

# A Mathematics Model for Determinating the Value of Ijarah Contract

**N Sumarti**

Industrial and Financial Mathematics, Institut Teknologi Bandung

novriana@math.itb.ac.id

**Abstract.** The contract of Ijarah (“providing services and goods temporarily for a wage” from Arabic) of Islamic Finance is an ownership of the right to the benefit of using an asset, or the usufruct, for a period of time. This contract is essentially of the same design as an instalment leasing agreement, in which the owner rents or leases his/her property or goods to a lessee for a specified number of periods for a fee. This paper explains the mathematical formulation in the Ijarah contract, which is needed in the simulation of Ijarah based on the various values of the depreciated methods of assets. The formulae are implemented into some examples. The result of the mix Ijarah and other contracts in a housing credit scheme could reveal the real bank’s margin, using some assumptions.

## 1. Introduction

Conceptually, Ijarah is a contract of exchange where one party enjoys the benefit arising from employment by another party in return for a consideration for the services rendered and from the use of an asset [1]. There are various definitions of the contract of Ijarah given by scholars of the schools of Islamic jurisprudence. One of views from a term of Islamic fiqh refers it as wages paid to a person in consideration of the services rendered by him/her. In the context of Islamic banking, Ijarah can be defined as a process by which the “usufruct/benefit of a particular property is transferred to another person in exchange for a rent claimed from him/her”. These definitions agree on the fact that the contract of Ijarah is a contract on using the benefits or services in return for compensation.

Analysis of risk management on Ijarah Muntahia Bittamleek (IMB) has been conducted in any IMB transaction [1]. The sequential steps certainly or possibly occurring in IMB contract are Agreement to Lease, Order/Receipt of Asset, Contract, Reject/Defective Goods, Early Settlement, Default and Maturity. In order to mitigate potential risks, Islamic law permits to apply Performance or Commercial Risk guarantees and some protection against damage or loss. There are 3 (three) important nature in guaranteeing the obligations; gratuitous, cancelable, not insurance. The latter means it concern the payment of an obligation rather than contingent losses.

In general, the implementation of Sharia compliance products is still a challenge. A critical view on how “Islamic” is the Islamic banking and how different are those banks from conventional banking are discussed in [2]. Having decades of implementation period from the introduction of the first Islamic bank in Egypt in 1963, there are still substantial divergences between Islamic Banking and Financing’s ideals and its practices, because many of them are functionally indistinguishable from conventional banking. A critical view of Ijarah contract, especially on a particular product named Al-Ijarah Thumma Al-Bay’(AITAB) in Malaysia, claims that this Islamic hire-purchase is not much different from



conventional hire-purchase, it has not been provided a satisfactory substitution of conventional products. The only difference is in the issue of interest (*riba*) and issuance of acceptance ('*aqd*') letter. It is lacking in Shariah regulatory framework. It is always referred to conventional regulations (Hire-Purchase Act 1967 or HPA) if there is any dispute arising from the transaction. There is only Shariah rules on the facility determined by Investment and Financing Instruments, Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI) of Bahrain. Since 2002, Malaysian government has issued Muamalah Hire-Purchase Bill in order to improve the Shariah properties of the product.

In Indonesia, Shariah financings including Ijarah in Islamic banking ruled by Bank of Indonesia [4]. The Ijarah *akad* (contract) can be used with other *akads*, for instance the soft-term liquidity financing for Sharia banks [5] and issuance of other Sharia financial instruments including Sharia Obligation or Sukuk [6]. Based on Regulation of Authority of Financial Services [6], Sukuk is a Sharia security in the form of certificate or proof of ownership of an equal value and represents an inseparable or undivided share on the underlying asset. Based on [7], Sukuk Ijarah has been more desirable than another dominant one; *mudharabah sukuk*, since 2004. It also happens in Islamic countries worldwide based on AAOIFI (the Accounting and Auditing Organization of Islamic Financial Institution).

