

Researches Regarding the Main Chain Vibrations Study of a Shaper

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Abstract. In this paper we highlight the importance of applying the integral transform for determining the partial dynamic response of the assembly tool – tool holder following the vibrations that appear during the working time.

1. Introduction

The paper [1] presents a mathematical model which allows the simulation of three dimensional behaviour of planetary/epicyclic and helical gears. The motion equations are solved by using a time-integration scheme and a contact algorithm simultaneously for all the networking.

A mechanical and mathematical model for a tool and the support unit of a milling device is presented in the paper [2]. In this way, a mechanical model with three freedom degrees was created for which there was applied the Lagrange formalism leading to a system with three differential equations of second degree. By using the integral transform Laplace, the mathematical model was reduced to a polynomial equation which was easily resolved.

Based also on the Laplace transform, unilaterally in relation with time, in [3] there were obtained the transversal displacements for kinematical elements that are in vibratory motion.

In [4] the longitudinal and transversal displacements fields were determined for the linear elastic rod of an R mechanism (RRT) by using an iterative method. In the paper [5] there were obtained the same elements, but for a linear elastic connecting rod, part of an parallelogram mechanism.

The dynamic behaviour modeling of a mechanical system is essential if it is desired to obtain vibration and acoustical acceptable levels. In this context, in [6], the authors affirm that there is a very large consensus in the researches domain to take into consideration the transmission errors fluctuations. Here, the authors develop the main techniques used and the hypotheses associated with these ones.

The essential problems of structural systems analysis subjected to time analysis were presented in [7]. More precisely, there were determined the solutions of these problems by using a method in which there were idealized these systems by using an assembly of discrete structural elements and by reaching to equations sets solved conveniently matrix algebraically operations.

The importance of vibrations study in the mechanical systems design processes or in the manufacturing processes is studied in [8] through different methods, by insisting over the noises occurrence as an immediate consequence of the vibrations.



The vibrations influence over the errors that may appear in the motion transmission process of the mechanical systems is studied in [9].

In [10] it is developed an analytical model of mechanical systems vibrations for motion transmission. This is used to investigate the natural frequencies and vibrations modes. The vibrations modes are classified in rotation and translation modes. The unique characteristics of each mode type are detailed investigated analytically.

Many of the calculus methods used are based on a matrix formalism easily implemented on computers. A good part of differential equations solutions, which describe, for example, the material systems behaviour at the internal and external factor influence that act upon them, can be obtained by using the computer.

In order to validate the essential mechanical models are the dynamic tests. In this way, an important role is played by the modal experimental analysis problems and the way in which the correspondence between the calculus – tests is made, but also the model actualization. Largely used in mechanics, particularly in vibration mechanics, are the inverse problems. Through the applications and methods used we can cite the sources inverse problems, shape optimization, devices active control for fluid transports or, in the end, the integral equations methods largely used in mechanics and acoustics. The present work can be enclosed on this direction.

2. Mechanical model. Mathematical model

Vibrations mechanical model of the main kinematical chain from a shaper should contain the tool holder masses, its support, the rotating device and of headwork, all considered as non-deformable solids. But, the basis vibrations elements analysis allows the elimination of partial systems with small influence over the elastic system dynamic behaviour in its assembly, reason for which there can be studied the vibratory partial system of tool holder assembly (2-1), like the one from the figure 1, of which its m_s mass is concentrated in its mass centre C [11].

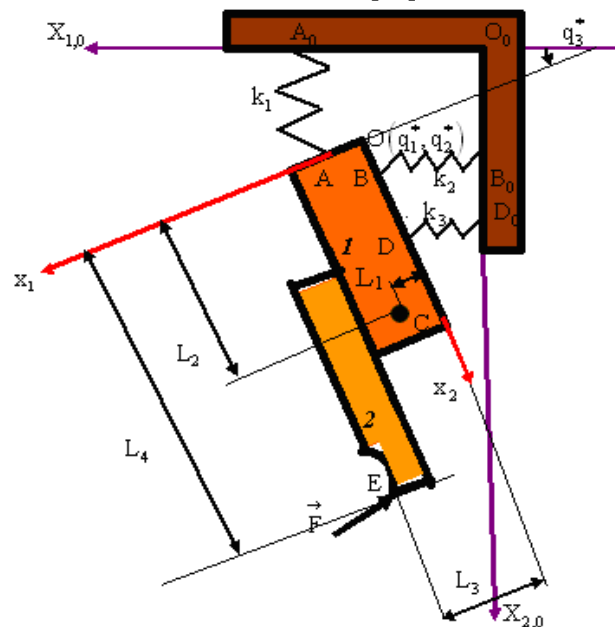


Figure 1. Mechanical model

In order to describe the mathematical model of motion, we will apply the Lagrange formalism by neglecting the resisting forces that are non-elastically, or, by other words, neglecting the damping and also the tool holder gravity force.

