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Parental Defence on the Offspring: Mathematical Model and Simulation

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Abstract. Parental defence of offspring is an important thing during a predator attack in many species. A higher level of defence will save the offspring but in the other hand it also exposes a higher risk for parent. In this article, the authors try to construct the model for parental defence in prey population. The model based on stage structured dynamical model which assumed that the offspring are entirely dependent on parental care for survival. A contact between parent and predator describe as a distance, not a direct bodily contact. The dynamic and stability analysis are done here to predict the optimal level of defence. Furthermore, the simulation are shown to understand the behaviour of prey population with a certain level of parental defence. The simulation showed that parental defence is increase the survival rate of the offspring.

1. Introduction
In an ecosystem of the animals, the interaction of one species with another plays an important role in maintaining the ecosystem in order to remain sustainable. The interactions can be either symbiotic mutualism or prey predators. With these interactions, each animal has a purpose in defending itself and protecting its community from outside attack, either from predators, or from competition factors with other species. In this article, the main focus is on parental defense on the offspring from predatory attacks. This defense is a safeguard for offspring so that they are able to survive longer and avoid the attack of predators [1].

In one hand, parental defense enhances security for offspring, but on the other hand, it also triggers a big risk to parents because it may cause injured or even death when they want to save the offspring. In order to prevent predator attacks and minimize the risk for parents, usually every animal has an instinct to deal with it, such as they avoid physical contact with predators so they keep the distance between the offspring and the predator for example by hiding or moving silently[2][3][4].

In this study, we made a simple model for the prey population by including the parental defense factor. The defense is expressed in terms of a certain distance from the prey's offspring to predator aiming to predict the dynamical population of prey.

2. Methods
In constructing a population dynamics of prey model, we will create several assumptions to help understand the model. This model is based on a stage-structured model in which one population is divided into two stages of population, the offspring and adult population where every individual in the

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offspring population can transition into adult individuals. The offspring are entirely dependent to the parent defence for survival[1]. This model is also follows the logistic function where every population has its own carrying capacity, so the population could not be grow infinitely or exponentially. Carrying capacity describes the natural resources which include foods and territorial area that support both population to live.

Let $\boldsymbol{E}$ represents the offspring population, $\boldsymbol{A}$ represents adult population, $\boldsymbol{\beta}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\gamma}$ are the number of offspring, number of adult, natural death rate in offspring population, and natural death rate in adult population respectively, $\boldsymbol{K}, \boldsymbol{C}$ are carrying capacity of offspring and adult population, $\boldsymbol{g}$ is transition rate from the offspring into adult, and $\boldsymbol{F}$ is the parental fitness. The parental fitness describe as a function of the parental survival probability ($\boldsymbol{P_1}$), offspring survival probability ($\boldsymbol{P_2}$), probability that the parent survives when employing offspring defence of level $\boldsymbol{d}$ against an attacking predator ($\boldsymbol{Q_1}$), and probability that the offspring survives a predator attack if protected by parental defence of level $\boldsymbol{d}$ ($\boldsymbol{Q_2}$). The function of $\boldsymbol{Q_1}$ and $\boldsymbol{Q_2}$ are important because those included the distance factor. Note that the $\boldsymbol{Q_1}$ is a straight line with negative slope. Therefore $\boldsymbol{Q_1 = 1 - d}$ where $\boldsymbol{d}$ is level of protection.

