

Abrasive slurry jet cutting model based on fuzzy relations

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Abstract. The cutting process of pre-mixed abrasive slurry or suspension jet (ASJ) is a complex process affected by many factors, and there is a highly nonlinear relationship between the cutting parameters and cutting quality. In this paper, guided by fuzzy theory, the fuzzy cutting model of ASJ was developed. In the modeling of surface roughness, the upper surface roughness prediction model and the lower surface roughness prediction model were established respectively. The adaptive fuzzy inference system combines the learning mechanism of neural networks and the linguistic reasoning ability of the fuzzy system, membership functions, and fuzzy rules are obtained by adaptive adjustment. Therefore, the modeling process is fast and effective. In this paper, the ANFIS module of MATLAB fuzzy logic toolbox was used to establish the fuzzy cutting model of ASJ, which is found to be quite instrumental to ASJ cutting applications.

1. Introduction

In recent decades, with the development of industrial processing field, high precision machining methods are becoming more and more mature. High-pressure waterjet cutting is one of the high-precision machining methods [1].

Water jet cutting is considered to be one of the best adaptive cutting methods [2-4]. Not only it can be used for cutting of hard alloy, ceramic, die steel, high silicon cast iron and high hardness material [5], but also it has a good performance for the cutting of viscoelastic materials such as titanium alloy and rubber. The earliest use of water jet is in the breaking of rock, the auxiliary drilling, hydraulic coal mining, and building demolition [6]. The addition of abrasive (slurry) makes the performance of water jet more powerful.

However, the cutting process of the abrasive water jet is a complex stochastic process affected by many parameters, which proper selection is very important. Since the determination of these parameters depend on the experience of operators, so it is difficult to establish an appropriate mechanism model.

In the current process databases, the conditions and data provided are limited and discrete. Therefore, many input data are not applicable for the process database output in practical applications, which brings inconvenience to end users. In order to solve these problems, the concept of "fuzzy logic" is introduced into the database system, so that in any conditions in the effective range of the corresponding process data can be obtained.

Adaptive neural fuzzy inference system (ANFIS) has a strong ability in dealing with the nonlinear, high complexity and discontinuous problems, and it also has many characteristics such as fast convergence speed, good stability, repeatability of the training process and higher precision of the predict. Therefore, it is especially suitable for the problems of engineering prediction. These problems



are highly complex because of their characteristics of diversity, spatial and temporal, but that's the good part of ANFIS predicting technology [7, 8].

To solve the problem of constructing an accurate cutting model, a cutting model of ASJ was developed in this study using the fuzzy inference system, which can provide the roughness prediction of the cutting process of abrasive slurry/water jet. In addition to incorporating the discrete values of the parameters into the training sample task, the model can also predict the output of any other value within the effective range via the membership function. Therefore, the fuzzy inference model established in this paper can be used for guiding the cutting process of ASJ, which can be lucrative for the intelligent control system of ASJ cutting.

2. Brief introduction to fuzzy theory

The fuzzy theory is to use fuzzy logic to describe the rank of things in real life, the concept of fuzzy is defined by the form of the fuzzy set, it can quantify the rank of events that belong to this collection then obtain the membership function, the membership degree is used to deal with various problems.

For a classic set A, any of the elements X in the space should correspond to $X \in A$ or $X \notin A$. This feature can be expressed as a function:

$$\begin{aligned}\chi_A(x) &= 1, x \in A \\ \chi_A(x) &= 0, x \notin A\end{aligned}\quad (1)$$

where $\chi_A(x)$ is the characteristic function of set A. If the characteristic function is extended to the fuzzy set, it is found that only 0 and 1 values are applied to the classical set, after being extended to the fuzzy set, the values are changed into the interval [0, 1].

In the domain of U, definition mapping μ_A :

$$\mu_A: U \rightarrow [0,1] \quad u \mapsto \mu_A(u) \quad (2)$$

It assumed that set U defines a fuzzy subset μ_A , record as A^* , called μ_A is the membership function of A^* . The value of μ_A at u is called the membership degree of u to A^* . Record as $\mu_A(u)$ or $A^*(u)$.