## 2. A Mathematical Model on Ijarah

There are 2 types of Ijarah; the first type is Ijarah Asset, consisting of Tangible Asset (Selling-Ijarah, Ijarah and Ijarah al Muntahia Bit Tamlik (IMB)), and of Intangible Asset (multi-services and continuing Ijarah). The second type is Ijarah Fee including Save Deposit Box and Gold Maintenance/Deposit. Firstly, a mathematical model of simple Ijarah (without buying option) is developed from the accounting theories and rules in [8, 9, 10]. The Ijarah with a buying option called IMB will be explained later.

In Ijarah Asset, the asset is returned to the owner after the period expires. In IMB, with an option to buy the leased object at a particular time according to *akad*. The lessee will pay rent money or *Ujrah*. Let  $n$  be the useful life of the asset,  $S$  be the salvage value of the asset, and  $A$  be acquisition cost of the asset. The salvage value is desired remaining book value of the asset at the end of Ijarah period, and  $S=0$  in general.

### 2.1. Depreciation values of the asset

The object value of Ijarah is depreciated during its useful life or economic age, which could be different from technical age. For instance, a permanent building's useful life is 20 years even though it can be used for more than that. The useful life of goods can be seen in Table 1.

**Table 1.** Useful life of tangible goods [10]

Classification	Useful life (years)
I. Not building	
- Group 1	4
- Group 2	8
- Group3	16
- Group 4	20
II. Building	
- permanent	20
-not permanent	10

Under Article 11 of the Income Tax Law, depreciable tangible assets are assets used to obtain, collect, and maintain income that has a useful life of more than 1 year [9]. They are depreciated in the form of expenditures for the purchase, establishment, addition, improvement, and change of tangible property. There are two depreciation methods; straight-line and declining balance methods. The first method is performed by reducing the acquisition value of the asset in equal parts over the designated useful life of the asset. The second method is done by applying the depreciation rate on the book value of the

remainder. However, at the end of the useful life, the book value is depreciated at once so it will leave zero value, provided that it is conducted consistently. Non building assets are depreciated using the straight line method or decline balance method. Building assets are depreciated only using the straight-line method.

Now we discuss the depreciation methods using assumption that  $S = 0$ . If  $S > 0$  then use  $A^* = A - S$  instead of  $A$ . Assume that the period of Ijarah is  $m \leq n$ . In the Straight-Line method, the depreciation amount at year- $i$  is

$$w_i = \alpha_i \frac{A}{n}$$

where a portion in a year of the useful life  $\alpha_i = 1$  if  $i = 2, 3, \dots, n-1$ ,  $\alpha_1 = 1 - \frac{j-1}{12}$ ,  $\alpha_n = \frac{j}{12}$  with the Ijarah contract occurs in the beginning of  $j$ th-month of the first year of the useful life. If it occurs in January or June, then  $j = 1$  or  $j = 6$  respectively. After the first year, the depreciation amount will be the same until year- $(n-1)$ . Let  $h_i$  be the book value at the end of  $i$ th-year, and  $h_0 = A$ . The book value at year- $i$  and the total depreciation amount are respectively:

$$h_i = \max\{A - iw_i, 0\} = \max\left\{A \left(1 - i \frac{\alpha_i}{n}\right), 0\right\}, i = 1, 2, \dots, n.$$

$$T_{SL} = \sum_{i=1}^m w_i = \frac{m}{144n} j(13-j)h_0.$$

Notice that if  $j = 1$  and  $m = n$  then  $T_{SL} = h_0$ .

