

# Modeling and simulation of cars in frontal collision

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**Abstract.** Protection of cars, mainly drivers and passengers in a collision are very important issues worldwide. Statistics given by "World Health Organization" are alarming rate of increase in the number of road accidents, most claiming with serious injury, human and material loss. For these reasons has been a continuous development of protection systems, especially car causing three quarters of all accidents. Mathematical modeling and simulation of a car behavior during a frontal collision leads to new solutions in the development of protective systems. This paper presents several structural models of a vehicle during a frontal collision and its behavior is analyzed by numerical simulation using Simulink.

## 1. Introduction

Modeling the behavior of a car during an accident is one of the most important issues in strength design. Impact tests are complex and difficult to achieve in practical terms, requiring qualified personnel, sophisticated equipment, time and money. For these reasons was developed a mathematical model for the analysis of a collision, which can give by provided results an answer as exactly over behavior of the car, response that can replace even the experimental results. Over the years, were developed two main approaches on the behavior of the car during the collision: focus parameters method known as Lumped Mass-Spring (LMS) and finite element method (FEM). Analysis with finite element method (FEM) requires a complete and detailed description of the component geometry and material properties associated, in contrast, analysis of focus parameters method (LMS) is based on several discrete masses and springs of materials whose mechanical properties are determined by dynamic tests. The vehicles structure design process is based on determining deceleration-time history, called pulse accident, history provided by both methods.

First model, relatively simple but powerful to simulate the response of a car following a frontal impact was developed by Kamal [1] in 1970. This model, know as Lumped Mass-Spring (LMS) has become widely used by accident engineers due to its simplicity and relative accuracy. The vehicle is approximated one-dimensional and includes mass-spring system, a simplification which is quite acceptable for basic modeling of some characteristics in frontal impact collision. Because of the simplistic representation of an accident, the LMS model requires a user with knowledge, understanding and experience in dynamic testing of structural materials. Several versions of the LMS models were successfully used to simulate the impact of frontal, side and rear vehicle crashes [2-4].

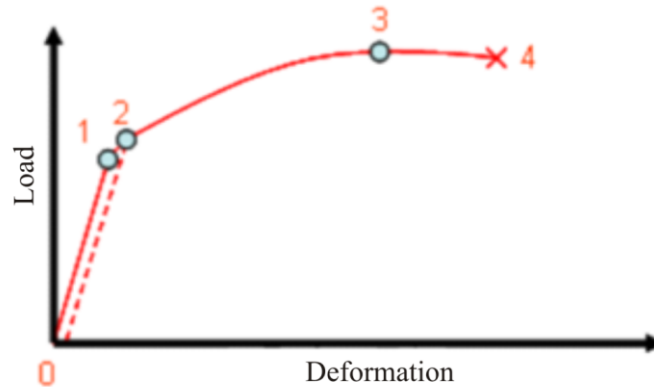
The purpose of this paper is to present the dynamic behavior of a car in a collision, starting from simple models with concentrated parameters (LMS) with one, two and three degrees of freedom, by numerical simulation using Simulink.



## 2. Lumped mass-spring models

### 2.1. Mass-elastic element collision model

Since the car's mechanical design phase, the focus is on passenger safety.

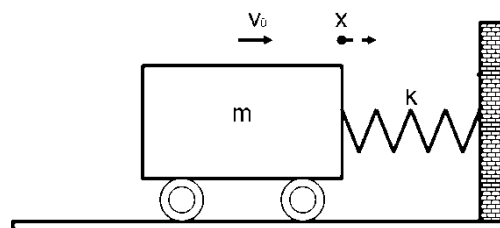


**Figure 1.** Load-deformation diagram

The car body is designed with special areas dedicated shock absorbing resulting from collisions. Deformation characteristics of the car structure by the collision, it can exemplify using load – deformation curve.

Depending on the speed and mechanical characteristics of vehicle and the object with which the collision occurs, the collision can be in character elastic, elasto-plastic or plastic.

Elastic collision (Figure 1, zone 0-1) usually occurs at very low speeds, the car has a low weight. In these cases, most of the time, activation of safety systems is not required.



**Figure 2.** Mass-spring model

For the model in Figure 2 can write the differential equation valid for a short period of collision:

$$m\ddot{x} = -kx \quad (1)$$

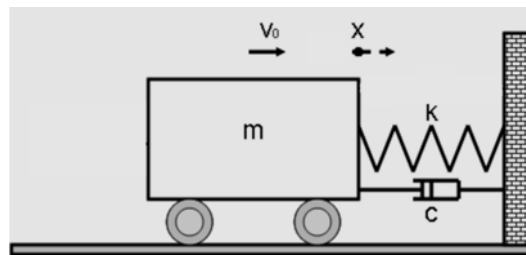
### 2.2. Mass-elastic damping element collision model

Most vehicle accidents are elasto-plastic (Figure 1, zone 1-2), where, due to forces acting on passenger protection systems must be enabled.

Plastic collision happens at high speeds, and the vehicle having significant weight (Figure 1, zone 2-3-4). Often in this situation the ability to protect passengers the systems offered insufficient protection, it is usually about accidents with fatal results. Taking account of the elasto-plastic collision, we find a similar behavior with mass-spring type models or elastic element wider sense. Due to its shape and the material it is made, spring turns (after deformation) the mechanical work ( $W$ ) in potential energy ( $E_p$ ).

An important factor in the study of the collision, of analyzed mechanical systems is the behavior of the kinetic energy dissipation by friction, also called dissipative systems. Basic mechanical subassemblies can be equated with dissipative systems, and by adding these dissipative subsystems behaviors can describe dissipative behavior of the vehicle.