3. Prediction model of upper roughness

The input parameters of roughness prediction model in this paper are outlet pressure of pipe of the water jet, cutting target distance, cutting speed and abrasive particle size, the output parameters are the roughness of the upper cutting surface. This system adopted in this paper is the Takagi-Sugeno (T-S) fuzzy logic system. As the output of this fuzzy system is single point fuzzy, so the multi-input and the multi-output system can be decomposed into multi-input and single-output system.

The conclusion of the T-S fuzzy model is a linear function of the input parameters, so it has these following advantages: high computational efficiency, be easy to use optimization and adaptive technology, guarantees the continuity of the output space and be suitable for mathematical analysis. The model uses the following "if-then" rule:

$$\begin{aligned}R^k: & \text{ if } x_1 \text{ is } F_1^k, \dots, x_n \text{ is } F_n^k, \text{ then} \\ y_k &= p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n = p_0^k + \sum_{i=1}^n p_i^k x_i\end{aligned}\quad (3)$$

where F_i^k is the fuzzy set of fuzzy system, p_j^k is the parameter of the fuzzy system, and y^k is the output according to fuzzy rule.

3.1. Model elaboration

The process of fuzzy modeling is a dynamic process that uses fuzzy systems to gradually approach the unknown nonlinear system, to achieve the goal of forcing the whole system. The network structure of the fuzzy system is shown in figure 1:

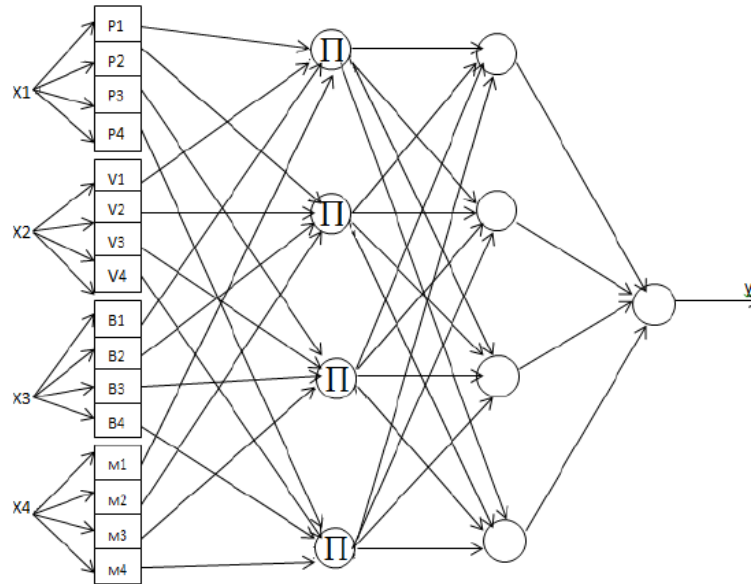


Figure 1. The network architecture of fuzzy systems of upper roughness.

There are four input parameters of the fuzzy system (x_1 , x_2 , x_3 , x_4), which are outlet pressure of pipe of the water jet, cutting speed, target distance, and abrasive particle diameter, respectively. After inputting the parameter values, they will be transferred to the first layer of the network. The range of each variable can be specified according to different applications.

The first layer is membership generating layer, the input variables will be fuzzy transformed in this layer, the number of nodes is determined by the number of fuzzy subsets of each input variables, where each node represents a subset (fuzzy linguistic variables). The number of nodes is based on a number of horizontal parameters, so the number of nodes in this layer is 16. The function of this layer is to compute the membership degree of each input component of the parameter to each fuzzy subset (linguistic variable value). The Gauss type membership function is considered here, as shown in equation (4):

$$\mu(x_{ij}) = \exp \left(- \left(\frac{x_j - c_{ij}}{b_{ij}} \right)^2 \right) \quad (4)$$