We insert the generalized coordinates (1), where $q_{i0}^*, i = \overline{1,3}$ give the stable static equilibrium position of the vibratory system when it is loaded with the stationary chipping force $\vec{F}_0 = -F_{10}\vec{i}_{10} - F_{20}\vec{i}_{20}$, in order to have a vibration around a stable equilibrium.

$$q_i = \Delta q_i^* = q_i^* - q_{i0}^*, i = \overline{1,3}, \quad (1)$$

The relations of moving the coordinates for some important points of the vibratory system, depending on the generalized coordinates, to the inherent coordinate system $T(X_1OX_2X_3)$, with the orthonormal base $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$, to the fixed coordinate system $T_0(X_{1,0}O_0X_{2,0}X_{3,0})$, with the orthonormal base $(\vec{i}_{1,0}, \vec{i}_{2,0}, \vec{i}_{3,0})$, in a matrix writing are given by (2), where by accepting the angular coordinate q_3 as being small, so with $\cos q_3 \cong 1$ and $\sin q_3 \cong q_3$, we obtain (3).

$$\begin{aligned} \begin{Bmatrix} X_{1,A} \\ X_{2,A} \end{Bmatrix} &= \begin{Bmatrix} q_1^* \\ q_2^* \end{Bmatrix} + [M] \begin{Bmatrix} l_1 \\ 0 \end{Bmatrix}; \begin{Bmatrix} X_{1,B} \\ X_{2,B} \end{Bmatrix} = \begin{Bmatrix} q_1^* \\ q_2^* \end{Bmatrix} + [M] \begin{Bmatrix} 0 \\ l_2 \end{Bmatrix}; \\ \begin{Bmatrix} X_{1,D} \\ X_{2,D} \end{Bmatrix} &= \begin{Bmatrix} q_1^* \\ q_2^* \end{Bmatrix} + [M] \begin{Bmatrix} 0 \\ l_3 \end{Bmatrix}; \begin{Bmatrix} X_{1,C} \\ X_{2,C} \end{Bmatrix} = \begin{Bmatrix} q_1^* \\ q_2^* \end{Bmatrix} + [M] \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix}; \\ \begin{Bmatrix} X_{1,E} \\ X_{2,E} \end{Bmatrix} &= \begin{Bmatrix} q_1^* \\ q_2^* \end{Bmatrix} + [M] \begin{Bmatrix} L_3 \\ L_4 \end{Bmatrix}; [M] = \begin{bmatrix} \cos q_3^* & -\sin q_3^* \\ \sin q_3^* & \cos q_3^* \end{bmatrix}, \end{aligned} \quad (2)$$

$$\begin{aligned} \sin q_3^* &= \sin(q_3 + q_{30}^*) = \sin q_3 \cos q_{30}^* + \cos q_3 \sin q_{30}^* \cong q_3 \cos q_{30}^* + \sin q_{30}^*, \\ \cos q_3^* &= \cos(q_3 + q_{30}^*) = \cos q_3 \cos q_{30}^* - \sin q_3 \sin q_{30}^* \cong \cos q_{30}^* - q_3 \sin q_{30}^*, \\ l_1 &= OA; l_2 = OB; l_3 = OC. \end{aligned} \quad (3)$$

The kinetic energy of the tool holder assembly is given by (4).

$$T = \frac{1}{2} m_s \vec{v}_c^2 + \frac{1}{2} J_{33} \omega^2 = \frac{1}{2} m_s \vec{v}_c^2 + \frac{1}{2} J_c \vec{\omega}^2. \quad (4)$$

In addition, the relations (5) and (6) are verified.

