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Transversely Isotropic Hyperelastic Constitutive Model of Short Fiber Reinforced EPDM Based on Tensor Function

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Abstract. Short fiber reinforced EPDM is a new kind of composite material used in solid rocket motor winding and coating. It has relatively large deformation under the small stress condition, and the physical non-linear characteristic is obvious. Due to the addition of fiber in the specific direction of the rubber, the macroscopic mechanical properties are expressed as transversely isotropic properties. In order to describe the mechanical behavior under the impact and vibration, the transversely isotropic hyperelastic constitutive model based on tensor function is proposed. The symmetry of the transversely isotropic incompressible material limits the stress tensor '$\boldsymbol{K}$' to be characterized as a function of 5 tensor invariants and 4 scalar invariants. The third power constitutive equations of the model give 12 independent elastic constants of the transversely isotropic nonlinear elastic material. The experimental results show that the non-zero elastic constants are different in the fiber direction and at the different strain rate. Number and value of adiabatic layer and related products R & D has a reference value.

1. Introduction

Ethylene Propylene Diene Monomer (EPDM) is a kind of polymer rubber which has good heat insulation performance. The thermal stability of EPDM can be attributed to its saturated main chain structure. EPDM exhibits outstanding resistance to oxidation, ozonization, weathering effects and excellent low temperature properties. EPDM also has the lowest density among elastomer (0.85 g/cm3) [1,2]. In order to adapt to the complex working environment, the mechanical properties of materials is improved by adding reinforcing agent, organic and inorganic filler[3,4,5]. The short fibre added to the isotropic rubber matrix can improve the tensile performance. Multiple theory model is used to describe the mechanical behavior of rubber materials, such as the Neo-Hookean model[6], the Mooney-Rivlin model[7], the Yeoh model[8], the Ogden model[9],etc. Due to the addition of fiber in the specific direction of the rubber, the macroscopic mechanical properties are expressed as transversely isotropic properties[10]. This paper presents a new constitutive model based on tensor function transverse isotropic elastic material.

2. Experiment and constitutive theory

The internal structure of short fiber reinforced EPDM material is shown in Fig.1. It defines parallel to the fiber direction of 0º, perpendicular to the fiber direction of 90º. The thickness of the thin film coating layer is 0.5mm. The strip specimen's size is 80mm×10mm. Its gage length is 40mm. The tensile testing was performed along the direction 0º and 90º with different tensile strain rates for 0.125/min, 0.5/min, 2.5/min under the condition of 293k and 50% humidity[11].

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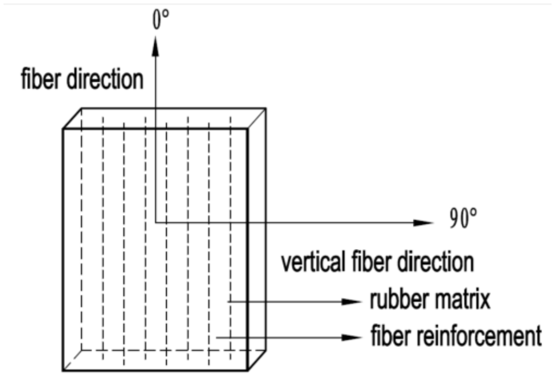

Fig.1 Schematic of short fiber reinforced EPDM.

Tensor function in describing nonlinear material mechanics behavior model is very effective. The general nonlinear constitutive theory research and the theory of tensor functions in the application of continuum mechanics began in Rivlin[12] and Reiner's[13] work, and the nonlinear tensor theory has carried on the detailed research by Mr Chen Li [14]. The symmetry of the transversely isotropic incompressible material limits the stress tensor '$\boldsymbol{K}$' to be characterized as the function of 5 tensor invariants and 4 scalar invariants in equation 1.

$$\boldsymbol{K} = \boldsymbol{K}(\boldsymbol{1}, \boldsymbol{E}, \boldsymbol{E}^2, \boldsymbol{a}\otimes\boldsymbol{a}, \boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}, \mathrm{tr}\boldsymbol{E}, \mathrm{tr}\boldsymbol{E}^2, \boldsymbol{a}\cdot\boldsymbol{E}\cdot\boldsymbol{a}, \boldsymbol{a}\cdot\boldsymbol{E}^2\cdot\boldsymbol{a}) \tag{1}$$

