

# Experimental and numerical analysis of the constitutive equation of rubber composites reinforced with random ceramic particle

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**Abstract.** Based on the four classical models of Mooney-Rivlin (M-R), Yeoh, Ogden and Neo-Hookean (N-H) model, a strain energy constitutive equation with large deformation for rubber composites reinforced with random ceramic particles is proposed from the angle of continuum mechanics theory in this paper. By decoupling the interaction between matrix and random particles, the strain energy of each phase is obtained to derive the explicit constitutive equation for rubber composites. The tests results of uni-axial tensile, pure shear and equal bi-axial tensile are simulated by the non-linear finite element method on the ANSYS platform. The results from finite element method are compared with those from experiment, and the material parameters are determined by fitting the results from different test conditions, and the influence of radius of random ceramic particles on the effective mechanical properties are analyzed.

## 1. Introduction

Some classical theoretical models, such as the Vogit upper-limit theory [1], Reuss lower-limit theory [2], Eshelby equivalent inclusion theory [3] and Mori-Tanaka method [4] and so on are satisfactory to analyze the mechanical property of particles composites with small deformation. However, this methods are not so precise for the analysis of rubber composites with large deformation. There are two kinds of methods to establish the constitutive equation for different materials, the first is the crystal slip model based on the meso-scopic plastic theory, which is limited for the complexity of slip direction. The second is the material constitutive relationship based on phenomenal theory, which is obtained by combining the mathematical method with experimental results. Based on the Neo-Hookean model, Guo Zaoyang [5] proposed a strain-energy function theory to analyze the mechanical properties of fiber composites with finite deformation, and the accuracy of the results were verified by numerical simulation. Huang Xiaoshuang [6] proposed an an-isotropic visco-super elastic constitutive model, and the parameters in the model are determined by uni-axial tension experiment, and the results are verified by the finite element method. By comparing the results from Matlab numerical simulation, Huang Jianlong [7] thought that the Yeoh strain-energy model is prior to the Mooney-Rivlin model to analyze the large deformation of rubber composites. In order to analyze the influence of reinforcement on effective mechanical properties of composites, Li Qing [8-9] added the carbon black with different volume fraction into the rubber materials to perform the mechanical experiment, the results show that the stiffness and initial modulus increases with the increase of volume fraction of carbon black. Dong Jinping [10] Put forward a kind of transverse isotropic hyperelastic constitutive model to describe the rubber composites reinforced with short fiber, combining with some test results, its application in numerical simulation is studied, and the result is satisfactory. According to the large



strain hardening phenomenon of the rubber, Liu Feng [11] proposed a strain energy function to describe the constitutive relationship of rubber material, and the least square method is applied to fit the constitutive parameters of rubber. The good consistence with the experimental results establishes a foundation for the study of the large deformation of rubber. In the above researches, the strain invariant is always taken as variant to establish the strain energy function, which is reliable to describe the rubber with isotropic and in-compressible, but it is inappropriate to analyze non-linear rubber composites. In this paper, a new strain energy function which considering the interaction between random particles and rubber matrix is proposed, and the parameters are determined by fitting the curves from the uni-axial tension, equal bi-axial tension and pure shear experiments, and the stress-strain relationship is simulated by finite element method on the ANSYS platform, and the results are good consistent with those from the experiment.

## 2. Classical models of strain energy functions for rubber materials

A plenty of hyper-elastic constitutive models for rubber materials were proposed in the past years, and most of them were based on the concept of strain energy function, and the second Piola-Kirchhoff theory is utilized to establish the relationship between the hyper-elastic constitutive equation and the strain energy function [8].

$$\sigma = 2 \frac{\partial W}{\partial C} \quad (1)$$

Where  $C$  is the strain tensor, and  $W$  is the strain energy function, which is determined by different supposed models.