$$\dot{q}_i^* = \dot{q}_i + \dot{q}_{i0}^* = \dot{q}_i, i = 1, 2, \frac{d}{dt}(\sin q_{30}^*) = 0, \frac{d}{dt}(\cos q_{30}^*) = 0, \vec{\omega} = \dot{q}_3^* \vec{i}_3 = \dot{q}_3 \vec{i}_3, \quad (5)$$

$$\begin{aligned} \vec{v}_c &= (\dot{i}_{1,0}, \dot{i}_{2,0}) \begin{Bmatrix} X_{1,C} \\ X_{2,C} \end{Bmatrix} = (\dot{i}_{1,0}, \dot{i}_{2,0}) \left(\begin{Bmatrix} q_1^* \\ q_2^* \end{Bmatrix} + \begin{bmatrix} -\dot{q}_3 \sin q_{30}^* & -\dot{q}_3 \cos q_{30}^* \\ \dot{q}_3 \cos q_{30}^* & -\dot{q}_3 \sin q_{30}^* \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} \right) \Rightarrow \\ \vec{v}_c &= (\dot{i}_{1,0}, \dot{i}_{2,0}) \begin{Bmatrix} \dot{q}_1 - \dot{q}_3 (L_1 \sin q_{30}^* + L_2 \cos q_{30}^*) \\ \dot{q}_2 + \dot{q}_3 (L_1 \cos q_{30}^* - L_2 \sin q_{30}^*) \end{Bmatrix}. \end{aligned} \quad (6)$$

By inserting the kinetic energy relation (4) in (5) and (6), there results the relation (4¹) for the kinetic energy of tool holder assembly.

$$\begin{aligned} T &= \frac{1}{2} m_s \left\{ \left[\dot{q}_1 - \dot{q}_3 (L_1 \sin q_{30}^* + L_2 \cos q_{30}^*) \right]^2 + \left[\dot{q}_2 + \dot{q}_3 (L_1 \cos q_{30}^* - L_2 \sin q_{30}^*) \right]^2 \right\} + \\ &+ \frac{1}{2} J_{33} \dot{q}_3^2 \Rightarrow \end{aligned}$$

$$T = \frac{1}{2} m_s \dot{q}_1^2 + \frac{1}{2} m_s \dot{q}_2^2 + \frac{1}{2} [m_s (L_1^2 + L_2^2) + J_c] \dot{q}_3^2 - \quad (4^1)$$

$$- m_s \dot{q}_1 \dot{q}_3 (L_1 \sin q_{30}^* + L_2 \cos q_{30}^*) + m_s \dot{q}_2 \dot{q}_3 (L_1 \cos q_{30}^* - L_2 \sin q_{30}^*)$$

The generalized forces are given by the relations (7).

$$Q_j = Q_{j,c} + Q_{j,e}, \quad j = \overline{1,3}, \quad (7)$$

In (7), $Q_{j,c}$ are the conservative generalized forces and $Q_{j,e}$ are the active generalized forces given by (8) and (9), where the chipping force \vec{F} is given by (10).

$$Q_{j,c} = -\frac{\partial V}{\partial q_j}, \quad j = \overline{1,3}, \quad (8)$$

$$Q_{j,e} = \vec{F} \frac{\partial \vec{r}}{\partial q_j} = \vec{F} \frac{\partial \vec{O_0 E}}{\partial q_j}, \quad j = \overline{1,3}, \quad (9)$$

$$\vec{F} = -F_{10} \sin(\omega t) \dot{i}_{10} - F_{20} \sin(\omega t) \dot{i}_{20}. \quad (10)$$

We mark with Δ the variation of a value between an instantaneous position and the static equilibrium position. By developing in Taylor series limited to the linear terms, we can write (11).

$$\Delta(\sin q_3^*) = \sin q_3^* - \sin q_{30}^* \cong q_3 \cos q_{30}^*, \quad \Delta(\cos q_3^*) \cong -q_3 \sin q_{30}^*. \quad (11)$$

The elastic potential can be calculated with (12).

$$\begin{aligned} V &= \frac{k_1}{2} (\Delta AA_0)^2 + \frac{k_2}{2} (\Delta BB_0)^2 + \frac{k_3}{2} (\Delta DD_0)^2 \Rightarrow \\ V &= \frac{k_1}{2} [q_1^2 + q_2^2 + l_1^2 q_3^2 + 2l_1 q_3 (q_2 \cos q_{30}^* - q_1 \sin q_{30}^*)] + \\ &+ \frac{k_2}{2} [q_1^2 + q_2^2 + l_2^2 q_3^2 - 2l_2 q_3 (q_1 \sin q_{30}^* + q_2 \cos q_{30}^*)] + \\ &+ \frac{k_3}{2} [q_1^2 + q_2^2 + l_3^2 q_3^2 - 2l_3 q_3 (q_1 \sin q_{30}^* + q_2 \cos q_{30}^*)] \end{aligned} \quad (12)$$

By using (8), (9), (10), (11) and (12) in (7) we obtain the generalized forces (7¹), (7²).