Dissipative systems connected to form a set of structural and functional elements, characterized by a number of parameters. Function of the dissipative system, like any dynamic system is to transform input into output sizes, useful technical (motion, force, energy). This transformation is achieved by dissipative system structure, i.e. the components which interact through processes of friction, which at the same time appearing not only friction loss, and tears.



**Figure 3.** Mass-spring-damper model

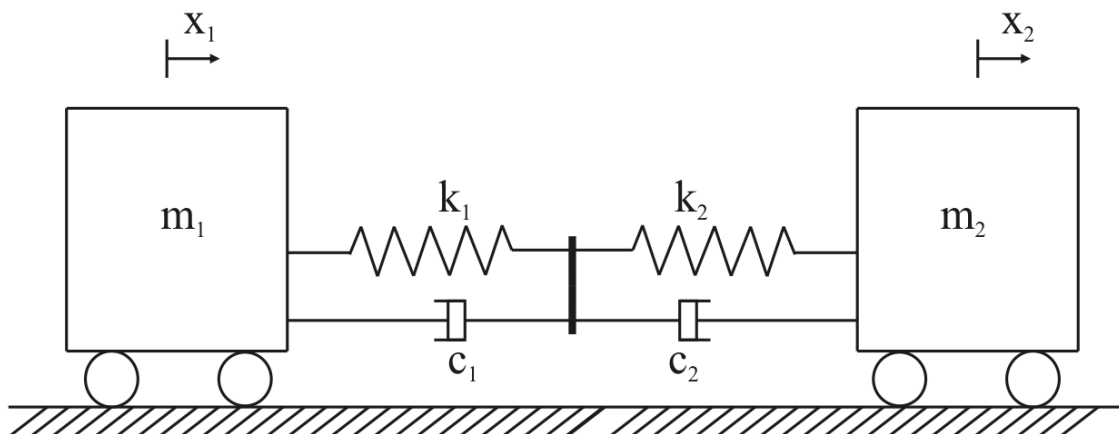
For the model in Figure 3 can write the differential equation governing the motion for collision interval:

$$m\ddot{x} = -c\dot{x} - kx \quad (2)$$

### 3. Lumped mass-spring-damping models for collision of two cars

#### 3.1. The behind crush simulation of two cars

In case of collision of two cars, there are two situations: the behind crash (Figure 4) and the front crash (Figure 5). To simulate dynamic behavior during collision using the SIMULINK module in MATLAB, the differential equations of motion will be determined. To determine these equations, Lagrange equations are used.



**Figure 4.** Behind collision of two cars

The kinetic energy of the system is:

$$E_c = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} \quad (3)$$

where  $m_1$  and  $m_2$  are the masses of the two cars,  $\dot{x}_1$  and  $\dot{x}_2$  are velocities of cars,  $x_1$  and  $x_2$  are their elastic deformations. The two elastic elements with the constants  $k_1$  and  $k_2$  will be considered in series. Also in the series, the two elements of amortization, having the  $c_1$  and  $c_2$  damping coefficients, will be considered connected. By  $k_e$ , it is noted the equivalent elastic constant of the two elastic elements, and with  $c_e$  is the equivalent damping constant of the two damping elements. With these notations the expression of the potential energy, is determined with the expression:

$$E_p = \frac{k_e (x_1 - x_2)^2}{2}; k_e = \frac{k_1 k_2}{k_1 + k_2}; \quad (4)$$

and the expression of the energy of dissipation is:

$$E_d = \frac{c_e (\dot{x}_1 - \dot{x}_2)^2}{2}; c_e = \frac{c_1 c_2}{c_1 + c_2} \quad (5)$$

Lagrange's two equations have the following form:

$$\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{x}_1} \right) - \frac{\partial E_c}{\partial x_1} = - \frac{\partial E_d}{\partial \dot{x}_1} - \frac{\partial E_p}{\partial x_1}; \quad (6)$$

$$\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{x}_2} \right) - \frac{\partial E_c}{\partial x_2} = - \frac{\partial E_d}{\partial \dot{x}_2} - \frac{\partial E_p}{\partial x_2}. \quad (7)$$

By replacing Lagrange's equations with expressions of kinetic, deformation and dissipation energies, two differential equations of the second order that govern the movement of the two bodies for the time interval they are in contact are obtained.

$$m_1 \ddot{x}_1 = -k_e (x_1 - x_2) - c_e (\dot{x}_1 - \dot{x}_2); \quad (8)$$

$$m_2 \ddot{x}_2 = k_e (x_1 - x_2) + c_e (\dot{x}_1 - \dot{x}_2) \quad (9)$$

Given that simulation in Simulink aims to integrate the two differential equations, it explains from each equation the accelerations of the two bodies during the collision.

$$\ddot{x}_1 = \frac{-c_e (\dot{x}_1 - \dot{x}_2) - k_e (x_1 - x_2)}{m_1}; \quad (10)$$