'$\boldsymbol{E}$' is strain tensor. '$\boldsymbol{a}$' is meaning of transverse isotropic materials on the surface of the isotropic normal vector, and it is an one-order tensor invariant. '$\boldsymbol{a}\otimes\boldsymbol{a}$' is meaning of a two-order tensor invariant. The nonlinear transverse isotropic incompressible elastic material constitutive in equation 1 can be described as follows:

$$\boldsymbol{K} = \varphi_1\boldsymbol{1} + 2\varphi_2\boldsymbol{E} + 3\varphi_3\boldsymbol{E}^2 + \varphi_4\boldsymbol{aa} + \varphi_5(\boldsymbol{aa}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{aa}) \tag{2}$$

$\{\boldsymbol{e}_i\}$ is the natural base vector strain tensor of Cartesian coordinate system.

$$\begin{aligned} &\boldsymbol{e}_\mathrm{a} := \boldsymbol{e}_1\otimes\boldsymbol{e}_1, \quad \boldsymbol{e}_\mathrm{b} := \boldsymbol{e}_2\otimes\boldsymbol{e}_2, \quad \boldsymbol{e}_\mathrm{c} := \boldsymbol{e}_3\otimes\boldsymbol{e}_3 \\ &\boldsymbol{e}_\mathrm{d} := \boldsymbol{e}_1\otimes\boldsymbol{e}_2 + \boldsymbol{e}_2\otimes\boldsymbol{e}_1, \quad \boldsymbol{e}_\mathrm{e} := \boldsymbol{e}_2\otimes\boldsymbol{e}_3 + \boldsymbol{e}_3\otimes\boldsymbol{e}_2, \quad \boldsymbol{e}_\mathrm{f} := \boldsymbol{e}_3\otimes\boldsymbol{e}_1 + \boldsymbol{e}_1\otimes\boldsymbol{e}_3 \end{aligned} \tag{3}$$

$\{\boldsymbol{e}_\alpha\}(\alpha = a,\cdots,f)$ make up the two-order symmetric tensor space Sym orthogonal basis, and $\boldsymbol{a} = \boldsymbol{e}_3$.

$$\begin{cases} \boldsymbol{1} = \boldsymbol{e}_a + \boldsymbol{e}_b + \boldsymbol{e}_c \\ \boldsymbol{E} = E_1\boldsymbol{e}_a + E_2\boldsymbol{e}_b + E_3\boldsymbol{e}_c + E_4\boldsymbol{e}_d + E_5\boldsymbol{e}_e + E_6\boldsymbol{e}_f \\ \boldsymbol{E}^2 = (E_1^2 + E_4^2 + E_6^2)\boldsymbol{e}_a + (E_2^2 + E_4^2 + E_5^2)\boldsymbol{e}_b + (E_3^2 + E_5^2 + E_6^2)\boldsymbol{e}_c \\ \quad +(E_1E_4 + E_2E_4 + E_5E_6)\boldsymbol{e}_d + (E_2E_5 + E_3E_5 + E_4E_6)\boldsymbol{e}_e \\ \quad +(E_1E_6 + E_3E_6 + E_4E_5)\boldsymbol{e}_f \\ \boldsymbol{a}\otimes\boldsymbol{a} = \boldsymbol{e}_3\otimes\boldsymbol{e}_3 = \boldsymbol{e}_c \\ \boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a} = E_{3k}(\boldsymbol{e}_3\otimes\boldsymbol{e}_k + \boldsymbol{e}_k\otimes\boldsymbol{e}_3) = 2E_3\boldsymbol{e}_c + E_5\boldsymbol{e}_e + E_6\boldsymbol{e}_f \end{cases} \tag{4}$$

$$\begin{cases} \bar{I}_1 = tr\boldsymbol{E} = E_1 + E_2 + E_3 \\ \bar{I}_2 = tr\boldsymbol{E^2} = E_1^{\ 2} + E_2^{\ 2} + E_3^{\ 2} + 2(E_4^{\ 2} + E_5^{\ 2} + E_6^{\ 2}) \\ J_1 = \boldsymbol{a}\cdot\boldsymbol{E}\cdot\boldsymbol{a} = E_3 \\ J_2 = \boldsymbol{a}\cdot\boldsymbol{E^2}\cdot\boldsymbol{a} = E_{3k}E_{k3} = E_3^{\ 2} + E_5^{\ 2} + E_6^{\ 2} \end{cases} \tag{5}$$