$$\begin{aligned} Q_1 &= Q_{1,c} + Q_{1,e} = -\frac{\partial V}{\partial q_1} + \vec{F} \frac{\partial \vec{O_0 E}}{\partial q_1} = -[(k_1 + k_2 + k_3)q_1 - (k_1 l_1 + k_2 l_2 + k_3 l_3)q_3 \sin q_{30}^*] + \\ &+ [-F_{10} \sin(\omega t) \dot{i}_{10} - F_{20} \sin(\omega t) \dot{i}_{20}] \cdot \frac{\partial}{\partial q_1} \{ [q_1 + q_{10}^* + L_3 (\cos q_{30}^* - q_3 \sin q_{30}^*) - \\ &- L_4 (\sin q_{30}^* + q_3 \cos q_{30}^*)] \dot{i}_{10} + [q_2 + q_{20}^* + L_3 (\sin q_{30}^* + q_3 \cos q_{30}^*) + \\ &+ L_4 (\cos q_{30}^* - q_3 \sin q_{30}^*)] \dot{i}_{20} \} \Rightarrow \\ Q_1 &= -[(k_1 + k_2 + k_3)q_1 - (k_1 l_1 + k_2 l_2 + k_3 l_3)q_3 \sin q_{30}^*] - F_{10} \sin(\omega t), \end{aligned} \quad (7^1)$$

$$\begin{aligned} Q_2 &= Q_{2,c} + Q_{2,e} = -\frac{\partial V}{\partial q_2} + \vec{F} \frac{\partial \vec{O_0 E}}{\partial q_2} = -[(k_1 + k_2 + k_3)q_2 - (k_1 l_1 + k_2 l_2 + k_3 l_3)q_3 \sin q_{30}^*] + \\ &+ [-F_{10} \sin(\omega t) \dot{i}_{10} - F_{20} \sin(\omega t) \dot{i}_{20}] \cdot \frac{\partial}{\partial q_2} \{ [q_1 + q_{10}^* + L_3 (\cos q_{30}^* - q_3 \sin q_{30}^*) - \\ &- L_4 (\sin q_{30}^* + q_3 \cos q_{30}^*)] \dot{i}_{10} + [q_2 + q_{20}^* + L_3 (\sin q_{30}^* + q_3 \cos q_{30}^*) + \\ &+ L_4 (\cos q_{30}^* - q_3 \sin q_{30}^*)] \dot{i}_{20} \} \Rightarrow \\ Q_2 &= -[(k_1 + k_2 + k_3)q_2 + (k_1 l_1 - k_2 l_2 - k_3 l_3)q_3 \sin q_{30}^*] - F_{20} \sin(\omega t), \end{aligned} \quad (7^2)$$

$$\begin{aligned}
Q_3 &= Q_{3,c} + Q_{3,e} = -\frac{\partial V}{\partial q_3} + \vec{F} \frac{\partial \vec{O_0 E}}{\partial q_3} = \\
&= -\left[(k_1 l_1^2 + k_2 l_2^2 + k_3 l_3^2) q_3 + (k_1 l_1 - k_2 l_2 - k_3 l_3) q_2 \sin q_{30}^* - (k_1 l_1 + k_2 l_2 + k_3 l_3) q_1 \sin q_{30}^* \right] + \\
&+ \left[-F_{10} \sin(\omega t) \dot{i}_{10} - F_{20} \sin(\omega t) \dot{i}_{20} \right] \cdot \frac{\partial}{\partial q_3} \left\{ [q_1 + q_{10}^* + L_3 (\cos q_{30}^* - q_3 \sin q_{30}^*) - \right. \\
&- L_4 (\sin q_{30}^* + q_3 \cos q_{30}^*)] \dot{i}_{10} + [q_2 + q_{20}^* + L_3 (\sin q_{30}^* + q_3 \cos q_{30}^*) + \\
&+ L_4 (\cos q_{30}^* - q_3 \sin q_{30}^*)] \dot{i}_{20} \Big\} \Rightarrow \\
Q_3 &= -\left[(k_1 l_1^2 + k_2 l_2^2 + k_3 l_3^2) q_3 + (k_1 l_1 - k_2 l_2 - k_3 l_3) q_2 \cos q_{30}^* - \right. \\
&- (k_1 l_1 + k_2 l_2 + k_3 l_3) q_1 \sin q_{30}^* \Big] + F_{10} (L_3 \sin q_{30}^* + L_4 \cos q_{30}^*) \sin(\omega t) - \\
&- F_{20} (L_3 \cos q_{30}^* - L_4 \sin q_{30}^*) \sin(\omega t). \tag{7^3}
\end{aligned}$$

Because the links are totally homonymic, the Lagrange equations are given by (13).