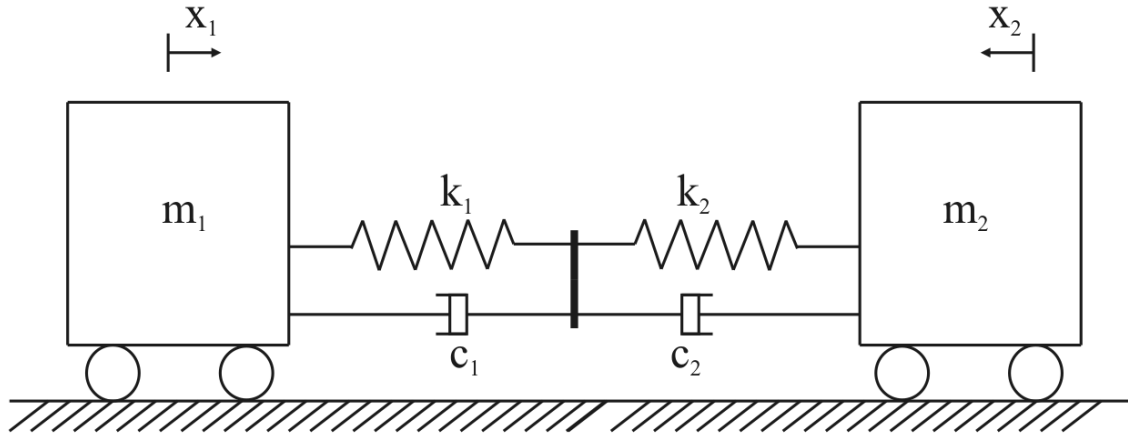
$$\ddot{x}_2 = \frac{c_e (\dot{x}_1 - \dot{x}_2) + k_e (x_1 - x_2)}{m_2}. \quad (11)$$

These two equations can be entered into the Simulink block diagrams, with multiplication, integration and introduction of the initial conditions, based on which the deformation, velocity and acceleration diagrams are obtained.

### 3.2. The frontal crush simulation of two cars

For the second case considered, the two cars collide frontal in opposite directions, making the deformation of the elastic elements equal to the sum of the individual deformations of the two machines, and similarly the speed taken into account in the expression of dissipation energy is equal to the sum of the speeds of the two machines for the time interval of the collision.

With these observations, kinetic, deformation and dissipation energies will have the following expressions:



**Figure 5.** Frontal collision of two cars

$$E_c = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} \quad (12)$$

$$E_p = \frac{k_e (x_1 + x_2)^2}{2} \quad (13)$$

$$E_d = \frac{c_e (\dot{x}_1 + \dot{x}_2)^2}{2} \quad (14)$$

Using the Lagrange equations that will have the same form as given in equations (6) and (7), are obtain the differential equations of the frontal collision of the two cars from opposite directions:

$$m_1 \ddot{x}_1 = -c_e (\dot{x}_1 + \dot{x}_2) - k_e (x_1 + x_2) \quad (15)$$

$$m_2 \ddot{x}_2 = -c_e (\dot{x}_1 + \dot{x}_2) - k_e (x_1 + x_2) \quad (16)$$

For the integration of the differential equations (15) and (16), the accelerations of the two cars must be explained during the collision, and based on them the model will be built in Simulink.

$$\ddot{x}_1 = \frac{-c_e (\dot{x}_1 + \dot{x}_2) - k_e (x_1 + x_2)}{m_1} \quad (17)$$

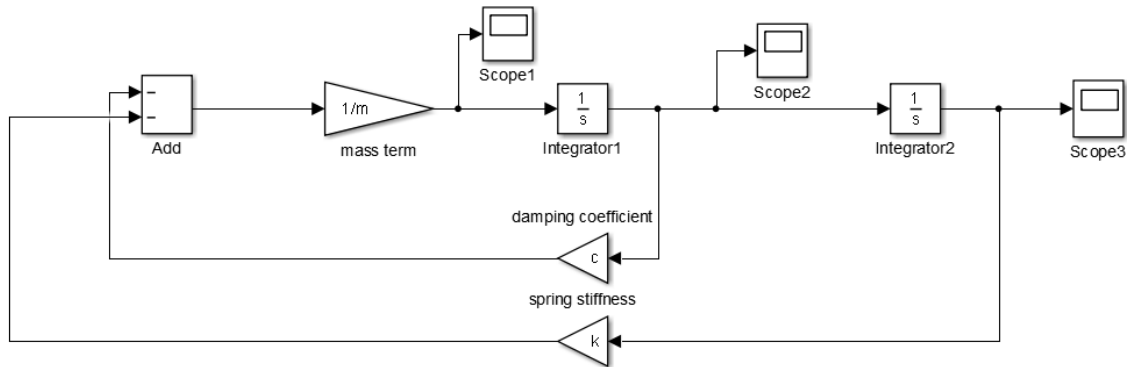
$$\ddot{x}_2 = \frac{-c_e (\dot{x}_1 + \dot{x}_2) - k_e (x_1 + x_2)}{m_2} \quad (18)$$

