

# Numerical prediction of the elastic characteristics of spatially reinforced composite materials

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**Abstract.** In this paper, the effective elastic characteristics of spatially reinforced composite materials is calculated. The problem is solved by the finite element method in the Ansys Workbench software using the periodicity cell of the material. When constructing the model, were considered the weaving scheme, the volume fraction of the reinforcing skeleton, and the geometric shape of the reinforcing filaments. The effective elastic modules are determined by the volume averaging method. To obtain nine independent characteristics of the orthotropic material, six calculation modes were used: tension along the axis and ashear in the planes. The stress-strain state of reinforcing filaments is analyzed.

## 1. Introduction

Currently, more attention is being paid to spatially reinforced composite materials (SRCM). These materials differ from traditional laminates by the reinforcement in third direction. In layered composite materials, the layers are joined together by a binder. In the SRCM manufacturing, additional bonds are created between the layers by introducing reinforcement in directions that differ from the direction of layer reinforcement. The use of such manufacturing method increases the specifications of composite materials in shear strength and tear resistance. Thus, these materials can be used in aircraft and rocketry to produce radar fairings (with glass fiber reinforcement), brake discs for aerospace industry (carbon-carbon materials) [1]. SRCM are used for the manufacturing of turbine blades, nozzle blocks, nose cones of rocket engines, sealing elements for hydraulic and pneumatic systems and many other structural elements that are operated under conditions of intense thermal (or cryogenic) and mechanic impact.

When designing structures from SRCM, it becomes necessary to develop methods for predicting the physico-mechanical properties of the materials. Existing experimental methods allow us to determine with sufficient accuracy only six elastic constants: elastic moduli  $E_1$ ,  $E_2$ , Poisson's ratio  $\mu_{12}$ , shear modulus in  $G_{12}$  plane and interlayer shear moduli  $G_{13}$  and  $G_{23}$  [2]. Therefore, the development of a technique for numerical prediction of the elastic characteristics of spatially reinforced composite materials is an urgent task.

## 2. Problem formulation

The subject of this study is the SRCM periodicity cell (Figure 1). The structure of the periodicity cell is a system of weft and sewing threads where parts of the reinforcing skeleton are connected by matrix. For carrying out a number of computational experiments, a geometric model was constructed

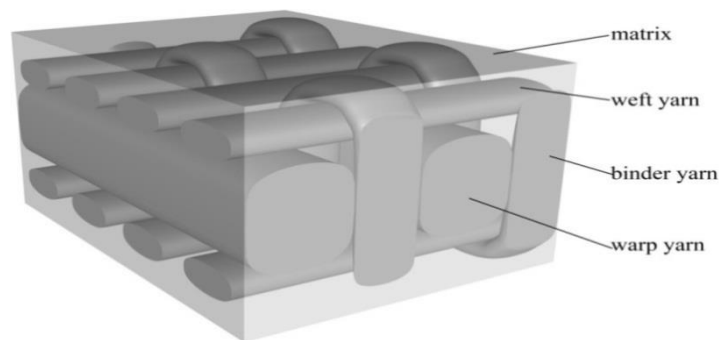


using the TexGen software package. The initial parameters (the cross-sectional area of the filaments and the distance between their centers), necessary for constructing the geometric model of the unit cell of the fabric, were determined on the basis of the specified parameters – the linear density of the filaments and the density for the base and weft. During the geometric model design, the following assumptions are used: solid state threads, the interaction of the filament fibers is ignored, there is no interfacial layer between the matrix and the filaments, technological dispersion in the geometry of the filaments is not taken into account. The deformation of threads during production process is also neglected.

Taking into account the given technological parameters of weaving, the dimensions of the unit cell of this fabric are 4.76 mm along the X axis, 3.68 mm along the Y axis and 2.57 mm along the Z axis. Assuming an ideal packing of fibers in the complex filament, its conditional cross-sectional area can be determined by the formula:

$$S = \frac{T}{pk} \quad (1)$$

where  $T$  is the linear density of the complex filament;  $p$  is the density of the yarn material,  $\text{kg/m}^3$ ;  $k$  is the degree of volume filling of the filament.