The second layer is the fuzzy inference layer, and each node of the layer represents a possible fuzzy rule. Taking the output of the second layer as the antecedent of this rule, the output of the fuzzy inference layer is the product of the membership of each part of the antecedent, which can be expressed by equation (5):

$$w(i) = m(x_{i1}) * m(x_{i2}) * m(x_{i3}) * m(x_{i4}) \quad (5)$$

The transfer function of each node of the third layer is a linear function, that is, the local linear model, so the output rule of the third layer is:

$$y_i = w(i) * (p_0^i + p_1^i x_1 + p_2^i x_2 + p_3^i x_3 + p_4^i x_4) / \sum_{i=1}^n w(i) \quad (6)$$

The set of parameter variables consisting of $\{p_0^i, p_1^i, p_2^i, p_3^i\}$ is called the parametric parameter of the conclusion.

The fourth layer: calculating the total output of the input parameter:

$$y = \sum_{i=1}^n \varpi_i y_i = \sum_{i=1}^n \omega_i y_i / \sum_{i=1}^n \omega_i \quad (7)$$

After determining the central values and widths (c, b) of the membership functions of each node of the second layers and the value of the connection weight (p) of the network post, the above fuzzy system can be determined

3.2. Process of model training

3.2.1. Acquisition of sample data. The sample data are the basis of the ANFIS model. The experiment of obtaining the sample data was carried out for an ASJ cutting machine. When selecting the sample, one needs to ensure the following points:

- In the training sample, the parameters of each variable should be balanced as much as possible to avoid bias to a certain factor or a horizontal value;
- In order to ensure the generalization ability of the network, the relevance of the data to be read should be as small as possible, even irrelevant;
- The input samples and output samples should have a nice relevance, and both these two values can be extracted or controlled;
- The interference of external causes should be avoided as much as possible in training sample selection.

Usually, the process of determining the capacity of sample data, too much capacity or too little capacity is unreasonable. Too much capacity will increase the difficulty of obtaining sample data, while too little capacity will not reflect the nature of the problem. Based on the cutting experiment of ASJ, 80% of the samples could be used for modeling, and 20% of the sample data would be used as model checking. In this paper, 6 datasets of experimental data were randomly selected as model checking data sets, and the remaining data were used as training samples for training adaptive fuzzy neural networks. Table 1 is part of the experimental data of ASJ cutting, which is used as the sample data of the model.

Table 1. Training data on 1060 aluminum.

No.	Pressure MPa	Velocity mm/s	Distance mm	Material	Upper roughness μm	Lower roughness μm
1	33	1.2	0.5	m1	3.274	4.115
2	33	0.9	0.5	m1	3.0815	3.888
3	30	1.2	1	m1	3.5856	4.562
4	30	0.9	1	m1	3.4465	4.321
5	25	0.6	2	m1	3.187	4.107
6	25	1.2	2	m1	3.54	4.476
7	27	0.6	1.5	m1	3.0522	3.853
8	27	1.2	1.5	m1	3.402	4.218
9	33	1.5	2	m1	3.1071	3.926
10	33	0.6	2	m1	2.6835	3.1325

3.2.2. Normalization of input and output data. Since the function depends on the inner product of the input sample vector, in this experiment, each attribute had a larger range of values, which makes the calculation more complex and the training time longer. In order to avoid the above situation, the parameters should be normalized.

Normalization is to limit the amount of data that need to be processed within a certain range

through an algorithm. After normalization, the data will be more convenient to use, and the convergence speed of the program can be accelerated. In MATLAB, there are 3 main methods for normalization.

a. Linear function transformation, the expression is as follows:

$$y = (x - x_{\min}) / (x_{\max} - x_{\min}) \quad (8)$$

where x and y are the values before and after conversion, respectively; $x_{\max}$ and $x_{\min}$ respectively are the maximum and minimum values of samples, respectively.

b. Logarithmic function transformation, the expression is as follows:

$$y = \log_{10}(x) \quad (9)$$

c. Inverse cotangent conversion, the expression is as follows:

$$y = \arctan(x) * 2 / \pi \quad (10)$$

By comparison, it can be found that the linear function transformation method has advantages of simpler operation and easier to implement compared with logarithm function transformation method and inverse cotangent transformation method. So linear function transformation method would be used as the normalized method.