## 4. Simulations and discussions

### 4.1. Simulation car to barrier collision

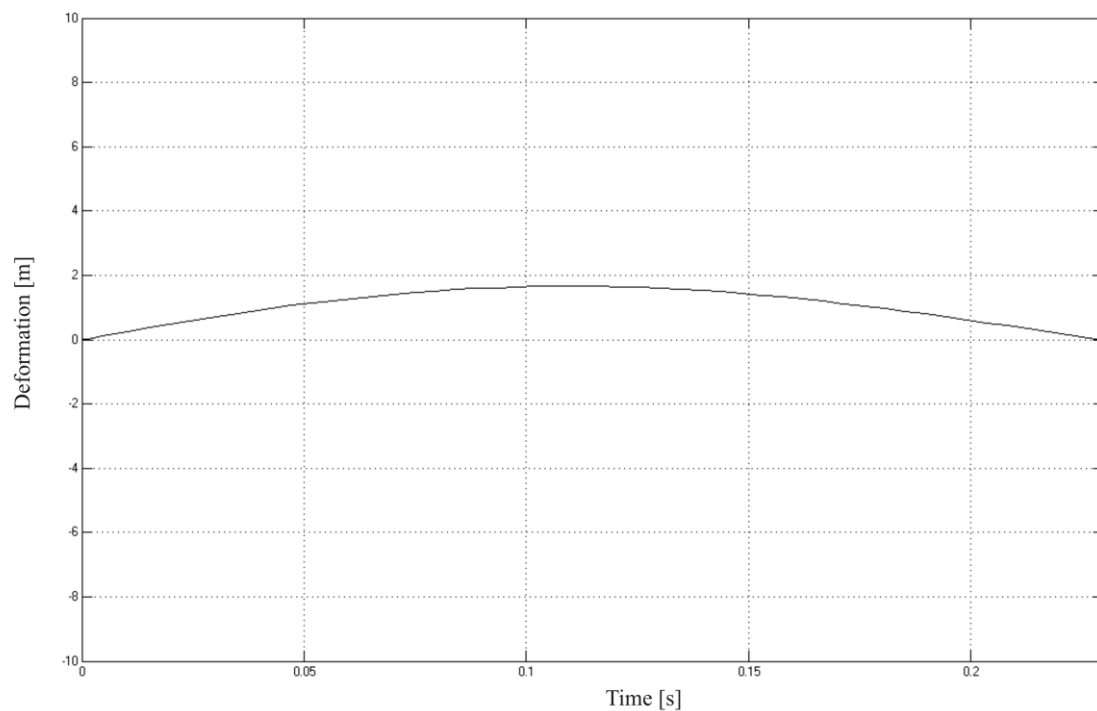
For simulation was considerate weight of one medium class vehicle,  $m=900\text{kg}$ , damper coefficient,  $c=1,500\text{Nm/s}$  and stiffness constant,  $k=170,000\text{N/m}$ ,  $v=25\text{m/s}$ .

These two models of collision were realized in numerical simulation software, Simulink. The scheme of these models is shown in Figure 6. The model was made following equation (1) and (2) of the system, for the first system, the damping ratio is zero,  $c=0$ .



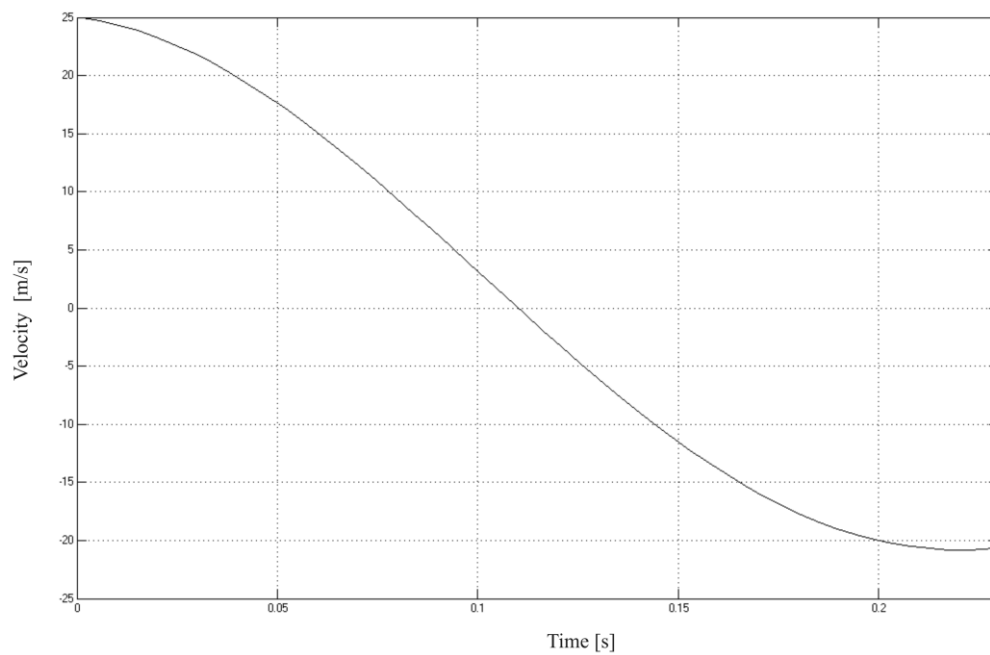
**Figure 6.** Mass-spring-damper model in Simulink

The simulation results can be seen in the diagrams of Figures 7a, 7b and 7c obtained on the scopes in the above scheme. In the figures are presented: deformation, velocity and acceleration during collision.



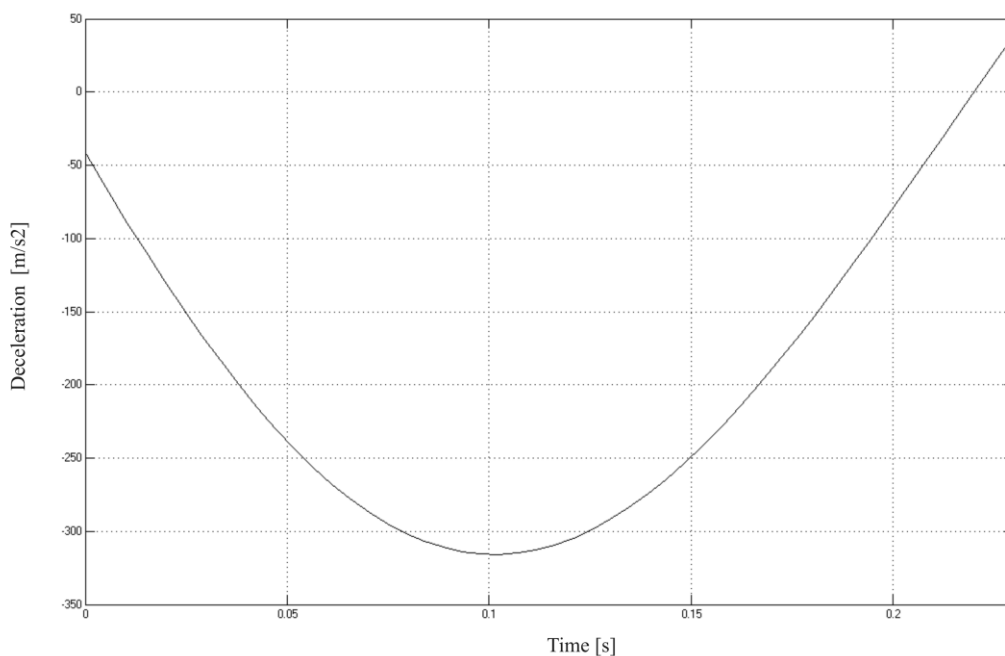
**Figure 7a.** Deformation of mass-spring-damper model