**Figure 1.** General view of the geometric model of the periodicity cell of a spatially reinforced composite material.

The mathematical formulation of the problem corresponds to the theory of elasticity of an anisotropic body. In the variational formulation this problem consists of finding the minimum of the Lagrange functional with additional conditions in the form of geometric Cauchy relations [3]. The variation of the functional in the absence of mass forces has the form:

$$\delta J_u = \int_V \varepsilon_{ij} C_{ijkl} \delta \varepsilon_{kl} dV - \int_{s_t} F_i \delta u_i dS \quad (2)$$

where  $\varepsilon_{ij}$  and  $\delta \varepsilon_{ij}$  are the tensor and the variation of the strain tensor correspondingly,  $C_{ijkl}$  is the tensor of elastic modules,  $\delta u_i$  is the variation of the displacement vector,  $F_i$  is the vector of external force.

Additional conditions for the functional (2) are geometric equations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (3)$$

Elastic constants of transversal-isotropic yarns based on carbon fibers and epoxy binder for the numerical simulation were used from papers [4-7].

The SRCM sample considered is orthotropic and is characterized by nine independent elastic constants: Young's modules  $E_X$ ,  $E_Y$ ,  $E_Z$ , Poisson's ratios  $\nu_{XY}$ ,  $\nu_{YZ}$ ,  $\nu_{XZ}$ , and shear modules  $G_{XY}$ ,  $G_{YZ}$ ,

$G_{XZ}$ . Where X is the direction of the base, Y is the direction of the weft. Between the interacting threads of the model the ideal contact interaction type was used. For the periodicity cell, six types of experiments are realized: stretching along the X, Y, Z axes to determine the effective Young's modules and Poisson's ratios and a shear in the planes XY, XZ, YZ in order to determine the shear modules [8]. Effective modules are obtained by averaging over the volume [9] using the following dependencies:

$$\begin{aligned} E_X^* &= \frac{\langle \sigma_X \rangle}{\varepsilon_X}, E_Y^* = \frac{\langle \sigma_Y \rangle}{\varepsilon_Y}, E_Z^* = \frac{\langle \sigma_Z \rangle}{\varepsilon_Z}, \\ G_{XY}^* &= \frac{\langle \tau_{XY} \rangle}{\gamma_{XY}}, G_{YZ}^* = \frac{\langle \tau_{YZ} \rangle}{\gamma_{YZ}}, G_{XZ}^* = \frac{\langle \tau_{XZ} \rangle}{\gamma_{XZ}}, \\ \nu_{XY} &= \left| \frac{\varepsilon_Y}{\varepsilon_X} \right|, \nu_{YZ} = \left| \frac{\varepsilon_Z}{\varepsilon_Y} \right|, \nu_{XZ} = \left| \frac{\varepsilon_Z}{\varepsilon_X} \right|, \end{aligned} \quad (4)$$

where deformations  $\varepsilon_X$ ,  $\varepsilon_Y$ ,  $\varepsilon_Z$  and  $\gamma_{ij}$  were predetermined in numerical experiments.

The average stresses were calculated from the following relationships:

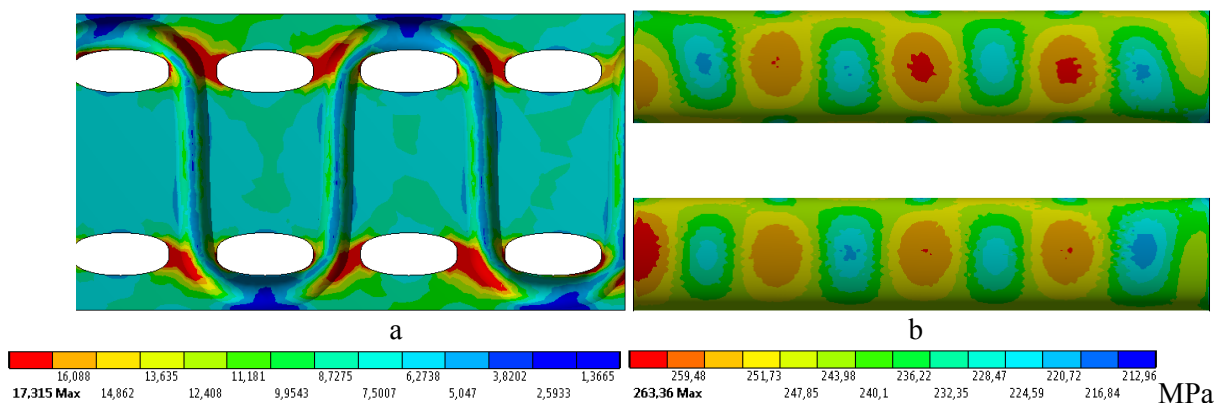
$$\langle \sigma_{ij} \rangle = \frac{\sum_{k=1}^n \sigma_{ij}^k V^k}{V}; i, j = x, y, z, \quad (5)$$

where n is the number of finite elements in a representative volume,  $\sigma_{ij}^k$  is the stress in the final element,  $V^k$  is the volume of the final element,  $V$  is the total volume of the model.

