

Numerical investigation on pulsating heat pipes with nitrogen or hydrogen

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Abstract. With flexible structure and excellent performance, pulsating heat pipes (PHP) are regarded as a great solution to distribute cooling power for cryocoolers. The experiments on PHPs with cryogenic fluids have been carried out, indicating their efficient performances in cryogenics. There are large differences in physical properties between the fluids at room and cryogenic temperature, resulting in their different heat transfer and oscillation characteristics. Up to now, the numerical investigations on cryogenic fluids have rarely been carried out. In this paper, the model of the closed-loop PHP with multiple liquid slugs and vapor plugs is performed with nitrogen and hydrogen as working fluids, respectively. The effects of heating wall temperature on the performance of close-looped PHPs are investigated and compared with that of water PHP.

1. Introduction

Cryocoolers have been widely used due to the growing requirement for low temperature, but they can only provide cooling power at the cold heads. In distributed cooling and long-distance cooling systems, a kind of flexible, high-efficiency heat transfer device connecting the cold heads and cooling objects is needed. Pulsating heat pipe (PHP) is regarded as a great option. When PHPs operate at 70-120 K or 18-30 K, nitrogen or hydrogen will be the working fluids.

When PHP is partially filled, a train of liquid slugs and vapor plugs will appear inside. They will be in the reciprocating oscillation, due to temperature difference between heating section and cooling section, which transfers the heat. Experiments have shown great performance of cryogenic PHP [1-5]. However, the modelling of cryogenic PHP is rarely seen and the understanding of its operating mechanism is limited. It is necessary to establish a model on cryogenic PHP to clarify the mechanism, which then can be used to help design or optimize the PHP.

One of the commonly used methods for numerical simulation of PHP is described below: assuming that the flow pattern in PHP is slug flow, and then the mass conservation equation, momentum



conservation equation and energy conservation equation are carried out and solved numerically. At last the movement and heat transfer characteristics of PHP can be obtained. For PHP filled with water, Shafii et al. [6] established the model of liquid-vapor slug flow. They found periodic movement of liquid slugs and periodic oscillation of vapor pressure and temperature. Sakulchangsattajai et al. [7], Senjaya and Inoue [8, 9], Mameli et al. [10, 11] and Manzoni et al. [12] improved the model by adding liquid film, nucleate conditions, and changing of tube wall temperature step by step. Their results with working fluids at room temperature agree well with experiments, which indicates the method is effective.

In this paper, a model of PHP is built with the method mentioned above. The performances of the PHP with hydrogen or nitrogen as working fluid are discussed and compared with those with water. The results show the similarities and differences between water PHP and cryogenic PHP, which will help researchers understand the mechanism.

2. Physical model

The structure of PHP in the model is shown in Fig.1. If the bend loss is ignored, PHP can be regarded as a straight tube where evaporating section and condensing section appear alternately. To solve the problem, the following assumptions are made:

- Flow pattern is slug flow;
- The liquid is incompressible and vapor behaves as ideal gas;
- The shear stress between vapor plug and liquid film are ignored;
- Between vapor plug and liquid film, only heat transfer due to phase change is considered;
- The influence of liquid film on momentum variation of liquid slugs is neglected.

The whole tube is divided uniformly into N control volumes, and each of them has the length of dx . The control volume can be classified into four types, as shown in Fig.2: (a) liquid control volume, (b) vapor control volume, (c) liquid-vapor control volume and (d) vapor-liquid control volume. Because the positions of liquid slugs and vapor plugs will change with time, the type of a certain control volume will also change with time. For 4 types of control volumes, mass conservation equation, momentum conservation equation and energy conservation equation are solved.