In the collision model of a vehicle with a barrier, in the absence of damping, the elastic collision ends after 0.16 s, and in the presence of the damping, with the values presented above, the collision ends after 0.232 s. When the velocity becomes zero, that is, after the time  $t = 0.116$ , the deformation phase is finished and follows the recovery phase, as can be seen in Figures 7a and 7b.



**Figure 7b.** Velocity of mass-spring-damper model

Therefore, the collision will have two phases. In the compression phase, the car deforms until the speed is canceled. In the second, deformation phase, a part of the deformation is recovered and held until the acceleration becomes zero. This happens after a time  $t = 0.232\text{s}$ .

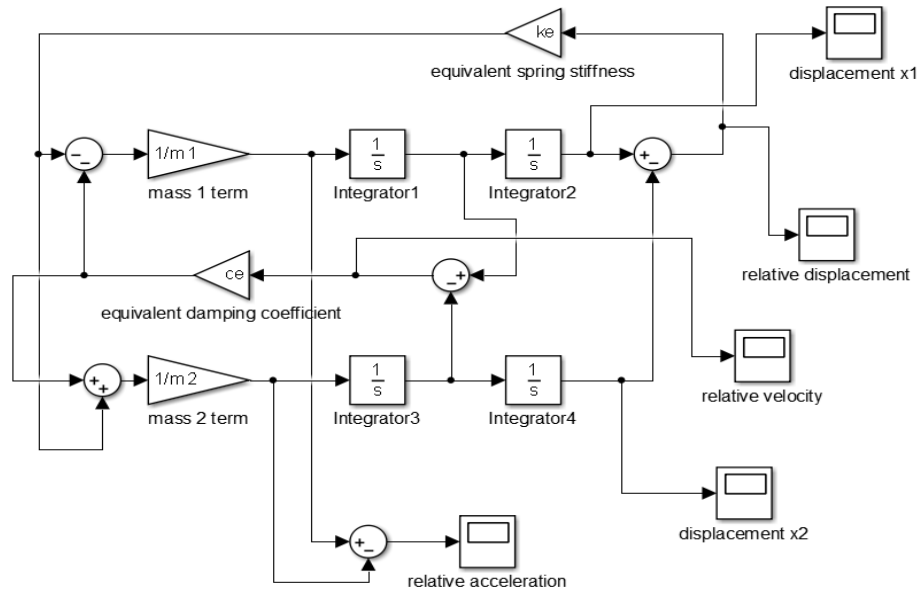


**Figure 7c.** Deceleration of mass-spring-damper model

It is found that in both cases there has been a sharp deceleration in 0.117 seconds for the first model and 0.1 seconds for second case. These decelerations are critical for vehicle passengers. Therefore, is the desire to increase the both of passive and active safety for new cars.

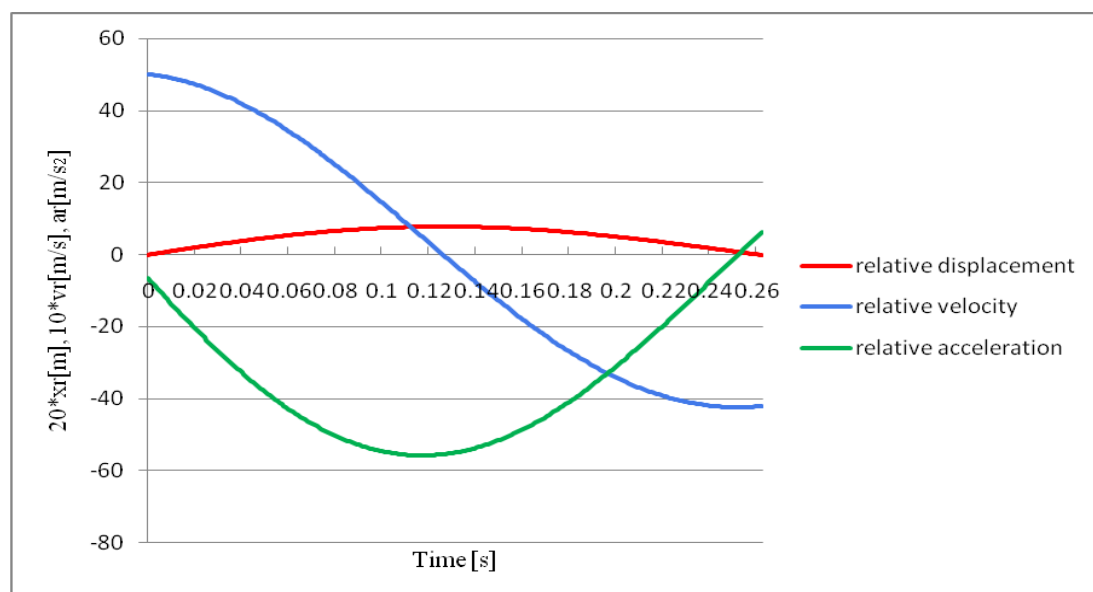
#### 4.2. Simulation behind collision of two cars