**Table 1.** Classical models of strain energy functions for rubber materials.

Model	Common expression	Parameter meaning
Mooney-Rivlin (M-R)	$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + \frac{1}{d}(J - 1)^2$	where $C_{10}$ , $C_{01}$ and $d$ are the Mooney constants, which are determined by experimental method, and $I_1$ , $I_2$ is the first and the second invariant, respectively. For in-compressible materials, $J=0$ .
Yeoh	$W = \sum_{i=1}^N C_{i0}(I_1 - 3)^i + \sum_{k=1}^N \frac{1}{d_k}(J - 1)^{2k}$	where $N, C_{i0}$ and $d_k$ are material constants, which are also determined by experimental method.
Ogden	$W = \sum_{i=1}^3 \frac{\mu_i}{\alpha_i} (\lambda_1^{\alpha_i} - 1)$	Where, $\alpha_1 = 1.3, \alpha_2 = 5.0, \alpha_3 = -2.0$ , $\mu_2 = 0.003\mu_0, \mu_3 = -0.0237\mu_0$ , $\mu_1 = 1.491\mu_0$ , are determined by Treloar test [12], $\mu_0$ is the initial poisson's ratio of rubber
Neo-Hookean (N-H)	$W = \frac{\mu}{2}(I_1 - 3)$	Where $\mu$ is the shear modulus.

The typical models for strain energy function showed in table 1 includes the polynomial model (M-R model), the first-order and third-order reduced polynomial model (N-H and Yeoh model), and the high-order polynomial model (Ogden model), and the difference among the four models were determined by different parameters in the equations, and the typical models and their parameters are shown in table 1. The purpose in this paper is to construct a nonlinear constitutive equation for rubber composites on the basis of the typical models, and the interaction between ceramic particles and

rubber matrix is considered in the proposed equation, and the corresponding parameters in the equation are determined by fitting the results from numerical simulation on the platform of ANSYS to those from the experiments.

### 3. Constitutive equation for rubber composites with random ceramic particles

The proposed constitutive model in this paper is

$$W = W^M(I_1, I_2, I_3) + W^F(I_0) + W^{FM}(I_{10}) \quad (2)$$

where  $w^M(I_1, I_2, I_3)$  is one of any classical model with a coefficient of  $A = (1 - \nu_p)\mu_m + \nu_p\mu_p$ , where  $\nu_p$  is the volume fraction of ceramic particles,  $\mu_m, \mu_p$  is the poisson's ratio of matrix and particles, respectively.  $W^F(I_0)$  is the strain energy of ceramic particle. The elastic modulus of random particles are much greater than that of rubber matrix, and the particles can be treated as rigid body, so  $W^F(I_0) = 0$ ,  $W^{FM}(I_{10})$  is the strain energy for the interaction between rubber and random ceramic particles.

In this paper, the random particles are considered as the infinitely small **and infinite density** orthogonality fabric, so the strain energy produced by the interaction between particles and matrix is the change of the angle between the orthogonal fibers for the particles (as shown in figure 1 and figure 2).

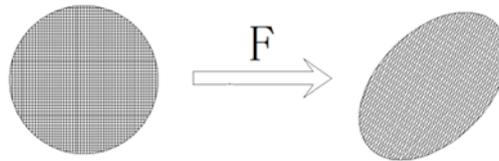


Figure 1. The particle with orthogonal woven.

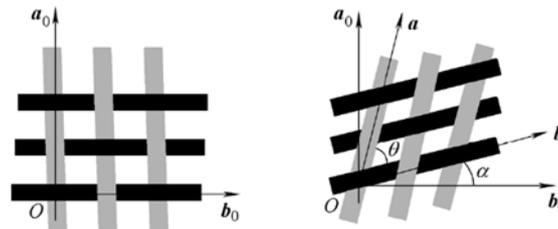


Figure 2. The change of the angle between fabrics Supposing.

$$I_{10} = \Delta\theta(I_4, I_6, I_8) \quad (3)$$

For the orthogonal fabric, since the angle has nothing to do with  $I_6$ , Equation (6) becomes

$$I_{10} = \Delta\theta \approx (I_4 I_8)^{-0.5} a_0 \cdot C \cdot b_0 - a_0 \cdot b_0 \quad (4)$$