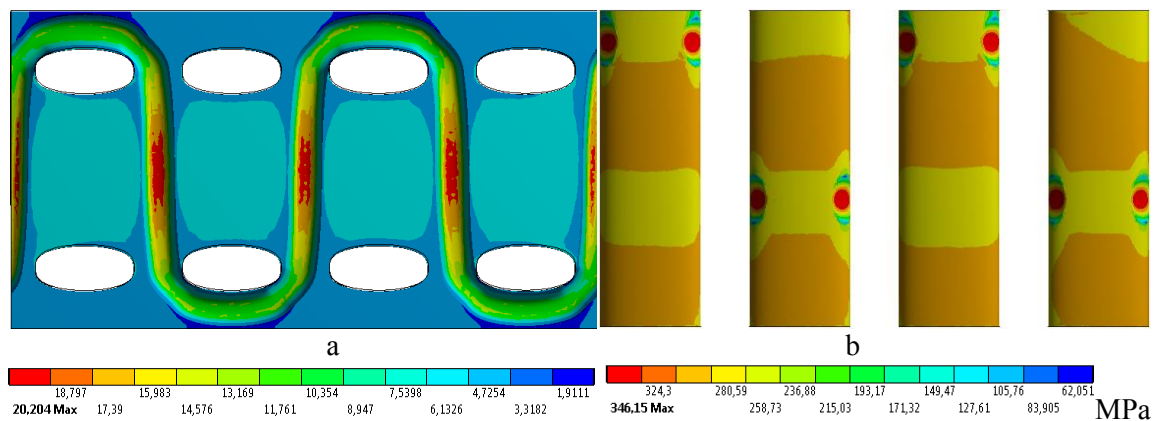
The discretization of the representative volume was carried out using the spatial finite elements SOLID186. The minimum size of the final element was 0.01 mm, the total number of finite elements was 3 million. It was taken into account that the numerical experiment is considered to converge, in the case when the result of numerical simulation does not change with further refinement of the finite element grid.

### 3. Results

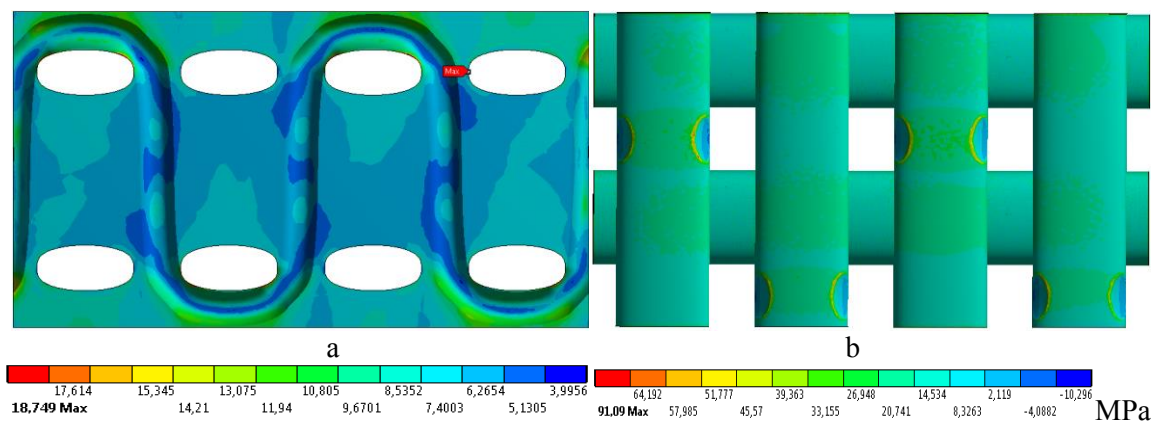
Inhomogeneous fields of equivalent, normal and shear stresses for the matrix and filaments in the local coordinate system were obtained from the results of numerical simulation of the periodicity cell stress-strain state (Figure 2-7). The inhomogeneity was caused by the three-dimensional weaving of SRCM. The macroscopic stresses and deformations were determined in the cell under study, and the effective elastic characteristics of SRCM were found at the ratio 4.5 (Table 1).