Equations 10 and 11 are numerically resolved and plotted using the Simulink module in MATLAB, following the construction of the bloc diagram in Figure 8. In the simulation, the masses of two cars were considered  $m_1=1,200\text{kg}$  and  $m_2=1,400\text{kg}$ . For the two elastic elements, the following values for the elastic constants were considered as  $k_1=170,000\text{N/m}$  respectively for  $k_2=210,000\text{N/m}$ , and for the damping elements were considered the following values of the damping coefficients  $c_1=1,500\text{Ns/m}$ , respectively  $c_2=2,000\text{Ns/m}$ . The speeds of the two cars in the moment of collision were considered to be  $v_1=25\text{ m/s}$ , respectively  $v_2=20\text{ m/s}$ .



**Figure 8.** The behind collision model of two cars in Simulink

Diagrams in Figure 9 are the relative deformation, relative velocity and relative acceleration. For reasons of the order of magnitude, relative deformation and relative velocity are multiplied by 20 and 10. They are represented in overlapping graphs to observe the two phases of the collision.

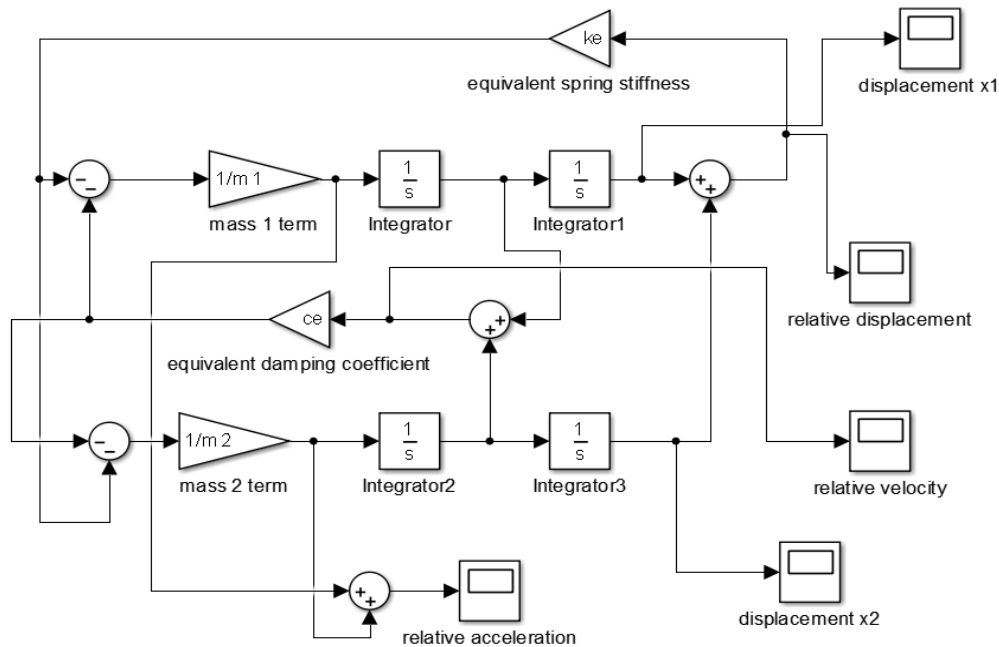


**Figure 9.** Relative displacement, velocity and acceleration diagrams, in behind collision of two cars



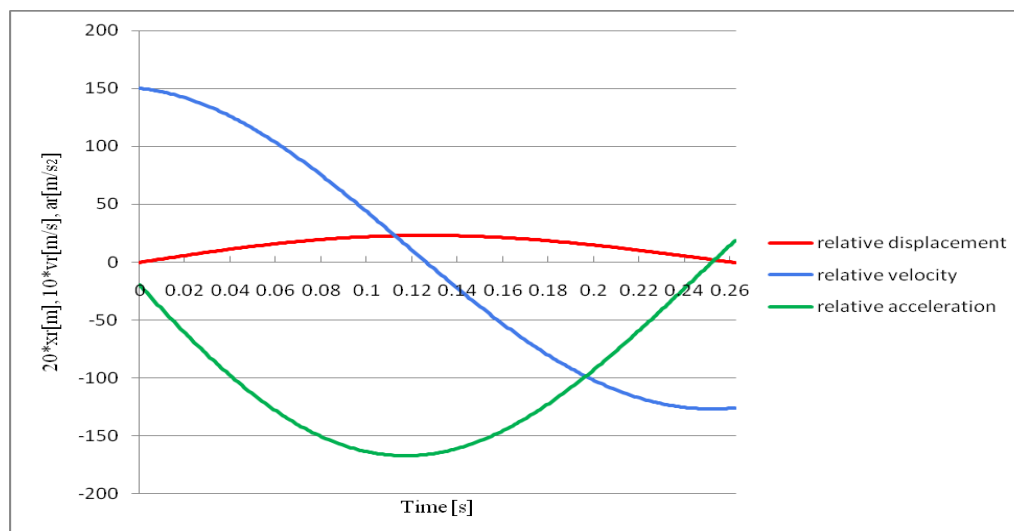
#### 4.3. Simulation frontal collision of two cars

In case of front collision, the characteristic elements of the two cars are the same as in the previous case, only the speeds are  $v_1=25$  m/s and  $v_2=10$  m/s. The frontal collision of two cars in Simulink is presented in Figure 10.