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = \overline{1,3}. \tag{13}$$

By replacing in the equation system (13) the kinetic energy given by (4¹) and the generalized forces given by (7¹), (7²) and (7³), there results the linear differential equations system of second degree order, which is the motion mathematical model (which was similarly obtained in [11]), under the matrix form (13¹).

$$\begin{aligned}
[A] \ddot{q} + [K] \{q\} &= \{Q_e\}, \tag{13^1} \\
[A] &= \begin{bmatrix} a_{11} & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}; [K] = \begin{bmatrix} k_{11} & 0 & k_{13} \\ 0 & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}; \{Q_e\} = \{Q_{1,e}, Q_{2,e}, Q_{3,e}\}^T; \\
\ddot{q} &= \{\ddot{q}_1, \ddot{q}_2, \ddot{q}_3\}^T; \{q\} = \{q_1, q_2, q_3\}^T; Q_{1,e} = -F_{10} \sin(\omega t); Q_{2,e} = -F_{20} \sin(\omega t); \\
Q_{3,e} &= [F_{10} (L_3 \sin q_{30}^* + L_4 \cos q_{30}^*) - F_{20} (L_3 \cos q_{30}^* - L_4 \sin q_{30}^*)] \sin(\omega t); \\
a_{11} &= a_{22} = m_s; a_{33} = m_s (L_1^2 + L_2^2) + J_c; \\
a_{13} &= -m_s (L_1 \sin q_{30}^* + L_2 \cos q_{30}^*); a_{23} = m_s (L_1 \cos q_{30}^* - L_2 \sin q_{30}^*); \\
k_{11} &= k_{22} = k_1 + k_2 + k_3; k_{33} = k_1 l_1^2 + k_2 l_2^2 + k_3 l_3^2; \\
k_{13} &= -(k_1 l_1 + k_2 l_2 + k_3 l_3) \sin q_{30}^*; k_{23} = (k_1 l_1 - k_2 l_2 - k_3 l_3) \cos q_{30}^*.
\end{aligned}$$

By applying the (13¹) system Laplace transform in relation with time and by accepting initial homogenous conditions, in a matrix writing, there results the algebraic system (13²).

$$\begin{aligned}
([K] + s^2 [A]) \tilde{q}(s) &= \{\tilde{Q}_e(s)\}, \quad \{\tilde{q}(s)\} = \{\tilde{q}_1(s), \tilde{q}_2(s), \tilde{q}_3(s)\}^T; \{\tilde{Q}_e(s)\} = \{\tilde{Q}_{1,e}(s), \tilde{Q}_{2,e}(s), \tilde{Q}_{3,e}(s)\}^T; \tag{13^2} \\
\tilde{Q}_{1,e}(s) &= -F_{10} \frac{\omega}{s^2 + \omega^2}, \quad \tilde{Q}_{2,e}(s) = -F_{20} \frac{\omega}{s^2 + \omega^2}, \\
\tilde{Q}_{3,e}(s) &= [F_{10} (L_3 \sin q_{30}^* + L_4 \cos q_{30}^*) - F_{20} (L_3 \cos q_{30}^* - L_4 \sin q_{30}^*)] \frac{\omega}{s^2 + \omega^2}.
\end{aligned}$$

The solution of (13²) system is (14).