The function “mapminmax” can make each row of the matrix normalized to [-1 1], and the syntax of the “mapminmax” statement is: [y1, PS] = mapminmax (x1). Where x1 is the normalized matrix, and y1 is the result of normalization. When it need restore the normalized data, using the following command: x1_again = mapminmax (‘reverse’, y1, PS)

3.2.3. Determination of the fuzzy system parameters. In the process of fuzzy modeling in the previous section, the membership function of the network structure of the second layer has been determined. Gauss type function is most widely used in MATLAB, and the output of the fuzzy inference system is a linear function of the input parameters, so it would be selected, and the expressions are as follows:

$$\mu(x_{ij}) = \exp \left(- \left(\frac{x_j - c_{ij}}{b_{ij}} \right)^2 \right) \quad (11)$$

$$y_i = w(i) * (p_0^i + p_1^i x_1 + p_2^i x_2 + p_3^i x_3 + p_4^i x_4) / \sum_{i=1}^n w(i) \quad (12)$$

In the equation, the values of c , b , and p are the parameter values of the system, and the fuzzy system is determined to determine the values of c , b and p of the system.

The program first assigned t for a column of 0.3, while the determination of b and c was realized by the system’s random assignment.

When determining the number of fuzzy iterations, iterations of 100, 200, 300, 500, 800 and 1000 times were set. The variance analysis of training data is shown in tables 2 and 3 below:

Table 2. Analysis of different iterations’ variance of the upper roughness model.

Times	100	200	300	500	800	1000
Variance	0.01334	0.00643	0.0056	0.00515	0.00371	0.00270

Table 3. Extrapolation analysis of variance of upper roughness model.

Times	100	200	300	500	800	1000
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Variance	0.06225	0.05425	0.05278	0.05075	0.05774	0.05942
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As can be seen from table 2, the error of the training data decreases gradually with the increased number of iterations.

In case depicted in table 3, the error decreases first and then increases with the number of iterations. For the number of iterations equal to 500, the error is the smallest.

Through a comprehensive analysis of tables 2 and 3, it was found that with the increase in the number of iterations, the degree of fitting of the training array would gradually improve, but then, due to the array extrapolation overfitting, the error became larger, which makes the model less adaptive to extrapolation arrays. Therefore, the number of iterations of the model was limited to 500.

Then, the system error has to be calculated. According to the calculation error, the system updated the values of membership parameters b and c , and the adjusted weight p of the model.

3.3. Analysis of the model realization results

When executing the prediction program via the model, the predicted and actual values of the 32 sets of training samples were displayed in the MATLAB command window, as well as the respective error estimations.

After the program realization, the actual values of the training data, the ANFIS prediction values, and their deviations/errors appeared in the MATLAB command window, which are partially listed in table 4.

Table 4. Deviation of the results predicted by the model from the actual ones.

Group No.	4	5	6	7	8	9	10	11	12
Roughness deviation	0.0923	0.0266	0.0485	-0.0140	0.0141	-0.0252	0.0162	-0.0463	-0.713

By the error values in table 4, it can be clearly seen that the error is at the 10^{-2} level. Compared with the surface roughness value, the error is very small, so the T-S fuzzy system can be used to predict the cutting surface roughness, and the result is accurate. The reliability of the T-S fuzzy system would be more visually and intuitively judged by comparing the actual values of the cut surface roughness with the predicted values, as shown in figure 2.