**Figure 10.** The frontal collision model of two cars in Simulink

The three diagrams representing relative displacement, relative speed and relative acceleration in the frontal collision of two cars, are represented in Figure 11 by overlapping to observe the two phases of the collision.



**Figure 11.** Relative displacement, velocity and acceleration diagrams, in frontal collision of two cars

#### 4.4. Coefficient of restitution

Coefficient of restitution  $C_R$  is a parameter used to estimate the lost energy during a collision. It is a measure of the elasticity of collision. There are three definitions for this parameter [5].

Kinematic definition of the restitution coefficient, given by Newton:

$$C_R = \frac{P_c}{P_r} = \frac{v_2' - v_1'}{v_1 - v_2} \quad (19)$$

where  $P_c$  is compression impulse,  $P_r$  is restitution impulse,  $v_2'$  and  $v_1'$  are the velocities after impact of two cars,  $v_1$  and  $v_2$  are the velocities before the impact.

Kinetic definition of the restitution coefficient, given by Poisson:

$$C_R = \frac{P_c}{P_r} \quad (20)$$

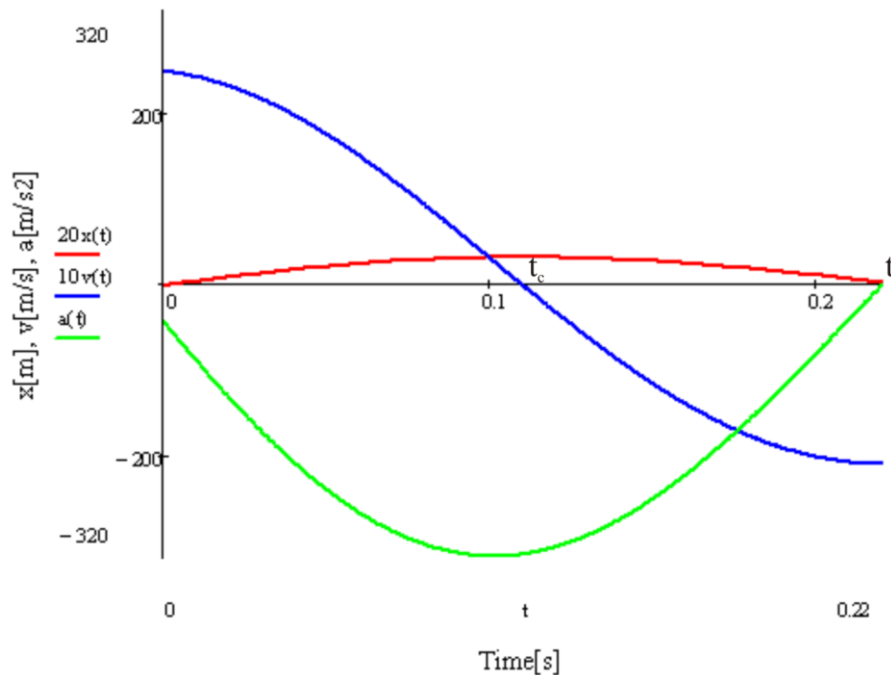
where  $P_c$  and  $P_r$  are as defined above.

Energetic definition of the restitution coefficient, given by Stronge [6]:

$$C_R^2 = -\frac{W_r}{W_c} \quad (21)$$

where  $W_r$  is the sum of the work done by both normal impulsive forces during the restitution the collision, and  $W_c$  is the sum of the work done by both normal impulsive forces during the compression phase of the collision.

The three expressions are similar if the impact is collinear, the collision is centered, and the friction is neglected for the short collision time [7]. These hypotheses were assumed in the modeling made in this paper.



**Figure 12** Deformation, velocity and acceleration of the mass, spring, damped model for  $v=25$  m/s

Taking into account the order of magnitude of the three kinematic elements of motion: displacement, velocity and acceleration, graphically represented in the diagrams in Figure 12 and to observe the behavior of the car during collision with a barrier, the displacement was multiplied by 20 and the speed was multiplied by 10.

In order to discuss and conclude on the three simulations presented in Simulink, the coefficient of restitution was calculated in the case of a collision of a car by a barrier. It is considered the model of Figure 3 whose equation (2) was written in the form:

$$\ddot{x} + 2\sigma\dot{x} + \omega_n^2 x = 0 \quad (22)$$

where  $\omega_n$ ,  $p$ ,  $\sigma$ , and  $\zeta$  are the circular frequency, the damped circular frequency, the damping factor and the damping ratio, having the following expressions:

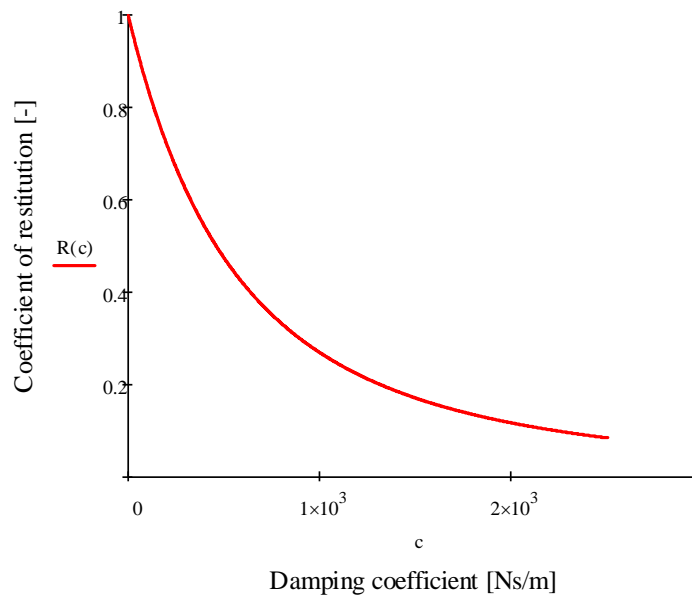
$$\omega_n = \sqrt{\frac{k}{m}}; p = \omega_n \sqrt{1 - \zeta^2}; \sigma = \frac{c}{2m}; \zeta = \frac{c}{2\sqrt{km}} \quad (23)$$