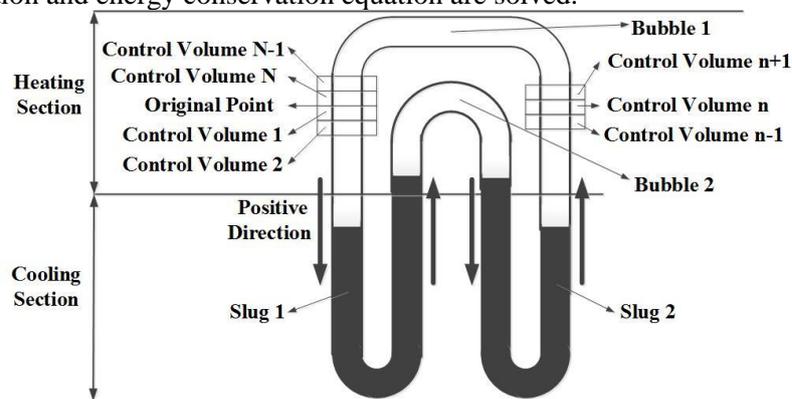


Figure 1. Structure of PHP

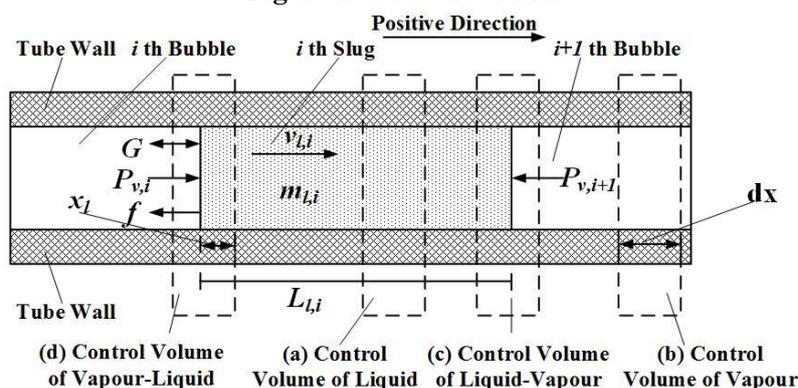


Figure 2. Control Volumes in PHP

2.1. Governing equations

The i^{th} bubble locates between i^{th} and $(i-1)^{\text{th}}$ liquid slugs. Assuming that the mass change of each bubble contributes equally to adjacent liquid slugs, the mass equation of i^{th} bubble follows

$$dm_{v,i} = -2dm_{l,\text{left},i} = -2dm_{l,\text{right},(i-1)} \quad (1)$$

In Equation (1), $dm_{l,\text{left},i}$ and $dm_{l,\text{right},(i-1)}$ refer to the mass change which happens on the left side of i^{th} liquid slug and on the right side of $(i-1)^{\text{th}}$ liquid slug, respectively.

Since the liquid is incompressible, it is easy to know that in the same liquid slug, the velocity at different place is identical at the same time. Velocity can be obtained from the momentum equation

$$\frac{dm_{l,i}v_{l,i}}{dt} = (P_{v,i} - P_{v,(i+1)})A_v - \pi dL_{l,i}\tau + \cos\theta_g m_{l,i}g \quad (2)$$

In Equation (2), θ_g is the angle between gravity direction and positive direction. τ refers to the shear stress between a liquid slug and tube wall, which can be obtained from Equation (3) and Equation (4)[13].

$$\tau = \frac{1}{2}C_1\rho_1v_{l,i}^2 \quad (3)$$

$$C_1 = \begin{cases} \frac{16}{\text{Re}} & \text{Re} \leq 1180 \\ 0.078\text{Re}^{-0.25} & \text{Re} > 1180 \end{cases} \quad (4)$$

Different from velocity, the temperature of liquid slugs depends on time and position. For liquid slugs, there are 3 types of control volumes: (a) (c) and (d). For each control volume, energy equations are needed. In type (a) control volume, liquid occupies all the space, thus the energy equation follows

$$\frac{\partial T_1}{\partial t} \rho_1 c_{pl} A_1 dx = -\frac{\partial T_1}{\partial x} v_{l,i} \rho_1 c_{pl} A_1 dx + \frac{\partial^2 T_1}{\partial x^2} \lambda_1 A_1 dx + h_1 \pi d (T_w - T_1) dx \quad (5)$$

For type (c) control volume, the length of liquid and boundary situation are different from type (a) control volume, so the energy equation is changed into

$$\frac{\partial T_1 x_1}{\partial t} \rho_1 c_{pl} A_1 = \rho_1 c_{pl} A_1 v_{l,i} (T_1 - \frac{\partial T_1 x_1}{\partial x} \frac{x_1}{2}) - \frac{\partial T_1}{\partial x} \lambda_1 A_1 + \frac{dm_{\text{right},i}}{dt} c_{pl} T_1 + h_1 \pi d (T_w - T_1) x_1 \quad (6)$$