In the Decline Balance method, the depreciation amount is decreasing because it is a percentage of the previous book value  $h_{i-1}$ . The formulae of depreciation amounts and the book value at year- $i$  are following.

$$w_1 = \alpha_1 \frac{1}{n} h_0 = \frac{1}{12n} (13-j)h_0,$$

$$h_1 = \max\{h_0 - w_1, 0\} = \frac{1}{12n} (12n - 13 + j)h_0$$

$$w_i = \alpha_i \frac{1}{n} h_{i-1} = \frac{1}{n} \left(1 - \frac{1}{n}\right) h_{i-2} = \frac{1}{12n^i} (12n - 13 + j)(n-1)^{i-2} h_0, \quad i = 2, 3, \dots, m-1.$$

$$h_i = \max\{h_{i-1} - w_i, 0\} = \max\left\{\frac{1}{12n^i} (12n - 13 + j)(n-1)^{i-1} h_0, 0\right\}, i = 1, 2, \dots, m-1.$$

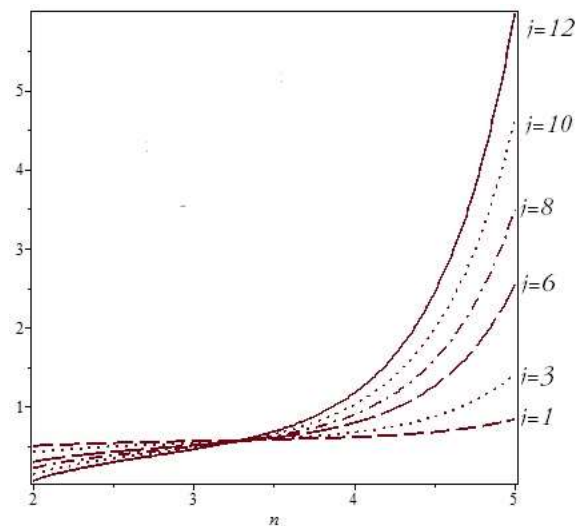
$$w_m = \alpha_m \frac{1}{n} h_{m-1} = \frac{j}{144n^m} (12n - 13 + j)(n-1)^{m-2} h_0$$

$$h_m = \max\{h_{m-1} - w_m, 0\} = \max\left\{\frac{(12n - 13 + j)(j - 12n)(n-1)^{m-2}}{144n^m} h_0, 0\right\}$$

The total depreciation amount at the end of the Ijarah period is following:

$$T_{DB} = \sum_{i=1}^m w_i = \frac{h_0}{144(n-1)^2} \left(144n^2 \left(1 - \left(1 - \frac{1}{n}\right)^m\right) + 12n \left(13 \left(1 - \frac{1}{n}\right)^m - j \left(1 - \frac{1}{n}\right)^m - 24\right) + j(12n(n-1) - 13 + j(n-1))^{m-2} n^{2-m} - 2j(12n(n-1) - 13 + j(n-1))^{m-1} n^{1-m} + j(12n(n-1) - 13 + j(n-1))^{m-2} n^{-m} + 144\right).$$

In Figure 1, the graphs of total amounts of depreciation values for  $n = 4, \dots, 20$  are plotted with different values of month— $j$  of the beginning year. It is straightforward that the longer the useful life, the larger the total of depreciation values. In the first interval up to  $n=3$ , the earlier the starting month, the higher the total of depreciation values. On the contrary in the interval greater than  $n=4$ , it shows that the earlier the starting month, the lower the total of depreciation values.



**Figure 1.** Total amount of depreciation values for useful life  $n = 4, \dots, 5$  and month  $j = 1, 3, 6, 8, 10, 12$ .

## 2.2. Computation of Ujrah

Now we discuss the Ujrah (rent cost) in an Islamic bank with and without buying option at the end of period. Note that Ijarah with buying option is called IMB. Let  $m$  years be the Ijarah period with  $m \leq n$ ,  $x$  be the amount of Ujrah per particular period, in this case per month, and  $y$  be the benefit per year taken by the lessor. Ujrah for Ijarah using the first depreciation method is defined as follows.

$$x_i = \frac{w_i + y}{12}, i = 1, 2, \dots, m.$$

Using the second depreciation method, one way is to make fix values despite using the decreasing values of Ujrah per month.

$$x = \frac{1}{12m} (T_{DB} + my)$$

where  $my$  is the total benefit for the bank.