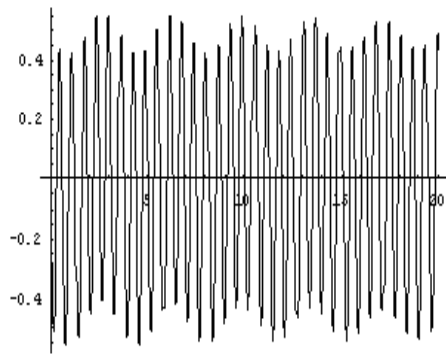
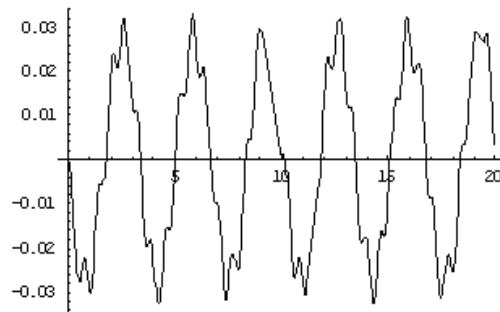
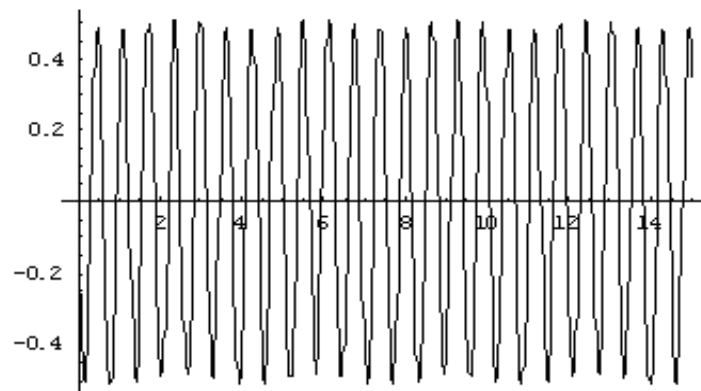
$$\begin{aligned}
\tilde{q}_1(s) &= \frac{P_1(s)}{P(s)}, \tilde{q}_2(s) = \frac{P_2(s)}{P(s)}, \tilde{q}_3(s) = \frac{P_3(s)}{P(s)}, \\
P(s) &= (s^2 + \omega^2)(a_{11}s^2 + k_{11}) \cdot \\
&\cdot [(-a_{13}^2 - a_{23}^2 + a_{11}a_{33})s^4 + (a_{33}k_{11} - 2a_{13}k_{13} - 2a_{23}k_{23} + a_{11}k_{33})s^2 - k_{13}^2 - k_{23}^2 + k_{11}k_{33}] \\
Q_1 &= -F_{10}, Q_2 = -F_{20}, Q_3 = [F_{10}(L_3 \sin q_{30}^* + L_4 \cos q_{30}^*) - F_{20}(L_3 \cos q_{30}^* - L_4 \sin q_{30}^*)] \\
P_1(s) &= -\omega(a_{23}^2 Q_1 - a_{11}a_{33}Q_1 - a_{13}a_{23}Q_2 + a_{11}a_{13}Q_3)s^4 - \\
&- \omega(-a_{33}k_{11}Q_1 + a_{13}k_{11}Q_3 - a_{23}k_{13}Q_2 + a_{11}k_{13}Q_3 + 2a_{23}k_{23}Q_1 - a_{13}k_{23}Q_2 - a_{11}k_{33}Q_1)s^2 - \\
&- \omega(k_{11}k_{13}Q_3 - k_{13}k_{23}Q_2 + k_{23}^2Q_1 - k_{11}k_{33}Q_1), \\
P_2(s) &= \omega(a_{13}a_{23}Q_1 - a_{13}^2Q_2 + a_{11}a_{23}Q_3)s^4 + \\
&+ \omega(a_{33}k_{11}Q_2 - a_{23}k_{11}Q_3 + a_{23}k_{13}Q_1 - 2a_{13}k_{13}Q_2 + a_{13}k_{23}Q_1 - a_{11}k_{23}Q_3 + a_{11}k_{33}Q_2)s^2 - \\
&- \omega(-k_{11}k_{23}Q_3 + k_{13}^2Q_2 + k_{13}k_{23}Q_1 + k_{11}k_{33}Q_2), \\
P_3(s) &= (a_{11}s^2 + k_{11})[\omega(a_{13}Q_1 + a_{23}Q_2 - a_{11}Q_3)s^2 - \omega(k_{11}Q_3 - k_{13}Q_1 - k_{23}Q_2)]
\end{aligned} \tag{14}$$

3. Dynamic response

By applying the Laplace transform inverse in relations (14), there results the vibratory system dynamic response with the form of time – functions $q_i = q_i(t)$, $i = 1, 3$, as written in (15).