In Equation (6), x_1 and x_v refer to the length of liquid part and vapor part in a certain control volume. They will change with time when the end of liquid slug moves. The energy equation of type (d) control volume is shown in Equation (7), which is similar to Equation (6). $dm_{\text{right},i}$ and $dm_{\text{left},i}$ refers to the mass change on the right and left side of the i^{th} slug respectively, while T_w is the temperature of tube wall.

$$\frac{\partial T_1 x_1}{\partial t} \rho_1 c_{pl} A_1 = -\rho_1 c_{pl} A_1 v_{l,i} (T_1 + \frac{\partial T_1 x_1}{\partial x} \frac{x_1}{2}) + \frac{\partial T_1}{\partial x} \lambda_1 A_1 + \frac{dm_{\text{left},i}}{dt} c_{pl} T_1 + h_1 \pi d (T_w - T_1) x_1 \quad (7)$$

In Equation (5)-(7), h_1 is the heat transfer coefficient between liquid slugs and tube wall, which can be calculated as Equation (8) [6].

$$Nu = \begin{cases} 4.364 & \text{Re} \leq 2200 \\ 0.012(\text{Re}^{0.87} - 280) \text{Pr}^{0.4} \left(\frac{\text{Pr}_m}{\text{Pr}_w}\right)^{0.11} & 2200 < \text{Re} < 10000 \\ 0.0263\text{Re}^{0.8} \text{Pr}^{0.43} \left(\frac{\text{Pr}_m}{\text{Pr}_w}\right)^{0.25} & \text{Re} \geq 10000 \end{cases} \quad (8)$$

The heat transfer coefficient between vapor and tube wall is very small compared to evaporating and condensing heat transfer coefficient, thus it is neglected. Assuming that the mass transfer only occurs

between liquid film and vapor, and the liquid film is supplied by adjacent liquid slugs, the mass change in a certain vapor plug will follow Equation (9). h_v is an adjustable parameter, which determines the mass transfer. The value of h_v is shown later. It should be pointed out that when both ends of the same bubble leave the evaporating section, the evaporation in this bubble will stop. It will restart when one end of this bubble moves into evaporating section again.

$$\frac{dm_{v,i}}{dt} = \sum_{\text{backposition},v,i}^{\text{frontposition},v,i} h_v \pi d (T_w - T_{v,i}) x_v / h_{fg} \quad (9)$$

The energy equation of i^{th} bubble is Equation (10), in which the pressure is determined by Equation (11).

$$c_{vv} \frac{d(m_{v,i} T_{v,i})}{dt} = \frac{dm_{v,i}}{dt} c_{pv} T_{v,i} - P_{v,i} \frac{dV_{v,i}}{dt} \quad (10)$$

$$P_{v,i} V_{v,i} = m_{v,i} R T_{v,i} \quad (11)$$

The total heat transfer of PHP can be separated into two categories: sensible heat and latent heat. In slug flow, when the bubble-wall convection is ignored, sensible heat will only result from convection between liquid slugs and tube wall, as shown in Equation (12). Latent heat is due to phase change, which should be calculated as Equation (13). The total heat is the sum of them.

$$Q_{\text{sensible}} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \sum_{\text{liquid}} h_1 \pi d (T_w - T_1) x_1 dt \quad (12)$$

$$Q_{\text{latent}} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \sum_{\text{vapour}} h_v \pi d (T_w - T_{v,i}) x_v dt \quad (13)$$

2.2. Numerical Procedure

In order to solve governing equations above, a self-written MATLAB code has been programmed. The code can run on MATLAB R2012b. The computation order for equations in every loop is followed as: Equation (2), Equation (5)-(7), Equation (11) and Equation (10).

In the process of calculation, the properties of liquid (specific heat, thermal conductivity and Prandtl number) and properties of vapor (specific heat and vaporization enthalpy) are treated as single-valued functions of temperature. The length of every control volume is set as 1 mm. The time step is 5×10^{-7} s. When the time step is decreased to 1×10^{-7} s, the difference on heat transfer is less than 2%. The equations are solved explicitly, which means using the value at time step n to calculate the value at time step $(n+1)$. Every 0.002 s, data is printed out to record the movement and heat transfer.

3. Results and discussion

3.1. Validation of the model

The results of the present model have been compared with those of Shafii et al. [6]. The working fluid is water. The dimensions of PHP and initial values in calculation are listed in Tab.1. The condensing temperature (T_c) and evaporating temperature (T_e) are set as 20°C (293.15 K) and 120°C (393.15 K) respectively, which is called Case 1. When movement of liquid slugs turns to be steady, the change trends of the position of liquid plugs are the same and the oscillation periods are both 0.11 s approximately.