Considering the initial conditions given by the moment of the collision with the barrier, when the car has the speed  $v_1$ , the three kinematic elements during collision, deformation, velocity and acceleration have the following analytical expressions:

$$x(t) = \frac{v_1}{p} e^{-\sigma t} \sin pt \quad (24)$$

$$v(t) = v_1 e^{-\sigma t} \left( \cos pt - \frac{\sigma}{p} \sin pt \right) \quad (25)$$

$$a(t) = v_1 e^{-\sigma t} \left[ -2\sigma \cos pt + \left( \frac{\sigma^2}{p} - p \right) \sin pt \right] \quad (26)$$



**Figure 13.** Coefficient of restitution versus damping coefficient

The compression phase lasts until the speed becomes zero this time is  $t_c$ , and the restitution phase lasts until the resulting force is zero, so the acceleration is canceled, this time being  $t_r$  (Figure 12). It can calculate the restitution coefficient in the collision of a car with a barrier, giving the expression:

$$C_R = \frac{v(t_r)}{v_1} = e^{-\frac{c}{\sqrt{4km-c^2}} \arccos \frac{c^2-2km}{2km}} \quad (27)$$

An analysis of this formula shows that the restitution coefficient depends on the three constituent parameters of the mechanical system mass  $m$ , stiffness  $k$  and damping  $c$ . By keeping constant the first

two parameters  $m$  and  $k$ , and by modifying the damping coefficient  $c$  the dependence of the restitution coefficient is obtained as in Figure 13.

It can be seen three important values for the coefficient of restitution:

1. For undamped system,  $c = 0$  and the coefficient of restitution is  $C_R = 1$ . The crash is perfectly elastic.
2. If the system has critical damping then,  $c = 2\sqrt{km}$ . After the elimination of the indetermination in the equation (27) the coefficient of restitution is:  $C_R = e^{-2} = 0.13533$ .
3. If the system is heavily over-damped then,  $C_R = 0$ . The collision is plastic.

## 5. Conclusions

To follow the collision of two cars in Simulink, the relative displacement, relative speed and relative acceleration are introduced. When the two velocities, before the collision, are in the same direction, the relative displacement is  $x_r = x_1 - x_2$ , and in the case of the front collision, the deformation is given by  $x_r = x_1 - x_2$ . Thus, practically the mechanical system can be considered with only one degree of freedom. In the diagrams given in Figures (9) and (11), it can be noticed that there is an identical behavior in the collision of two cars with the behavior of a collision of a car with a barrier. It should be noticed that here by  $t_c$  it can be understand the time after which the relative speed is zero, and the two cars arrive at an equal common velocity, followed by the restitution phase, until the moment  $t_r$ , when the relative acceleration is null. Similarly, it can be calculated the restitution coefficient that will have the same expression given by the relationship (27), where the mass  $m$  will be the equivalent mass  $m_e$ , the elastic constant  $k$  will be replaced by the equivalent elastic constant  $k_e$  and the damping coefficient  $c$  with the equivalent damping coefficient.

A structural analysis [8], that takes into account all the components of the front vehicle: torque box, front frame, drive line, sheet metal, wall radiator, fan and grille and transmission mount can bring a much better characterization of the behavior of a vehicle collision and sad simulation results close to real if the mechanical characteristics of these components are know.

These simulations help developed new technologies in vehicle crash safety, saving several resources. In simulation can bring a much better characterization of the behavior of a vehicle collision simulation results close to real if the mechanical characteristics of these components are know.

## References

- [1] Kamal M M 1970 *Analysis and Simulation of Vehicle to Barrier Impact*, SAE Paper No. 700414
- [2] Magee C L 1988 *Design for Crash Energy Management- Present and Future developments*, The Seventh International Conference on Vehicle Structural Mechanics, Detroit, Michigan, USA, 2-4 April, 142-154
- [3] Lim G G and Paluszny A 1988 *Side Impact Research*, XXII Fisita Congress- SAE Publication P-211, Paper No. 885055
- [4] Green J E 1977 *Computer Simulation of Car-To-Car Collisions*, SAE Paper No. 770015
- [5] Ahmad M, Ismail K A and Mati F 2016 Impact models and coefficient of restitution: A review, *ARPJ Journal of Engineering and Applied Sciences* **11**(10) 6549-6555
- [6] Stronge W J 2000 *Impact Mechanics*. Cambridge: Cambridge University Press
- [7] Daily J, Strickland R and Daily J 2006, *Crash Analysis with Under-rides and the Coefficient of Restitution*, Institute of Police Technology and Management's 24 th Annual Special Problems in Traffic Crash Reconstruction
- [8] Prasad P and Belwafa J E 2004 *Vehicle Crashworthiness and Occupant Protection*, American Iron and Steel Institute, Town Center, Southfield, Michigan