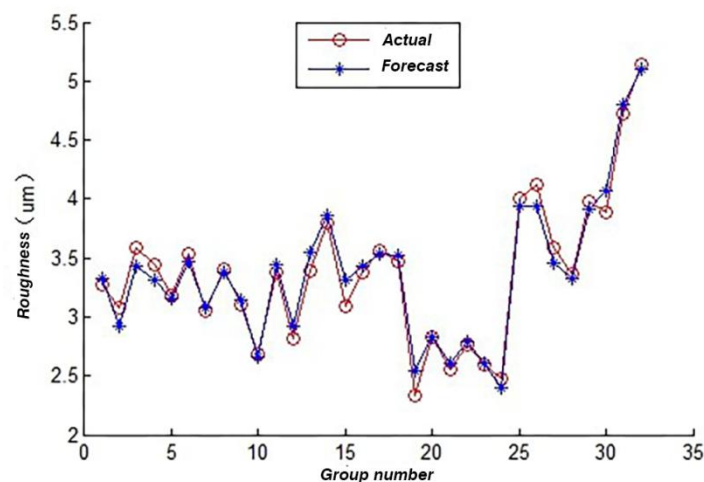


Figure 2. The actual and predicted roughness of training data.

The red line in figure 2 represents the actual result of the roughness of the training sample, and the blue line is the predictive value of the upper roughness model. As can be seen from figure 2, the actual

and predictive values of the 32 sets of sample data used for training are relatively close, and the error is small, which shows that the model has a good fitting effect, and the system training is very effective.

The model was validated by the additional six sets of data that were not involved in the previous training process. After the input of the above six datasets, the program was run and the outputs were displayed in the MATLAB command window, as shown in table 5:

Table 5. The roughness of the additional data.

Output data	3.342	3.2936	3.7637	3.3751	3.4060	1.9240
Test validation data	3.6762	3.5356	3.9025	3.3087	3.5209	1.7216

The integrated fitting results of the training and validation datasets are depicted in figure 3.

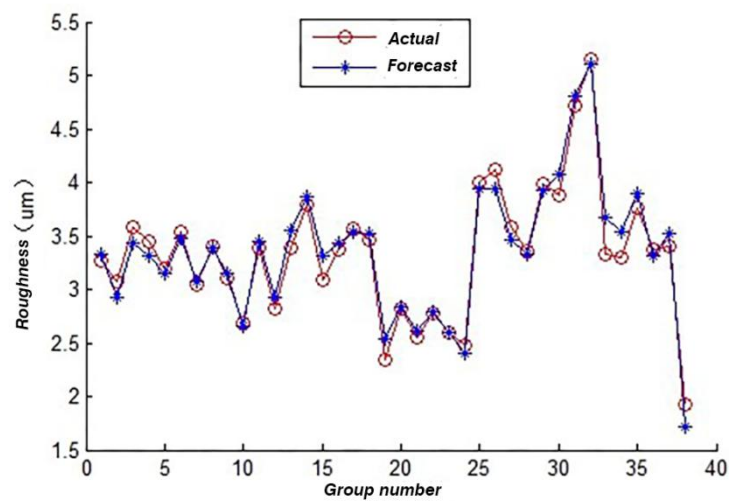


Figure 3. The actual and predicted roughness of training and validation data.

The six datasets on the right side of figure 3 correspond to the validation array. It can be observed that the difference is small in the fitting process with the training array, and the predicted values are close to the actual results. Therefore, the model has better generalization function, and it has certain guiding significance for the cutting process of ASJ. Therefore, the ANFIS can be used to predict the quality of ASJ cutting.

4. Prediction model of the lower roughness

The modeling of lower section roughness and the realization process of the ANFIS program are similar to those of the upper section roughness, with the only exception: in the process of data input, one needs to select the lower roughness for training purposes. Therefore, the description of modeling and training processes are omitted for brevity sake.

When determining the number of fuzzy iterations, their number was set to 100, 200, 300, 500, 800, and 1000. The variance analysis of training data is shown in tables 6 and 7.

Table 6. Analysis of different iterations variance of lower roughness model.