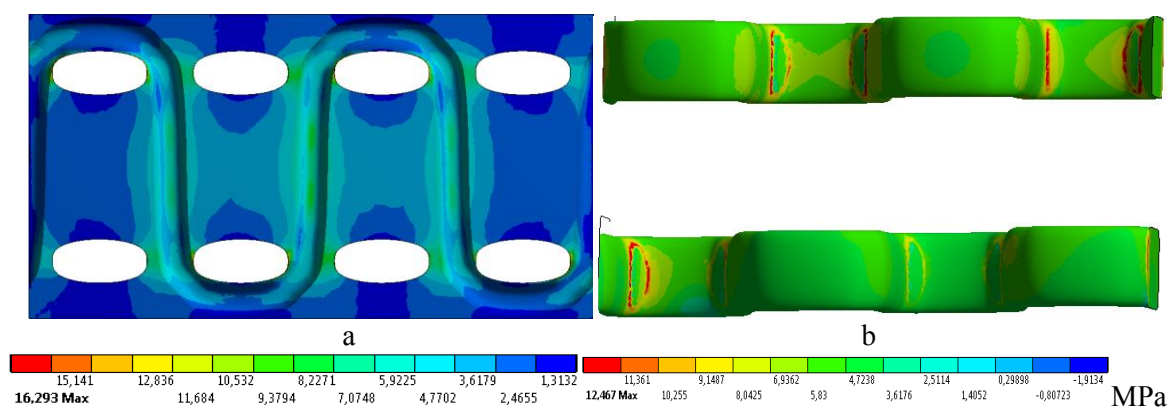
**Figure 2.** Distribution of stress fields when stretching along the X axis: a – equivalent von Mises stresses in matrix; b – normal  $\sigma_{11}$  in warp threads.



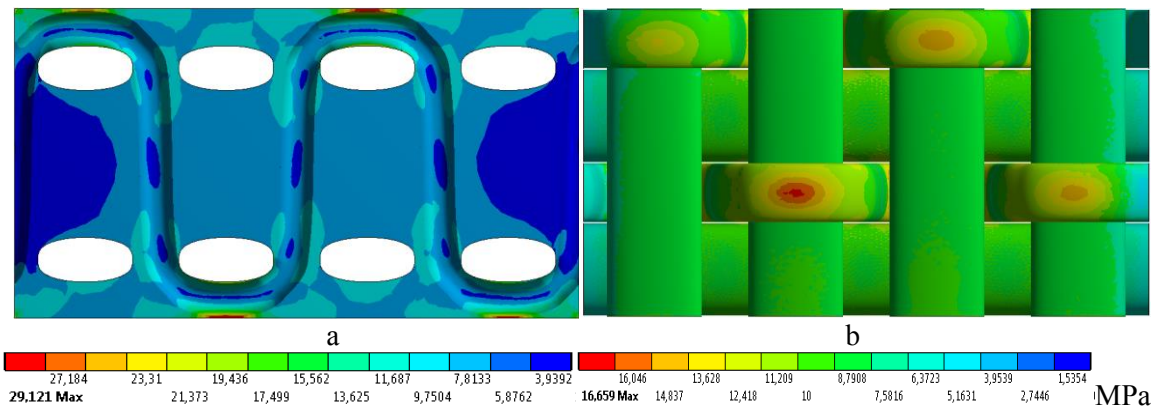
**Figure 3.** Distribution of stress fields when stretching along the Y axis: a – equivalent von Mises stresses in matrix; b – normal  $\sigma_{22}$  in weft threads.