Table 1. Initial values for PHP with water

Item	Value	Item	Value	Item	Value
Fluid	Water	Heating Section Length	0.1 m	Condensing Heat Transfer Coefficient	200 W/(m ² K)
Turns	2	Total Length of PHP	1.14 m	Initial Filling Ratio	64.2%
Cooling Section Temperature	20°C	Inner Diameter of PHP	1.5 mm	Initial Liquid Temperature	20°C

Cooling Section Length	0.37 m	Evaporating Heat Transfer Coefficient	300 W/(m ² K)	Initial Vapor Temperature	80°C
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The model is also used to calculate the following two cases. Except for the values listed below, other initial values remain the same as table.1. These two cases are:

Case 2: inner diameter is 1.5 mm; T_c is 20°C; T_e is 90°C;

Case 3: inner diameter is 3 mm; T_c is 20°C; T_e is 120°C.

The total heat transfer of Case 1, Case 2 and Case 3 is compared with that of Shafii et al.. The results are shown in Tab.2, which shows that our calculations are in good agreement with theirs. It means that this model is reliable to reflect the behavior of long liquid slugs in PHP. The properties of nitrogen and hydrogen will be applied to the model. Then behavior of long liquid slugs in these PHPs can be obtained and discussed.

Table 2. Comparison of total heat transfer

Case	Inner Diameter(mm)	Heating Section Temperature(°C)	Total Heat Transfer from Our Model(W)	Total Heat Transfer of Shafii et al.'s Model(W)	Difference
1	1.5	120	24.81	24.95	0.56%
2	1.5	90	7.66	8.36	8.37%
3	3	120	75.96	79.37	4.30%

3.2. PHP with Hydrogen

PHP with hydrogen usually operate between 20 K and 30 K. The dimensions of PHP are the same as in Tab.1, except for inner diameter of tubes. The inner diameter is determined by Bond number, which is shown in Equation (14). In PHP, the Bond number should be small enough to make sure the surface tension forces is strong enough, so that liquid slugs can form inside the tube. Usually, Bond number should be less than 2. For the PHP with water at room temperature, the diameter of 1.5 mm is reasonable, which corresponds to Bond number as 0.554(the properties of water is set as the values at 293 K). For hydrogen at 20 K, the diameter of 1.5 mm matches the Bond number of 0.884.

$$Bond = d \sqrt{\frac{g(\rho_l - \rho_v)}{\sigma}} \quad (14)$$

Initial values are set the same as that of water PHP except that the initial temperature and condensing temperature are set as 20 K. Initially, there are two long liquid slugs in PHP, whose length is determined by initial filling ratio. 20% pressure difference is set, which is the initial power of movement. When T_e is 27 K, the oscillation of liquid slugs becomes steady after 2 s, which is similar to that of water PHP. The oscillation is shown in Fig.3. The movement of long slugs is similar to a sinusoidal curve (actually it is not a sinusoidal curve), so we can define the amplitude and period of the oscillation. Both of the two slugs have nearly the same amplitude of 32 mm, and the period is about 0.0133 s.

To understand the oscillation clearly, the oscillation of two long liquid slugs between 2.70 s and 2.76 s is shown in Fig.3(a), and the pressure is shown in Fig.3(b). Taking the oscillation between 2.701 s and 2.713 s for example, the oscillation process can be described as below.

At about 2.701 s, the left end of the first slug and the right end of the second slug move into heating section almost at the same time. Evaporation happens in bubble 1 and increases its pressure (P_1). Meanwhile condensation decreases the pressure in bubble 2(P_2). The velocity of slug 1(v_1) remains negative and the velocity of slug 2(v_2) remains positive, which means bubble 1 is compressed and bubble 2 is expanded. This process also contributes to rising P_1 and reducing P_2 . Gradually, v_1 slows down to 0 and then reverses the direction at 2.704 s. Before v_2 becomes opposite, P_1 reaches its maximum and P_2 reaches its minimum. Then, slug 1 moves in positive direction and slug 2 moves in negative direction. Although the evaporation in bubble 1 and the condensation in bubble 2 are still taking place, the movement of slugs tends to expand bubble 1 and compresses bubble 2. Resulting from these two factors simultaneously, P_1 and P_2 behave like that in 2.704-2.706 s. From 2.706 s to 2.709 s, the behavior of bubble 2 is similar to that of bubble 1 between 2.701 s and 2.704 s. This means two bubbles exchange

their function and state. At 2.709 s, P_2 reaches its maximum and P_1 reaches its minimum. Nearly at the same time, liquid slugs start reverse motion. Until 2.714 s, the status of system turns back to that at 2.701 s. This is a complete cycle of oscillation. The pressure difference between bubbles is the motivation of oscillation, and it will trace back to the evaporation and condensation phenomenon, even motion itself.