In IMB scheme, the lessee has an option to buy the underlying asset with price  $P$ . In this case, we can substitute the value of  $n$  by  $m$  in the formula of depreciation amount  $w_i$ .

### 3. Examples of Ijarah

Now we implement the models into two examples, where the latter is extracted from a real offer from an Islamic bank in Indonesia.

#### 3.1. Example 1

In January 2017, Mr S takes Ijarah financing of a car with buy option in an Islamic bank with price Rp 140 million. The bank applies the ujah Rp 3.2 million per month for 36 months, all maintenance cost and others will be a responsibility of the costumer. The car is depreciated using Straight Line method for 5 years. At the end of year 3, the buying price is Rp 65 million. We calculate the total profit for the bank and its percentage.

The amount of depreciation is  $w = \frac{140 \text{ million}}{5 \times 12} = 2.333 \text{ million per month}$ . If Ujah value is  $x = 3.2 \text{ million per month}$ , the total Ujah for 3 years is 115.2 million. The bank's profit from Ujah per month is  $(x - w) = 866,666.67$  or  $\frac{866,666.67}{2,333,333} = 37\%$ . The total profit for 3 years is 31,200.012 million. The amount of depreciation for 3 years:  $36 \times w = 83,999.988 \text{ million}$ , and the book value at year-3 is 56 million. If the buying price is 65 million, then the trading profit is 9 million. Total bank's profit from Ujah and sell price is 40.2 million or  $\frac{40}{140} = 28.7\%$  per 3 years of 9.57% per year.

#### 3.2. KPR (Kredit Perumahan Rakyat or People's housing credit) scheme

An Islamic bank and an individual or a corporate customer enter into a financing agreement named *Musarakah Mutanaqishah* (MMQ) agreement, within 3 years in the form of KPR. If the total capital of MMQ partnership is Rp M million where the Bank's portion is P million (as in Table 2 column 1) and the customer portion is Rp (M-P) million with a *nisbah* (profit sharing ratio) of  $n_b : n_c$ , where  $n_b$  is the bank's nisbah and  $n_c$  is the costumer's nisbah. The house is leased by the costumer using Ijarah contract, so the value of payment contains Ujah and the profit sharing portion for the bank. The information containing two groups of years (group 1: years 1-2, group 2: years 3 – the end) is in Table 2 where the payment per month is fix only in one group. It is stated that the bank's margin percentage of the Ujah is 9.5% for years 1-2 but there is no information on the bank's margin for group 2 [11]. We will breakdown the detail calculation on the bank's nisbah for each group and approximate the bank's margin for group 2.

**Table 2.** Example of KPR scheme of an Islamic Bank

Plafond (Million Rp)	Fix Payment per month					
	5 years		10 years		15 years	
	Years 1 – 2	Years 3 - 5	Years 1 - 2	Years 3 - 10	Years 1 - 2	Years 3 - 15
150	3,150,279	3,254,730	1,940,963	2,169,652	1,566,337	1,908,598
200	4,200,372	4,339,639	2,587,951	2,892,870	2,088,449	2,544,797
250	5,250,465	5,424,549	3,234,939	3,616,087	2,610,562	3,180,996

**Table 3.** Depreciation values

Plafond	5 years	10 years	15 years
150	2,500,000.00	1,250,000.00	833,333.33
200	3,333,333.33	1,666,666.67	1,111,111.11
250	4,166,666.67	2,083,333.33	1,388,888.89

The depreciation values for each scheme are shown in Table 3. We assume that the values of nisbah  $n_b$  are each constant for the same Ijarah period; which are 5, 10 and 15 years. If all entries in Table 2 are arranged as matrix A, for instance  $A_{11} = 3,150,279, A_{12} = 3,254,730, A_{33} = 3,234,939$  etc. Depreciation values in Table 3 is made as matrix B. Denote  $mg_1 = 0.095$  be the Ujah margin for the group 1. Now we show the calculation of the bank nisbah for  $P = 150, n = 5$  in group 1.