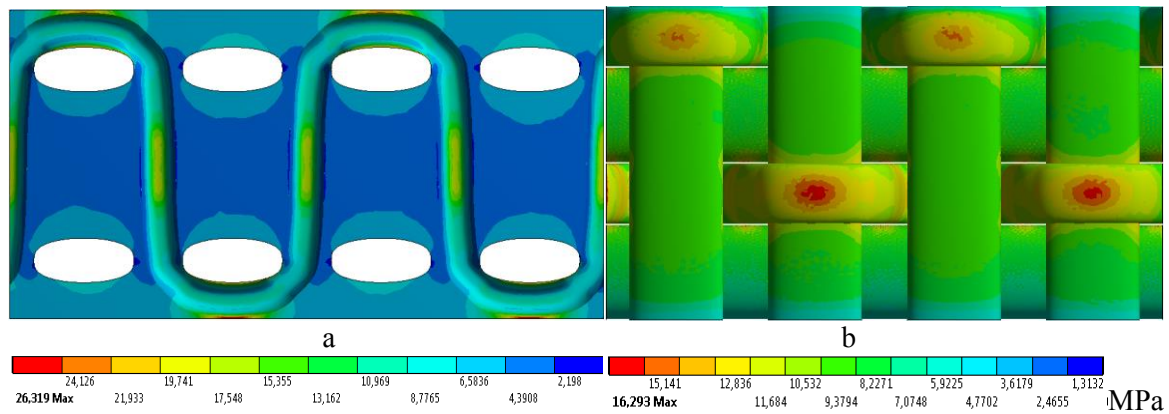
**Figure 4.** Distribution of stress fields when stretching along the Z axis: a – equivalent von Mises stresses in matrix; b – normal  $\sigma_{33}$  in the warp and weft yarns.



**Figure 5.** Distribution of stress fields with a shear in the XY plane: a – equivalent von Mises stresses in matrix; b – tangents  $\tau_{12}$  in the sewing threads.



**Figure 6.** Distribution of stress fields with shear in the plane XZ: a – equivalent von Mises stresses in matrix; b – the tangent  $\tau_{13}$  in the threads of the weft and the base.



**Figure 7.** Distribution of equivalent stress fields in the matrix by the Mises criterion in the longitudinal sections along the center: a – the warp threads; b – the sewing thread, with shear in the YZ plane.

**Table 1.** Effective elastic properties of a textile-reinforced composite.

$E_x$ (GPa)	$E_y$ (GPa)	$E_z$ (GPa)	$\nu_{xy}$	$\nu_{yz}$	$\nu_{xz}$	$G_{xy}$ (GPa)	$G_{yz}$ (GPa)	$G_{xz}$ (GPa)
62.4	42.3	7	0.08	0.3	0.28	4.52	2.55	2.49

Analysis of equivalent stress fields of the matrix revealed that the maximum values 29 MPa are observed with a shear in the XZ plane, near bend of the sewing threads, and equal to approximately 29 MPa. These stresses will be the most dangerous, determining the safety margin of a structure made using this type of weaving. The places of localizations of maximum stresses in the framework were also revealed. When stretching along the X axis, the maximum normal stresses  $\sigma_{11}$  were detected in the warp yarns, in the regions between the base threads and weft threads and amounted to approximately 263 MPa. The greatest normal stresses  $\sigma_{22}$  under tension along the Y axis are observed in the contact areas of the weft yarns and bonding yarns and are 346 MPa. The analysis of stresses along the Z axis shows that the maximum stresses  $\sigma_{33}$  occur in the binding zone of the bonding yarns and equal 91 MPa. With transversal shear in the XY plane, the maximum shear stresses  $\tau_{12}$  are localized at the bends



of the connecting yarns at the contact points with weft threads and are about 12.5 MPa. With shear in the XZ and YZ planes, the greatest shear stresses are observed at the upper inflection point of the bonding yarns and equal to 16.6 MPa and 16.3 MPa for  $\tau_{12}$  and  $\tau_{23}$  correspondingly

In the framework of verification of the developed numerical model, the results obtained were compared with the results of mechanical tests. Comparison showed the following differences from natural tests: 2.2% when stretching along the base ( $E_x$ ), 15% when a shearing across the main threads ( $G_{xy}$ ), 12% for the Poisson's ratio ( $\nu_{xy}$ ). The difference can be explained by the spread of the characteristics of the filaments and the binder and by the presence of technological defects in the full-scale specimens.

#### 4. Conclusion

Thus, a numerical model and a technique for calculating the effective properties of SRCM have been developed. The model takes into account the properties of the material components, their volume content, and the spatial framework architecture, which is determined by the technological parameters of the weaving (number of basic and filling layers, stacking density, interlock machine type, etc.). Effective elastic characteristics for a spatially reinforced composite material are obtained. Comparison of the results of numerical simulation with mechanical tests is performed, good matching of the experimental and calculated values of the effective properties of SRCM is obtained. Analysis of stresses in the SRCM matrix for six calculated cases revealed that the most dangerous, determining the safety factor of a structure made using this type of weave, are the stresses  $\tau_{13}$  observed under shear in the XZ plane.

#### Acknowledgments

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