Based on the assumptions that have been made, the mathematical model for the case above is as follows:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \beta E\left(1 - \frac{E}{K}\right) - \mu E - gE + F \tag{1}$$

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \alpha A\left(1 - \frac{E}{C}\right) + gE - \gamma E \tag{2}$$

where

$$F = Q_1 P_1 + Q_2 P_2 Q_1$$

Substituting F and Q_1 to the (1) we obtain the new form model such that

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \beta E\left(1 - \frac{E}{K}\right) - \mu E - gE + (1-d)P_1 + (1-d)P_2 Q_2 \tag{3}$$

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \alpha A\left(1 - \frac{E}{C}\right) + gE - \gamma E \tag{4}$$

This model has equilibrium point denoted by (E^*, A^*) that ends up with the expression

$$E^*(A) = \frac{A(-\alpha C + \alpha A + \gamma C)}{gC} \tag{5}$$

$$A^* = s_4 A^4 + s_3 A^3 + s_2 A^2 + s_1 A + s_0 = 0 \tag{6}$$

where A^* is four degree polynomial equation with

i. $s_4 = \beta\alpha^2$
ii. $s_3 = 2C\beta\alpha(-\alpha + \gamma)$
iii. $s_2 = C(\alpha\mu K g - gK\beta\alpha + \gamma^2 C\beta + \alpha g^2 K - 2\alpha C\beta\gamma + \alpha^2\beta C)$
iv. $s_1 = C^2 K g(\mu + g - \beta)(-\alpha + \gamma)$
v. $s_0 = C^2 K g^2 (Q_2 P_2 + P_1)(d - 1)$

The exist condition for equilibrium point generally given by

i. $-\alpha C + \alpha A + \gamma C > 0$
ii. The roots of equation (6) must be positive

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Furthermore, we will analyze the stability of the system by the eigenvalues of Jacobian matrix for. Jacobian matrix of the system generally is given by

$$J = \begin{bmatrix} \beta\left(1-\dfrac{E^*}{K}\right)-\dfrac{\beta E^*}{K}-\mu-g & 0 \\ g & \alpha\left(1-\dfrac{A^*}{C}\right)-\dfrac{\alpha A^*}{C}-\gamma \end{bmatrix}$$

Which is obtained the characteristic polynomial such that

$$\lambda^2+\frac{(-K\alpha C+2K\alpha A^*+KC\gamma-CK\beta+2CE^*\beta+CK\mu+CKg)}{CK}\lambda+\frac{(-\alpha C+2\alpha A^*+\gamma C)(-\beta K+2\beta E^*+\mu K+gK)}{CK}$$

The eigenvalues can be obtained by solving the quadratic equation above. Stable coexistence eigenvalue will be analyzed by the roots of quadratic equation. Let λ_1 and λ_2 are the eigenvalues and both of them must be negative to guarantee that the equilibrium point is stable. By the rule of addition and multiplication in quadratic equation, $\lambda_1+\lambda_2<0$ and $\lambda_1\lambda_2>0$. Because the coefficient of λ^2 is positive, so we should make sure that $(-\alpha C+2\alpha A^*+\gamma C)(-\beta K+2\beta E^*+\mu K+gK)>0$ and $(-\alpha C+2\alpha A^*+\gamma C)(-\beta K+2\beta E^*+\mu K+gK)>0$. So this is clear enough to show that equilibrium point stable under certain conditions [5][6]. The next step we would try to simulate the model and describe it according to the simulation result.

3. Results and discussion

To investigate the result of the analysis, here we provide the numerical simulation to illustrate the behavior of the population. The simulation is done by MAPLE 13 conclude dynamical simulation, phase portrait, and sensitivity analysis for both offspring and adult population. The parameters have been chosen based on the Table 1.

Table 1. Parameters that chosen to simulate the population dynamic, phase portrait of the stability, and sensitivity analysis for both population

Parameters	Description	Interval
E	Offspring population	$E>0$
A	Adult population	$A>0$
α	Number of adults	$\alpha>0$
β	Number of offspring	$\beta>0$
μ	Natural death rate of offspring population	$0<\mu<1$
γ	Natural death rate of adult population	$0<\gamma<1$
K	Carrying capacity of offspring population	$K>0$
C	Carrying capacity of adult population	$C>0$
g	Transition rate from offspring population to adult population	$0<g<1$
P_1	Parental survival probability	$0<P_1<1$
P_2	Offspring survival probability	$0<P_2<1$
Q_2	Probability that the offspring survives from a predator attack protected by parental defence of level d	$0<Q_2<1$