Making the equation 4,5 into equation 2, get 1 to 3 power of constitutive equation 6,7,8.

$$\begin{aligned} \boldsymbol{K}_{(1)} &= \varphi_1\boldsymbol{1} + 2\varphi_2\boldsymbol{E} + \varphi_4\boldsymbol{a}\otimes\boldsymbol{a} + \varphi_5(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \\ &= (k_1\bar{I}_1 + k_2J_1)\boldsymbol{1} + 2k_3\boldsymbol{E} + (k_2\bar{I}_1 + k_4J_1)\boldsymbol{a}\otimes\boldsymbol{a} + k_5(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \end{aligned} \tag{6}$$

$$\begin{aligned} \boldsymbol{K}_{(2)} &= \varphi_1\boldsymbol{1} + 2\varphi_2\boldsymbol{E} + 3\varphi_3\boldsymbol{E}^2 + \varphi_4\boldsymbol{a}\otimes\boldsymbol{a} + \varphi_5(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \\ &= (k_1\bar{I}_1^{\ 2} + k_2\bar{I}_2 + 2k_3\bar{I}_1J_1 + k_4J_1^{\ 2} + k_5J_2)\boldsymbol{1} + 2(k_2\bar{I}_1 + k_6J_1)\boldsymbol{E} + 3\mathrm{k}_7\boldsymbol{E}^2 \\ &\quad + (k_3\bar{I}_1^{\ 2} + k_6\bar{I}_2 + 2k_4\bar{I}_1J_1 + k_8J_1^{\ 2} + k_9J_2)\boldsymbol{a}\otimes\boldsymbol{a} + (k_5\bar{I}_1 + k_9J_1)(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \end{aligned} \tag{7}$$

$$\begin{aligned} \boldsymbol{K}_{(3)} &= \varphi_1\boldsymbol{1} + 2\varphi_2\boldsymbol{E} + 3\varphi_3\boldsymbol{E}^2 + \varphi_4\boldsymbol{a}\otimes\boldsymbol{a} + \varphi_5(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \\ &= (k_1\bar{I}_1^{\ 3} + 2k_2\bar{I}_1\bar{I}_2 + k_3\bar{I}_1 + 3k_4\bar{I}_1^{\ 2}J_1 + k_5\bar{I}_1J_1^{\ 2} + 2k_6\bar{I}_1J_2 + k_7\bar{I}_2J_1 + k_8J_1^{\ 3} \\ &\quad + k_9J_1J_2 + k_{17}\bar{I}_1^{\ 2} + k_{18}\bar{I}_2)\boldsymbol{1} + 2(k_2\bar{I}_1^{\ 2} + k_{10}\bar{I}_2 + k_7\bar{I}_1J_1 + k_{11}J_1^{\ 2} + k_{12}J_2)\boldsymbol{E} \\ &\quad + 3(\mathrm{k}_3\bar{I}_1 + k_{13}J_1)\boldsymbol{E}^2 + (k_4\bar{I}_1^{\ 3} + k_7\bar{I}_1\bar{I}_2 + k_{13}\bar{I}_1 + k_5\bar{I}_1^{\ 2}J_1 + 3k_8\bar{I}_1J_1^{\ 2} + k_9\bar{I}_1J_2 \\ &\quad + 2k_{11}\bar{I}_2J_1 + k_{14}J_1^{\ 3} + 2k_{15}J_1J_2 + k_{19}\bar{I}_1^{\ 2} + k_{20}\bar{I}_2)\boldsymbol{a}\otimes\boldsymbol{a} + (k_6\bar{I}_1^{\ 2} + k_{12}\bar{I}_2 + k_9\bar{I}_1J_1 \\ &\quad + k_{15}J_1^{\ 2} + k_{16}J_2)(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \end{aligned} \tag{8}$$