If the temperature of condensing section remains 20 K, the effect of T_e on total heat transfer and sensible heat ratio is shown in Fig.4. Sensible heat ratio is the ratio of sensible heat to total heat transfer. When T_e rises, total heat transfer in hydrogen PHP will increase. This conclusion agrees with the experiment results [2, 3]. The sensible heat ratio will increase when T_e rises, and the sensible heat ratio is always bigger than 90%. In the calculations of Shafii et al., they announced that sensible heat ratio will be 90% if the flow pattern is slug flow [6]. The sensible heat is dominant for hydrogen in slug flow, which is similar to the result of water.

The effect of T_e on amplitude and period of oscillation is shown in Fig.5. Increasing T_e will increase the amplitude, and decrease the period. The increased amplitude means the time-averaged heat transfer area is enlarged. The decreased period means increasing the frequency and time-averaged velocity of the motion. It improves heat transfer by rising heat transfer coefficient. Both of the two reasons lead to the enhancement of heat transfer when T_e increases.

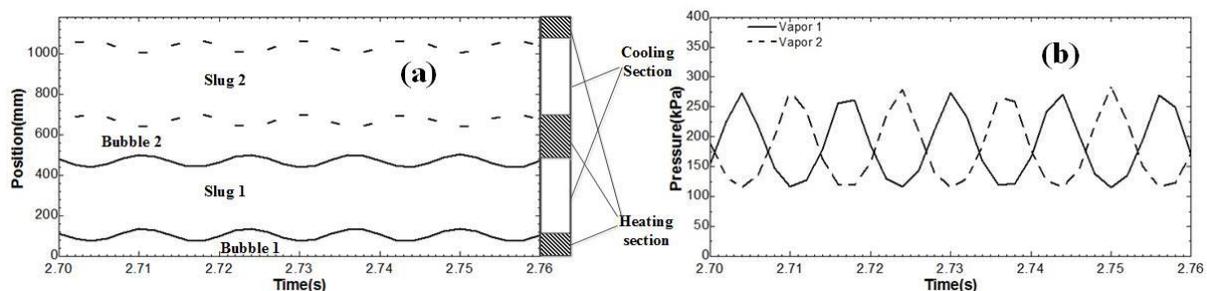


Figure 3. The variation of (a) position of slugs and (b) pressure of bubbles (hydrogen, $T_c=20$ K)

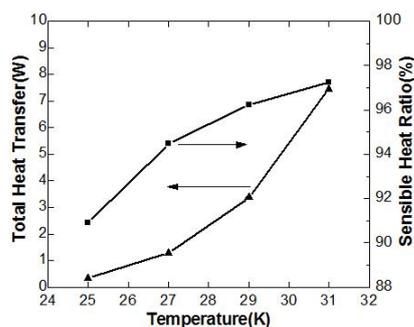


Figure 4. Effect of T_e on total heat transfer and sensible heat ratio (hydrogen, $T_c=20$ K)

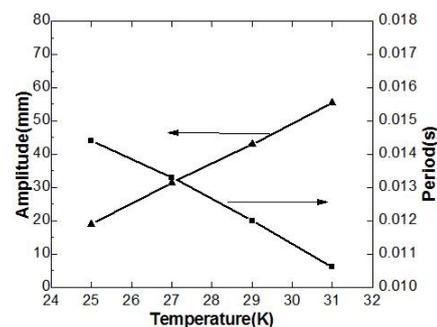


Figure 5. Effect of T_e on amplitude and period of oscillation (hydrogen, $T_c=20$ K)

3.3. PHP with Nitrogen

PHP with nitrogen usually operate between 80 K and 120 K. Inner diameter of nitrogen PHP is determined in the same way with hydrogen PHP. Initial temperature and T_c are 77 K, while other initial values are the same as that in Tab.1. The periodic oscillation is also obtained when T_e is 87K, as shown in Fig.6. The effect of T_e on total heat transfer and sensible heat ratio is shown in Fig.7. Its effect on amplitude and period of oscillation is in Fig.8. The trend of total heat transfer, amplitude and period are similar to those of hydrogen PHP. When T_e is higher than 83 K, sensible heat ratio is over 90%. Similar to hydrogen and water, the sensible heat is dominant for nitrogen PHP in slug flow.