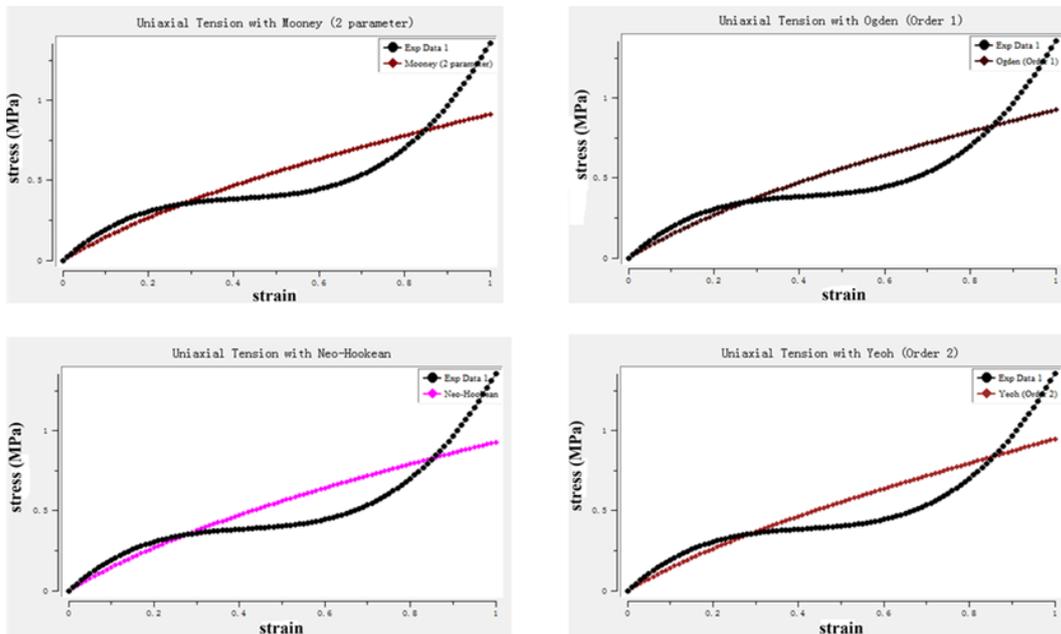
and the strain energy for the interaction between rubber and random particles can be written as

$$W^{FM}(I_{10}) = W^{FM}(\Delta\theta) \approx (I_4 I_8)^{-0.5} a_0 C b_0 - a_0 b_0 = \sum_2^4 s_i (I_{10})^i \quad (5)$$

In which,  $C$  is the shear strain tensor, and

$$I_4 = a_0 \cdot C \cdot a_0 = \lambda_a^2, \quad I_8 = b_0 \cdot C \cdot b_0 = \lambda_b^2 \quad (6)$$

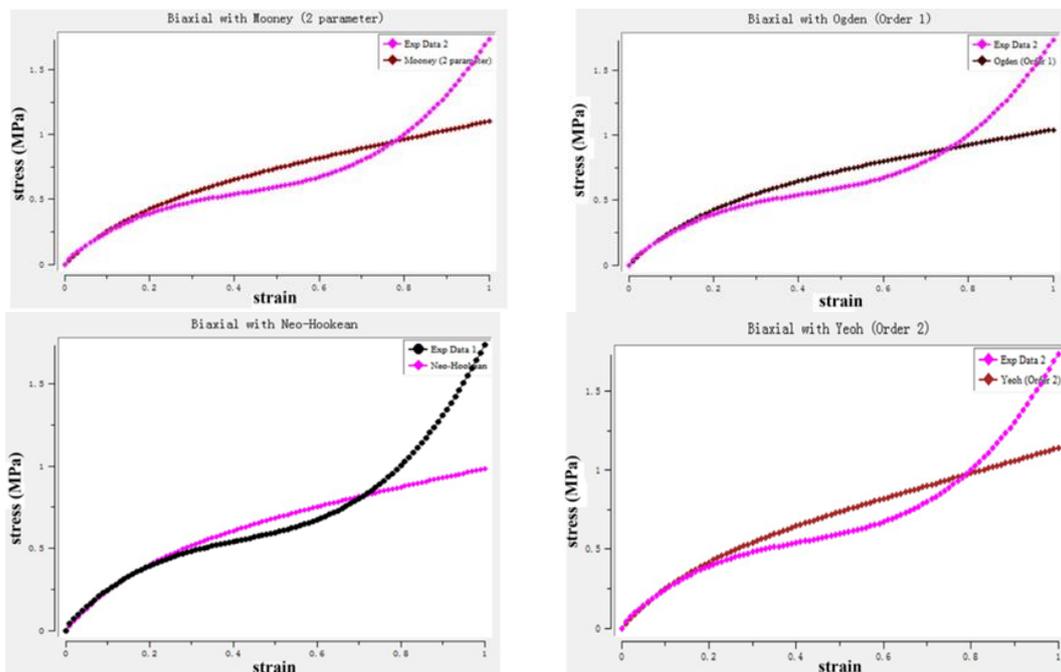
Where,  $a_0$  and  $b_0$  is the unit vector along the fiber direction for the orthogonal fabric as shown in figure 2, and  $\lambda_a, \lambda_b$  are the tensile rate of the orthogonal fibers,  $I_4, I_6$  and  $I_8$  all are the invariant. It is important to determine the shear parameters by experimental methods.