$$\begin{aligned} \boldsymbol{K} &= \boldsymbol{K}_{(1)} + \boldsymbol{K}_{(2)} + \boldsymbol{K}_{(3)} \\ &= (k_1\bar{I}_1^{\ 3} + 2k_2\bar{I}_1\bar{I}_2 + k_3\bar{I}_1 + 3k_4\bar{I}_1^{\ 2}J_1 + k_5\bar{I}_1J_1^{\ 2} + 2k_6\bar{I}_1J_2 + k_7\bar{I}_2J_1 + k_8J_1^{\ 3} \\ &\quad + k_9J_1J_2 + k_{17}\bar{I}_1^{\ 2} + +k_{18}\bar{I}_1J_1 + k_{19}\bar{I}_2 + k_{20}J_1^{\ 2} + k_{21}J_1 + k_{22}J_2)\boldsymbol{1} + 2(k_2\bar{I}_1^{\ 2} + k_{10}\bar{I}_2 \\ &\quad + k_7\bar{I}_1J_1 + k_{11}J_1^{\ 2} + k_{12}J_2 + k_{23} + k_{24}\bar{I}_1 + k_{25}J_1)\boldsymbol{E} + 3(k_3\bar{I}_1 + k_{13}J_1 + k_{26})\boldsymbol{E}^2 \\ &\quad + (k_4\bar{I}_1^{\ 3} + k_7\bar{I}_1\bar{I}_2 + k_5\bar{I}_1^{\ 2}J_1 + 3k_8\bar{I}_1J_1^{\ 2} + k_9\bar{I}_1J_2 + 2k_{11}\bar{I}_2J_1 + k_{13}\bar{I}_1 + k_{14}J_1^{\ 3} \\ &\quad + 2k_{15}J_1J_2 + k_{27}\bar{I}_1^{\ 2} + k_{28}\bar{I}_2 + k_{29}\bar{I}_1J_1 + k_{30}J_1^{\ 2} + k_{31}J_1 + k_{32}J_2)\boldsymbol{a}\otimes\boldsymbol{a} + (k_6\bar{I}_1^{\ 2} \\ &\quad + k_{12}\bar{I}_2 + k_9\bar{I}_1J_1 + k_{15}J_1^{\ 2} + k_{16}J_2 + k_{33} + k_{34}\bar{I}_1 + k_{35}J_1)(\boldsymbol{a}\otimes\boldsymbol{a}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{a}\otimes\boldsymbol{a}) \end{aligned} \tag{9}$$

To simplify the equation 9 to uniaxial tensile state specific equation are obtained.

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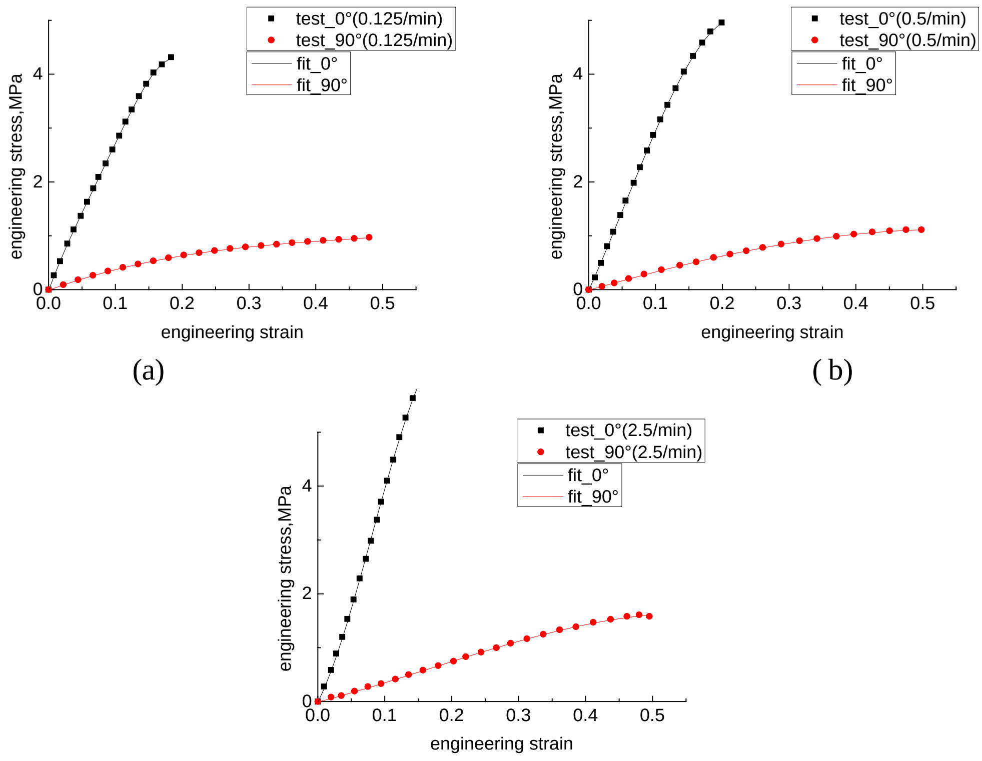
$$K_{ii}=(k_1\bar{I}_1+k_2\bar{I}_2+k_3\bar{I}_1^{\,2}+k_4\bar{I}_1J_1+k_5\bar{I}_1^{\,3}+k_6\bar{I}_1\bar{I}_2)+2(k_7+k_8\bar{I}_2+k_9J_2+k_{10}\bar{I}_1^{\,2})\varepsilon_{ii}$$
$$+3(k_{11}+k_{12}\bar{I}_1)\varepsilon_{ii}^2 \qquad (10)$$