Number of iterations	100	200	300	500	800	1000
Variance	0.01774	0.01300	0.01066	0.00796	0.00692	0.00626

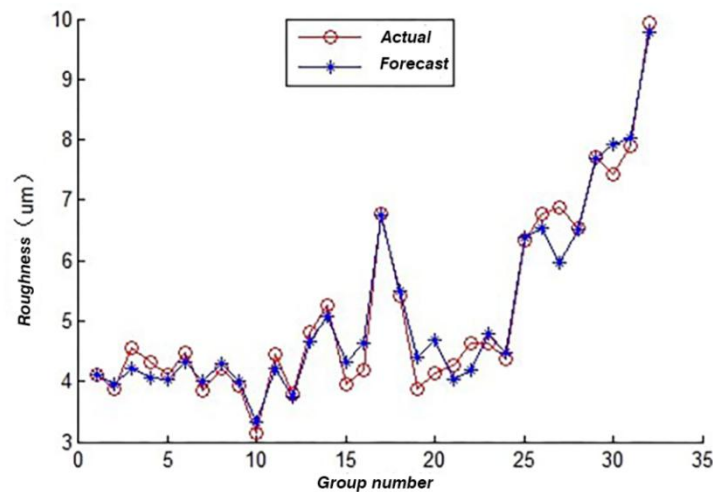
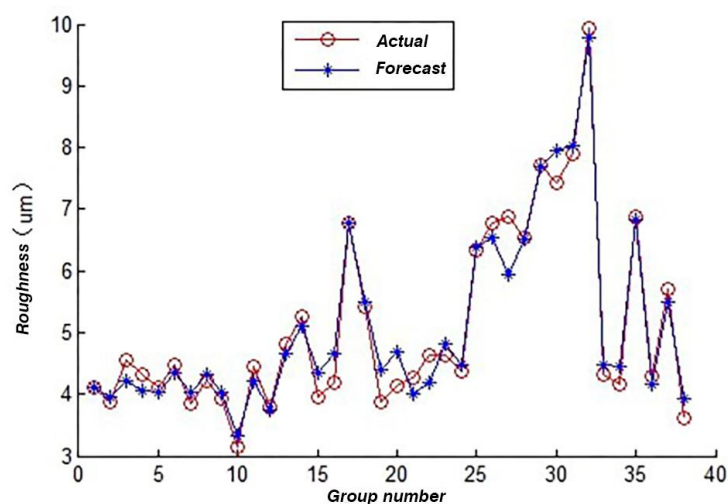
As seen from table 6, the training data error decreases gradually with the number of iterations.

Table 7. Extrapolation analysis of variance of the lower roughness model.

Times	100	200	300	500	800	1000
Variance	0.09277	0.06841	0.05834	0.04371	0.04576	0.04866

As can be seen from table 7, as the number of iterations increases, the error decreases first and then increases. When the number of iterations is 500, the error is the smallest.

In tables 6 and 7, it is seen that with the increase in the number of iterations, the array fit training is improved, but because of the phenomenon of the array extrapolation overfitting, the error became larger, which makes the model less adaptive to extrapolation arrays. Therefore, the number of iterations of the model was set to 500. After the input of training data (lower roughness), the program was run, the lower roughness values predicted by the model were obtained and compared with the actual values, the comparison results being shown in figure 4.

**Figure 4.** The actual and predicted lower roughness values of training data.**Figure 5.** The actual and predicted lower roughness of training and validation data.

Similar to the upper surface roughness prediction model, after the input of six validation datasets not used for training, the program execution resulted in adequate fitting results of the training and validation datasets, as shown in figure 5.

A joint analysis of figures 4 and 5 has revealed that the lower roughness model can also well predict the plane roughness, and the fitting results on trained and validation datasets meet the basic requirements. Therefore, the model can be incorporated into the ANFIS fuzzy inference system to predict the cross-sectional roughness value of ASJ cutting process.

5. Conclusions

- Based on the theory of fuzzy inference system, combined with the characteristics of ASJ, this article established the roughness prediction model, selected appropriate membership functions, the output of the system as a linear combination of the input.
- Through the comparison between forecast value and actual value of the analysis by using the dates in the real experiment as training samples of the cutting model, it is found that the training effect of the upper and lower roughness prediction model is good. The roughness prediction model has good function of training and generalization, so it has a good guidance to the experiment of ASJ cutting.

Acknowledgments

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