$$\begin{aligned}
q_1(t) &= \frac{\sqrt{2}}{4a_1^2 \sqrt{\frac{c_1}{a_1}(b_1^2 - 4a_1c_1)}} \left(b_1^2 d_1 - 2a_1c_1d_1 - b_1d_1\sqrt{b_1^2 - 4a_1c_1} + a_1b_1d_2 - a_1\sqrt{b_1^2 - 4a_1c_1}d_2 - 2a_1^2d_3 \right) \cdot \\
&\cdot \left[\sqrt{\frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{a_1}} \sinh \left(\sqrt{\frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1}} t \right) + \sqrt{\frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{a_1}} \sinh \left(\sqrt{\frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1}} t \right) \right], \\
q_2(t) &= \frac{\sqrt{2}}{4a_1^2 \sqrt{\frac{c_1}{a_1}(b_1^2 - 4a_1c_1)}} \left(b_1^2 D_1 - 2a_1c_1D_1 - b_1D_1\sqrt{b_1^2 - 4a_1c_1} + a_1b_1D_2 - a_1\sqrt{b_1^2 - 4a_1c_1}D_2 - 2a_1^2D_3 \right) \cdot \\
&\cdot \left[\sqrt{\frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{a_1}} \sinh \left(\sqrt{\frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1}} t \right) + \sqrt{\frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{a_1}} \sinh \left(\sqrt{\frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1}} t \right) \right], \\
a_1 &= -a_{13}^2 - a_{23}^2 + a_{11}a_{33}; b_1 = a_{33}k_{11} - 2a_{13}k_{13} - 2a_{23}k_{23} + a_{11}k_{33}; c_1 = -k_{13}^2 - k_{23}^2 + k_{11}k_{33}; \\
A_1 &= \omega a_{13}F_1 + \omega a_{23}F_2 - \omega a_{11}F_3; A_2 = -\omega F_3k_{13} + \omega F_1k_{13} + \omega F_2k_{23}; \\
d_1 &= -\omega(a_{23}^2Q_1 - a_{11}a_{33}Q_1 - a_{11}a_{23}Q_2 + a_{11}a_{33}Q_3); \\
d_2 &= \omega(-a_{33}k_{11}Q_1 + a_{13}k_{11}Q_3 - a_{23}k_{13}Q_2 + a_{11}k_{13}Q_3 + 2a_{23}k_{23}Q_1 - a_{13}k_{23}Q_2 - a_{11}k_{33}Q_1); \\
d_3 &= \omega(k_{11}k_{23}Q_3 - k_{13}k_{23}Q_2 + k_{23}^2Q_1 - k_{11}k_{33}Q_1); D_1 = \omega(a_{11}a_{33}Q_1 - a_{13}^2Q_2 + a_{11}a_{33}Q_2 - a_{11}a_{23}Q_3);
\end{aligned}$$

$$\begin{aligned}
q_3(t) = & \frac{1}{4\omega c_1(\omega^4 a_1 - \omega^2 b_1 + c_1)\sqrt{b_1^2 - 4a_1 c_1}} \left\{ -\omega A_1 c_1 \left[b_1 \sqrt{2}\Omega_1 \sin(\Omega_1 t) - \sqrt{2}\Omega_2 \sin(\Omega_2 t) \right] - \right. \\
& - 2\sqrt{2}\omega^2 a_1 \left[-\sqrt{2}\Omega_1 \sin(\Omega_1 t) + \sqrt{2}\Omega_2 \sin(\Omega_2 t) \right] - \\
& - \sqrt{b_1^2 - 4a_1 c_1} \left[4\omega \sin(\omega t) + 2\Omega_1 \sin(\Omega_1 t) + 2\Omega_2 \sin(\Omega_2 t) \right] + \\
& + 4A_2 c_1 \sin(\omega t) \sqrt{b_1^2 - 4a_1 c_1} + \sqrt{2}\omega A_2 a_1 \left[-\sqrt{2}\Omega_1 \sin(\Omega_1 t) + \sqrt{2}\Omega_2 \sin(\Omega_2 t) \right] - \\
& - b_1 \sqrt{2}\omega(\omega^2 a_1 - b_1) \left[\sqrt{2}\Omega_1 \sin(\Omega_1 t) - \sqrt{2}\Omega_2 \sin(\Omega_2 t) \right] + \\
& \left. + \sqrt{2}\omega(\omega^2 a_1 - b_1) \sqrt{b_1^2 - 4a_1 c_1} \left[\sqrt{2}\Omega_1 \sin(\Omega_1 t) + \sqrt{2}\Omega_2 \sin(\Omega_2 t) \right] \right\} \\
\Omega_1 = & \sqrt{\frac{\sqrt{b_1^2 - 4a_1 c_1} - b_1}{2a_1}}; \Omega_2 = \sqrt{\frac{b_1 + \sqrt{b_1^2 - 4a_1 c_1}}{2a_1}}; D_3 = \omega(-k_{11}k_{23}Q_3 + k_{13}^2 Q_2 + k_{13}k_{23}Q_1 + k_{11}k_{33}Q_2) \\
D_2 = & \omega(-k_{11}a_{33}Q_2 + a_{23}k_{11}Q_3 - a_{23}k_{13}Q_1 + 2a_{13}k_{13}Q_2 - a_{13}k_{23}Q_1 + a_{11}k_{23}Q_3 - a_{11}k_{33}Q_2);
\end{aligned} \tag{15}$$