The dynamic simulation illustrates the relationship between the two populations with time t. Based on the figure, it can be seen that initial condition for the offspring population is on 10 individuals, while the adult population is on 5 individuals. We set that the parent provides full defense to the offspring, that means the offspring has the highest probability to survive. Both the offspring and adult carrying capacity sets in level 40 individuals. Transition rate is automatically increased the adult population, because there are many offspring become the adult. The result of the first simulation, shows that in the beginning of time, the population of offspring and the adult increased which is influenced by the logistic equation. So both population try to reach their own carrying capacity but carrying capacity that has been reached is less than the actual one, in here the actual carrying capacity is 40 individuals while the carrying capacity according to simulation is around 19 individuals for the offspring, and 22 individuals for the adult (Figure 1). Later in time, the curve of both population remains constant so the condition for both population is stable coexistence.

A stability simulation is performed to check whether the equilibrium point is stable or not[7]. Based on the results of the analysis of eigenvalues obtained that the equilibrium point is stable conditionally. To check the results of the analysis, it will be performed numerical simulation for the equilibrium point by taking any different initial conditions. The result obtained that the equilibrium point is properly stable under certain condition. Based on the simulation results, the two populations will be stable when the offspring population is at number 19 and the adult population is at number 22 (Figure 2).

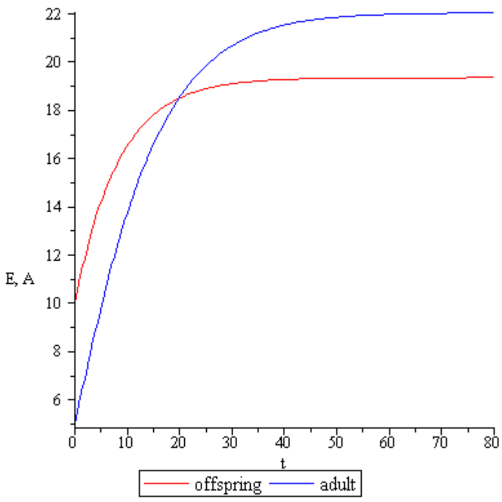

Figure 1. The dynamics of both population with $Q_2 = 0.8, P_1 = 0.9, P_2 = 0.8, d = 1, n = 0.01$.

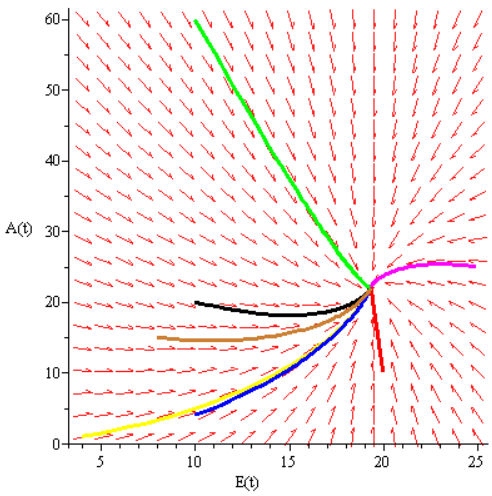

Figure 2. Phase portrait of the equilibrium point with the same parameters in Figure 1 and various initial condition.

The next simulation that we are going to show is sensitivity analysis. Here we should pick two any important parameters to show the relationship between those two parameters and how one parameter influence the other. Let the important parameter in parental defense model is the parental survival probability (P_1), probability that the offspring survives from a predator attack if protected by parental defence of level d (Q_2) , and the level of protection or level of defense from the parent (d). Sensitivity analysis is intended to know the effect of protection level to parameters (P_1) and (Q_2). Therefore, we divide it into sensitivity analysis for adult populations and offspring populations.