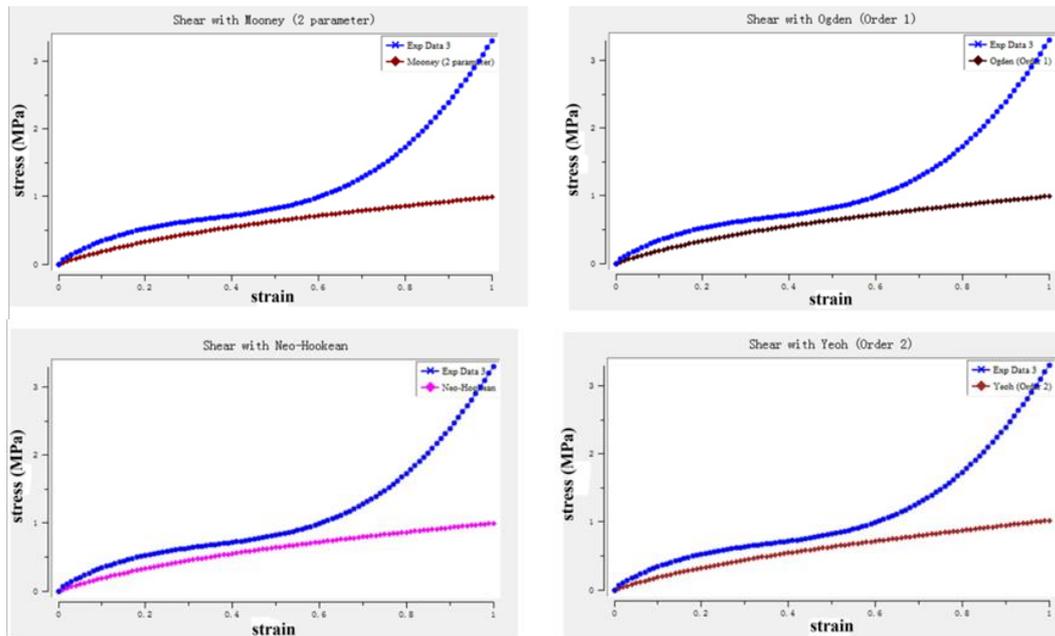
**Figure 3.** Fitting the test results to numerical simulation for different models with uni-axial tension.

#### 4. Determination of material parameters

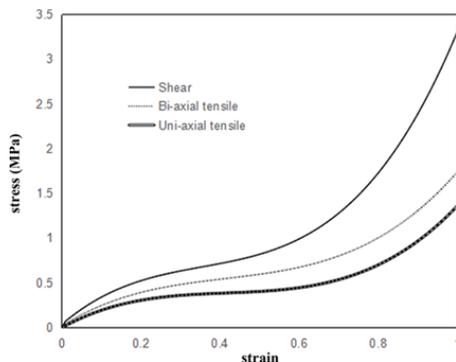
The test results for rubber materials under the uni-axial tensile, equal bi-axial tensile and pure shear are selected from reference [9]. By substituting the constitutive models into the ANSYS platform, the finite element results are simulated, and the non-linear curve are selected to fit the stress-strain relationship for both from test and numerical simulation as shown from figure 3 to figure 5, and the average curves for each test are shown in figure 6, then the material parameters are obtained in table 2.



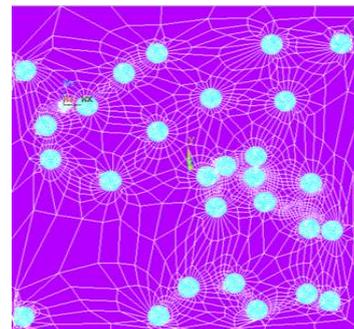
**Figure 4.** Fitting the test results to numerical simulation for different models with Equal bi-axial tension.