Figure 2. $q_1 = q_1(t)$.Figure 3. $q_2 = q_2(t)$ Figure 4. $q_3 = q_3(t)$

In the case of a numerical application, in which are given the next initial data:

$$\begin{aligned}
m_s = 0.056 \left[\frac{N \cdot s^2}{cm} \right], \quad l_1 = 20 [mm], \quad L_4 = 250 [mm], \quad l_2 = 30 [mm], \quad J_c = 2.04 [N \cdot cm \cdot s^2], \\
L_1 = 37 [mm], \quad L_2 = 168 [mm], \quad L_3 = 70 [mm], \quad k_1 = 28 \cdot 10^4 \left[\frac{N}{cm} \right], \quad l_3 = 50 [mm], \quad k_3 = 11 \cdot 10^6 \left[\frac{N}{cm} \right],
\end{aligned}$$

$k_2 = 28 \cdot 10^4 \left[\frac{N}{cm} \right]$, we obtain the graphics of time – functions $q_i = q_i(t)$, $i = \overline{1,3}$, from the 2, 3 and 4 figures.

4. Conclusions

The dynamic behaviour of mechanical systems is often affected by the non-linear effects, which, in many cases, are situated at the interfaces levels of coupling between different parts (dynamic excitations generated by the contact between the movements transmitting parts and the efforts, the instability related to the friction in mechanical systems) which may lead to unwanted effects (noises) or can harm the mechanism functioning (excessive wear).

In our days, the research activities are focused on the experimental parts development, theoretically and numerically multidisciplinary inserted when the structures, machine parts or, generally the mechanical systems, are designed. The research purpose is to improve the knowledge regarding the materials and structures behaviour, to develop models and instruments useful in the design process of machines and structures and to exploit a technical culture regarding the analysis, conception and fabrication analysis.

In this context, from the design stage, there must known the vibrations influence in the products manufacturing processes, and, in definitive, the machine dynamic behaviour, fact that is high lightened by the above study.

References

- [1] Abousleiman V and Velez P 2006 *A hybrid 3d finite element/lumped parameter model for quasi-static and dynamic analyses of planetary/epicyclic gear sets* (Mechanism and Machine Theory 41(6)), pp 725–748
- [2] Băgnaru D Gh Stănescu M M Cuță P 2008 *Vibrations of the tool and tool holder unit for a shaper device* (8th International Conference "Research and Development in Mechanical Industry" RaDMI 2008, 14-17 September 2008, Užice, Serbia) pp 202-205
- [3] Băgnaru D Gh 1997 *Sur l'analyse des vibrations des barres viscoelastique, les elements constitutifs des mecanismes plans* (SYROM 97 București) pp 25-30.
- [4] Băgnaru D Gh Hadăr A Bolcu D Stănescu M M Cuță P 2009 *The vibrations influence on the field of accelerations of the linear-elastic connecting rod of a mechanism connecting rod lug* (Annals of DAAAM for 2009 & Proceedings 20(1)) pp 833-835.
- [5] Băgnaru D Gh Stănescu M M Bolcu D Cuță P 2008 *Approximate method in order to determine the displacement fields for the kinematic elements standing in vibration* (Second international congres automotive, safety and environment, SMAT, Craiova, 23-26 October) pp 9-12.
- [6] Bard C 1995 *Modelisation du comportement dynamique des transmissions par engrenages* (L'institut National des Sciences Appliquees de Lyon)
- [7] Craig R R Bampton M C C 1968 *Coupling of substructures for dynamics analysis* (AIAA Journal, 6(7)) pp 1313–1319
- [8] Gafițanu M Focșa V Merticaru V Biborosch L 1980 *Vibrations and noises* (Junimea Publishing, Iași).
- [9] Kahraman A 1994 *Planetary gear train dynamics* (Journal of mechanical design, 116(3)) pp 713–720
- [10] Lin J Parker RG 1999 *Analytical characterization of the unique properties of planetary gear free vibration* (*Journal of vibration and acoustics*, 121(3)) pp 316–321.
- [11] Băgnaru D Gh Stănescu M M 2017 *Aplicații ale transformatei Laplace în abordare teoriilor moderne în domeniul vibrațiilor unor sisteme mecanice* (Universitaria Publishing Craiova ISBN 978-606-14-1230-3)