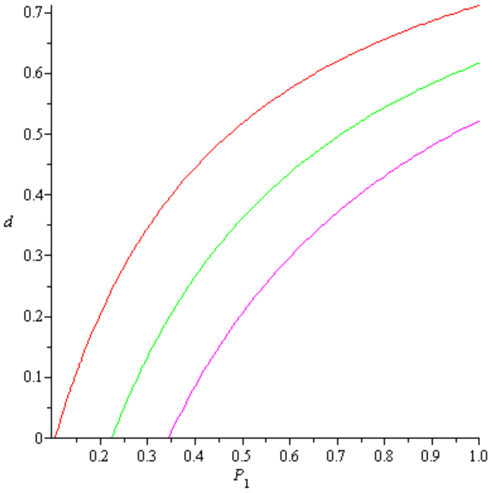

Figure 3. Sensitivity analysis between parameter P_1 and d with several initial condition

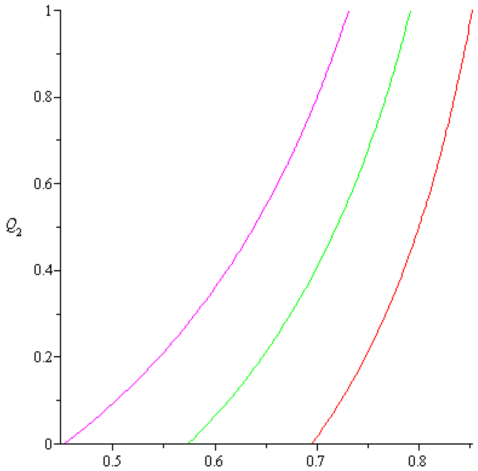

Figure 4. Sensitivity analysis between parameter Q_2 and d with various initial condition

If parental survival probability is low, the parent cannot defend the offspring, so the resistance level is low. Conversely, if parental survival probability is high, the parent is able to defend the offspring. As a result, the level of defense will increase (Figure 3). The red curve in Figure 3 describe the adult population in 19 individuals, while magenta curve in 21 individuals. Therefore, the number of individuals is also important. When the number of individuals decrease, the parental survival probability must be higher as well as level of protection to maintain the population from the extinction. Now we try to simulate the sensitivity analysis that influenced to the offspring population. Suppose that the offspring given by low level of defense by the parent, then the survival probability of the offspring automatically decrease because in this case we assumed that the offspring entirely dependent by the level defense from the parent. By taking different initial condition, the level of defense will work differently depend on the number of individuals on the offspring population (Figure 4). The magenta curve describe the number of offspring in 10 individuals, while red curve in 12 individuals. The result show that the parent should provide less protection when there are only 10 offspring, but they should have more protection as the number of offspring is higher.

Based on the results obtained, this model only discusses the simple model of population dynamics for both population which are offspring population and adult population. In addition to the survival probability of the offspring and the parent, here we make the death rate as small as possible[8][9]. It has been done so that the population dynamics of both populations can survive and stable for long periods of time and do not experience natural extinction. In particular, we demonstrated the influenced of parental defense to the survival probability of the offspring. This model still needs the improvement, such as including time delay because in the real life, there is time delay when the offspring transition to the adult population. Also stage structured dynamical model could be more detail and include the whole generations of population such as from egg, offspring, juvenile, then adult population[7], [10].

4. Conclusion

In conclusion, we suggest that the strategy in providing defence to the offspring needs to be addressed[11]. Nevertheless, this model is good enough to analyse the effect of level of defence in increasing the survival rate of the offspring. The most exciting conclusion that the level defense of the parent increased the survival probability of the offspring. But first the parent should have high parental survival probability so they could provide high protection or defense for their offspring. Note that the number of both population are also important as describe in the result of sensitivity analysis simulation.

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However, this is important because the parent should provide the higher level of defense if the number of offspring is bigger.

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