**Figure 5.** Fitting the test results to numerical simulation for different models with pure shear.



**Figure 6.** Stress-strain relationship for different test conditions (shear, uni- and bi-axial tensile).



**Figure 7.** Finite element model.

**Table 2.** Material parameters for different models with different test methods.

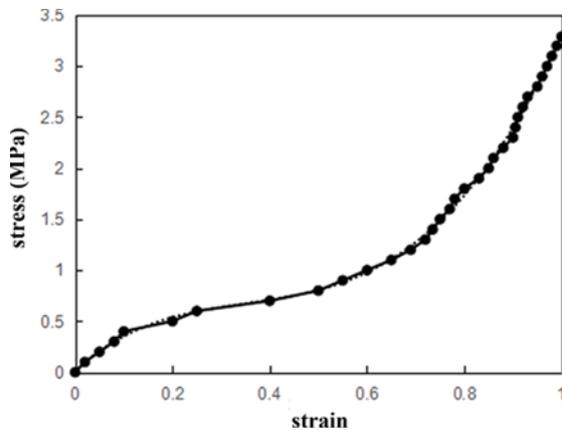
	Uni-axial tensile	Pure shear	Bi-axial tensile	Three test methods
M-R	$C_{10} = 0.227$	---	$C_{10} = 0.183$	$C_{10} = 0.258$
	$C_{01} = 0.057$	---	$C_{01} = 0.038$	$C_{01} = 0.0055$
Yeoh	$C_{10} = 0.232$	$C_{10} = 0.343$	$C_{10} = 0.225$	$C_{10} = 0.259$
	$C_{20} = 0.013$	$C_{20} = 0.069$	$C_{20} = 0.0116$	$C_{20} = 0.0031$
Ogden	$\alpha_1 = 0.41$	$\alpha_1 = 0.338$	$\alpha_1 = 0.291$	$\alpha_1 = 0.537$
	$\mu_1 = 2.281$	$\mu_1 = 3.889$	$\mu_1 = 3.243$	$\mu_1 = 1.98$
N-H	$\mu = 0.525$	$\mu = 0.98$	$\mu = 0.554$	$\mu = 0.531$

## 5. Results and discuss

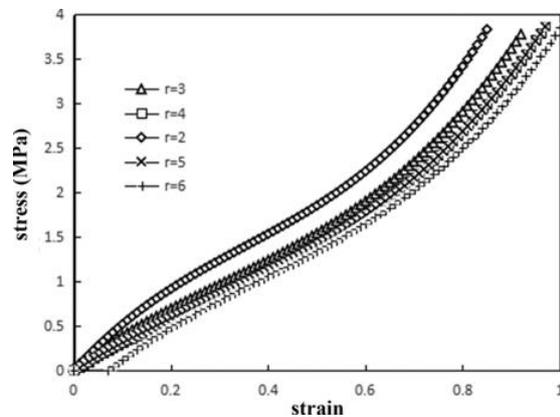
In order to verify the reliability of the supposed model, an example is tested here. The rubber composites are reinforced by random ceramic particles with the volume fraction of 8%, and the radius of the particles are changed from 2nm to 6nm, and the elastic modulus and Poisson ratio for the ceramic particles are 700GPa, 0.499, and for the rubber matrix are 4MPa, 0.23, respectively. The Finite Element model is established in figure 7, and the experimental result for pure shear is shown in figure 8, so the shear parameters in Eq.(7) are obtained from the test results:

$$s_2 = -10.752, s_3 = 11.386, s_4 = -1.675 \quad (7)$$

Substituting the obtained shear parameters into Eq.(7), the stress-strain relationship for different particle's radius are described in figure 9. It shows that the tendency of stress-strain change is consistent with the experimental result in figure 8, and the stress decreases with the increase of the particle's radius, which show that the method is reliable to combine the test result with numerical simulation, and the supposed constitutive equation is verified effectively.



**Figure 8.** Stress-strain relationship for shear deformation.



**Figure 9.** Stress-strain relationship for different particle's radius.

## 6. Conclusions

In this paper, a strain energy constitutive equation with large deformation for rubber composites reinforced with random ceramic particles is proposed from the angle of continuum mechanics theory. The results show that it is reliable to fit the material parameters for three kinds of typical experimental conditions (uni-axial tension, equal bi-axial tension and pure shear) with finite element method, and the proposed constitutive relationship is satisfactory to describe the non-linear stress-strain curves for rubber composites reinforced with random ceramic particles. The radius of random particles has a significant effect on the mechanical properties for rubber composites.

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