3. Experimental analysis and discussion

The engineering stress and strain curve are obtained through 0º and 90º direction of uniaxial tensile test, and the elastic constants of the nonlinear elastic constitutive equation is obtained by REGRESS function in MATLAB software [15]. The elastic constants 'k_1' –'k_{12}' are shown in Tab.1 and Tab.2 . Non-zero elastic constants are 'k_7', 'k_8', 'k_{11}'. Among them 'k_7' is a low order elastic constant in constitutive equation, and it occupies an important position in the material mechanics performance of elastic deformation. Its value decreases with the increasing of tensile strain rate. 'k_8' and 'k_{11}' are high order elastic constants. Their values are affected by the nonlinear mechanical properties of the material, could be positive or negative. It can draw stress and strain curve through bringing these strain data and the constants 'k_1' –'k_{12}' into equation 10. Better fitting effect as can be seen from Fig.2. The equation 10 for describing transverse isotropic materials under uniaxial tensile constitutive relation is very effective.



(c)

Fig 2. Fitting results of hyperelastic parameters at different tensile strain rates.

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Table 1. The non-zero elastic constants of short fiber reinforced EPDM 0° at different tensile strain rates.

	0.125/min (MPa)	0.5/min (MPa)	2.5/min (MPa)
k_7	14.2339	13.74827	12.25978
k_8	-69.9581	-117.13	-366.889
k_{11}	3.884324	18.17015	85.62068

Table 2. The non-zero elastic constants of short fiber reinforced EPDM 90° at different tensile rates.

	0.125/min (MPa)	0.5/min (MPa)	2.5/min (MPa)
k_7	2.269142	1.766896	1.604102
k_8	1.924606	-0.82582	-3.06809
k_{11}	-2.63598	-0.4654	1.496165

4. Conclusions

The following formula shows the short fiber reinforced EPDM uniaxial tensile constitutive equation.

$$K_{ii} = (k_1 \bar{I}_1 + k_2 \bar{I}_2 + k_3 \bar{I}_1^{\,2} + k_4 \bar{I}_1 J_1 + k_5 \bar{I}_1^{\,3} + k_6 \bar{I}_1 \bar{I}_2) + 2(k_7 + k_8 \bar{I}_2 + k_9 J_2 + k_{10} \bar{I}_1^{\,2})\varepsilon_{ii} + 3(k_{11} + k_{12} \bar{I}_1)\varepsilon_{ii}^2$$

It only has 3 elastic constants in direction of specimen 0°and 90° uniaxial tensile in 0.125 / min to 2.5 / min tensile strain rate of short fiber reinforced EPDM.

The low order term elastic constants of 'k_7' value decreases with the increasing of tensile strain rate.

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Responsible author; Chen Li, Professor of Taiyuan University of Science and Technology, China. tydz_lc@126.com.

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