$$n_b = \frac{A_{11} - B_{11}}{(1 + mg_1)B_{11}}$$

Given constant  $n_b$ , the Ujrah margin for group 2 can be calculated from the equation as follows.

$$mg_2 = \frac{A_{12} - B_{12}}{n_b B_{12}} - 1$$

The calculated bank's nisbah  $n_b$  for 5 years, 10 years and 15 years are respectively 23.75%, 50.48% and 80.33%. Using these values of bank's nisbah, the bank's margin percentages for years 3 to the end are respectively 27.09%, 45.74% and 60.63%. Note that they are far higher than the stated margin for years 1 – 2, which is only 9.5%. This KPR offer using our assumption above can be considered as containing high margins for the bank.

#### 4. Conclusion

A mathematical model has been developed to calculate the depreciation values of underlying asset used in Ijarah contract. The model can be used to breakdown the detail calculation of an existing offer. This will show the real margin for the bank's profit applied in the offer so we could consider it as expensive or not. The model is a useful tool to simulate potential product of Ijarah with various conditions so we can determine the appropriate bank's margin.

#### References

- [1] M Vejzagic 2014 Ijarah Muntahia Bittamleek (IMB): A risk management perspective *Merit Research Journal of Accounting, Auditing, Economics and Finance* **2** 1 001-007
- [2] F Khan 2010 How 'Islamic' is Islamic Banking? *Journal of Economic Behavior & Organization* **76** 805–820
- [3] N I Abdullah and A W Dusuki 2004 A Critical Appraisal of Al-Ijarah Thumma al-Bay' (AITAB) operation: Issues and Prospects *Paper presented at 4th International Islamic Banking and Finance Conference* Monash University of Malaysia. Kuala Lumpur, November 13<sup>th</sup> – 14<sup>th</sup>
- [4] Regulation of Bank of Indonesia, No. 7/46/PBI/2005 on Contracting and Channelling of Funds for Sharia Compliance Banks. [http://www.ojk.go.id/id/kanal/syariah/regulasi/peraturan-perbankan-syariah-pbi-dan-sebi/Documents/pbi\\_074605\\_1395131491.pdf](http://www.ojk.go.id/id/kanal/syariah/regulasi/peraturan-perbankan-syariah-pbi-dan-sebi/Documents/pbi_074605_1395131491.pdf). Retrieved on July 2017.
- [5] Regulation of Bank of Indonesia, No. 19/4/PBI/2017 on Sharia Liquidity Financing in short term for Sharia Banks. [http://www.bi.go.id/id/peraturan/ssk/Documents/PBI\\_190417.pdf](http://www.bi.go.id/id/peraturan/ssk/Documents/PBI_190417.pdf). Retrieved on July 2017.
- [6] Regulation of Authority of Financial Services, No. 18/POJK.04/2015 on Publishing and Requirements of Sukuk. <http://www.ojk.go.id/id/kanal/syariah/regulasi/peraturan-ojk-terkait-syariah/Default.aspx>. Retrieved on July 2017.
- [7] D A Fatah 2011 Perkembangan Obligasi Syariah (Sukuk) di Indonesia : Analisis Peluang dan Tantangan (The Development of Syari'ah Bonds (Sukuk) in Indonesia: Analysis of Opportunities and Challenges), *Innovatio X*
- [8] Muljono Djoko 2015 *Buku Pintar Akuntansi Perbankan dan Lembaga Keuangan Syariah (A Guide Book on Accounting in Sharia Banking and Financial Institution)* Jakarta: Andi
- [9] Ketentuan Penyusutan, <http://www.pajak.go.id/content/22115121-ketentuan-penyusutan>. Retrieved on July 2017.
- [10] Metode Penyusuta Aktiva Tetap <http://www.pajakonline.com/engine/learning/view.php?id=267>. Retrieved on July 2017
- [11] E Herlinawati, Khairunnisa F R, O Daniel 2017 *Group Final Report on Ijarah Islamic Finance* Course ITB