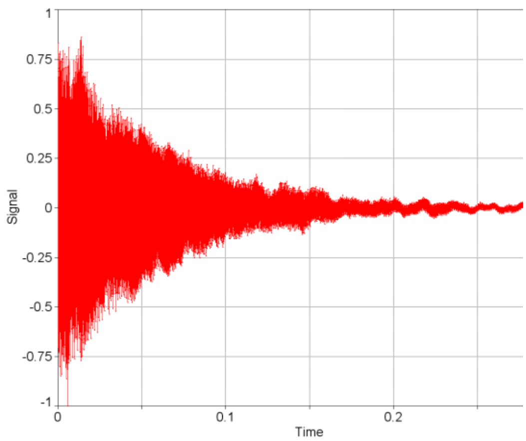

Figure 10. Signal in time domain for single simple supported rotor shaft

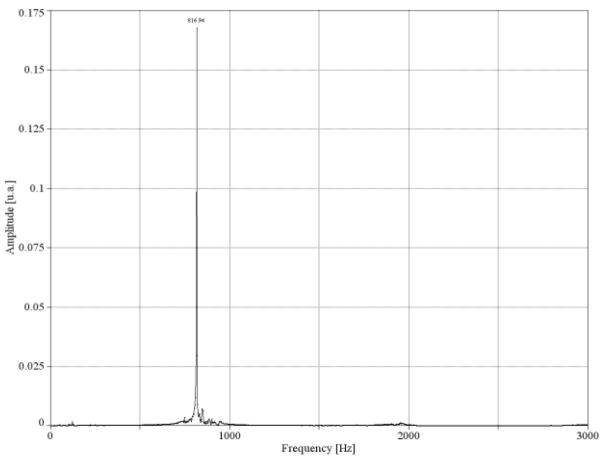

Figure 11. Signal in frequency domain for single simple supported rotor shaft

The Figure 10 shows the recorded signal from the experimental sample, consisting of a single-rotor shaft, simply supported. The spectrum of the recorded signal is shown in the Figure 11. Manual excitation was done by impulsive mini hammer consisting of a metal sphere and a plastic handle.

To retain only the component corresponding to the fundamental frequency of the vibration signal, it was filtered through a band pass filter.

Figure 12 shows the recorded signal from the experimental sample, consisting of a three-rotor shaft, simply supported. The spectrum of the recorded signal is shown in the Figure 13.

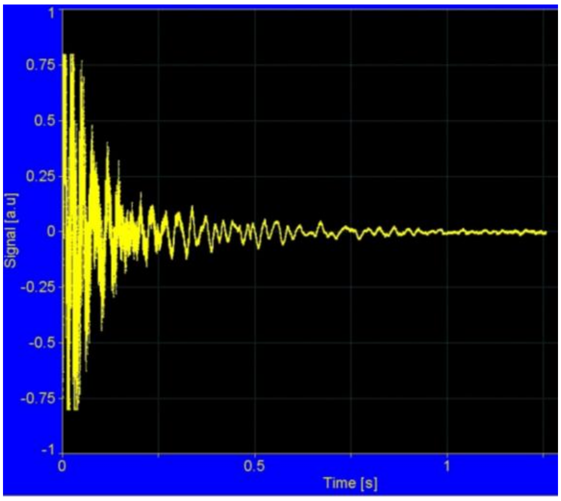

Figure 12. Signal in time domain for three-rotor simple supported shaft

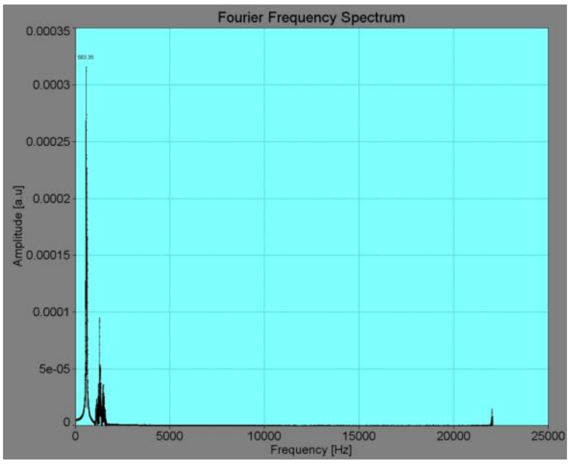

Figure 13. Fourier Frequency Spectrum Signal for three-rotor simple supported shaft

5. Results and conclusions

For experimental determinations of the critical speeds of a single rotor shaft by pulse excitation technique, the sample was made of a steel shaft having the length L=294 mm and the diameter d=20 mm. The rotor is in the form of a disc and was made of the same material having a diameter D=130 mm and a thickness h=10 mm. The material was stainless steel with Young's modulus E=210 GPa, and density ρ=7850 kg/m3. The mass of the rotor-disc and of the shaft has been measured by a precision digital balance which is accurate to one-hundredth of a gram. Average after six weighing, for the disc was $m=m_d=800.333$g, and $m_s=\rho AL=700.231$ g, for the shaft.

By making the ratio between equation (24) and equation (23), i.e. between the critical speed of the rotor in which the mass of the shaft is negligible with respect to the disc rotor, and the critical speed of the same rotor taking into account the mass of the shaft, it is ascertained that since the relative deviation of the two speeds, to be less than 1%, the ratio between the shaft mass and the disc mass must be less than 1/25. As can be seen from the mass measurement for the sample considered in the experiment, it does not answer to this requirement.

Table 1. The fundamental frequencies and critical speeds of the single rotor shaft in two planes

Type of analysis	Fundamental frequency in Oxy plane [Hz]	Critical speed in Oxy plane [rpm]	Fundamental frequency in Oxz plane [Hz]	Critical speed in Oxz plane [rpm]
Analytical analysis. Shaft massless	712.32	42739,2	712.32	42739,2
Analytical analysis. Shaft with mass	594.12	35647,2	594,12	35647,2
Numerical analysis	596,23	35773,8	596,66	35799,6
Experimental analysis	605,17	36310,2	607,03	36421,8

For experimental determinations of the critical speeds of a thre-rotor shaft by pulse excitation technique, the sample was made of a steel shaft having the same dimensions as in the first experiment. The midle rotor is noted here with m_2 and it is also the same like in first experiment. The mass of the rotor-disc and of the shaft has been measured by a precision digital balance which is accurate to one-hundredth of a gram. Average after six weighing, for the disc was $m=m_d=800.333$g, and $m_s=\rho AL=700.231$ g, for the shaft.

Table 2. The fundamental frequencies and critical speeds of the three-rotor shaft in two planes

Type of analysis	Foundamental frequency in Oxy plane [Hz]	Critical speed in Oxy plane [rpm]	Foundamental frequency in Oxz plane [Hz]	Critical speed in Oxz plane [rpm]
Analytical analysis. Shaft massless	660,34	3962,04	660,34	3962,04
Aalytical analysis.Shaft with mass	574,21	34454,6	574,21	34454,6
Numerical analysis	577,77	34666,2	578,27	34696,2
Experimental analysis	581,17	34870,2	583,34	35000,4

In conclusion, comparing the results given in Table 1 and Table 2 it can be said that the experimental method validates numerically or analytically results**.** The negligence of the shaft mass, when it is comparable to the rotor mass, causes large deviations in determining the critical speeds.

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