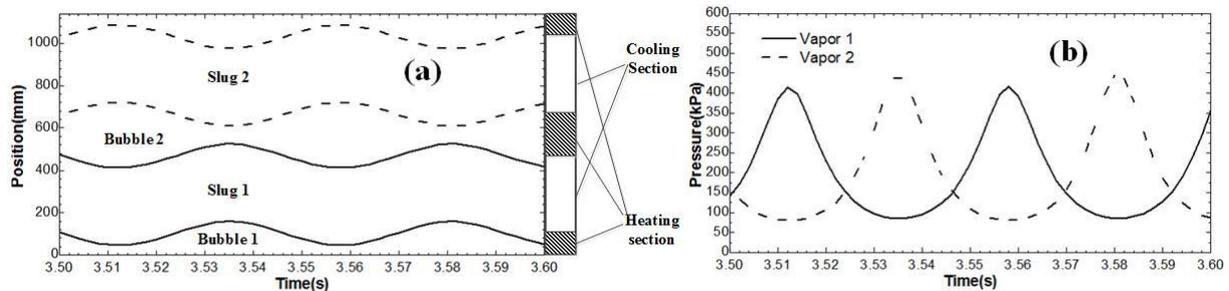


Figure 6. The variation of (a) position of slugs and (b) pressure of bubbles (nitrogen, $T_c=77$ K)

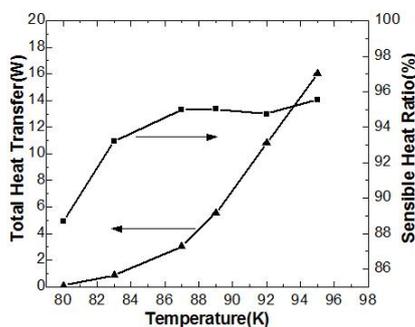


Figure 7. Effect of T_e on total heat transfer and sensible heat ratio (nitrogen, $T_c=77$ K)

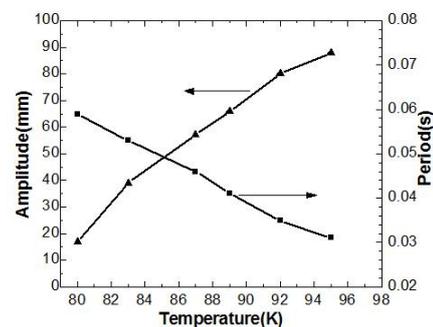


Figure 8. Effect of T_e on amplitude and period of oscillation (nitrogen, $T_c=77$ K)

3.4. Comparison between Hydrogen, Nitrogen and Water

The results of several cases with hydrogen, nitrogen and water are shown in Tab.3. The total heat transfer is nearly the same in Case 4 of water and Case 5 of nitrogen, as well as in Case 6 of water and Case 7 of hydrogen. The oscillation amplitude of long liquid slugs for nitrogen and hydrogen is 5.3 times and 4 times as much as that for water. The period of nitrogen is 30% of period of water, while the corresponding quantity for hydrogen is only 10% that of water. There are two reasons for the phenomenon. The first is that the latent heat of vaporization of nitrogen and hydrogen is much smaller than that of water (10% and 15% approximately), which means small temperature difference can lead to severe evaporation. A large amount of liquid will turn into vapor and the pressure change can be intense. The second is the friction force of nitrogen and hydrogen is smaller than that of water, which benefits to the amplitude of movement. Therefore, the temperature difference, required for PHP with nitrogen and hydrogen operating, is smaller.

Table 3. Comparison of PHP with hydrogen, nitrogen and water

Case	Fluid	Temperature Difference(K)	Total Heat Transfer(W)	Period(s)	Amplitude(mm)
4	Water	80	11.500	0.115	15
5	Nitrogen	15	10.823	0.035	80
6	Water	70	7.661	0.119	14
7	Hydrogen	11	7.455	0.011	56

4. Conclusions and prospect

In order to explore the oscillation and heat transfer performance of long liquid slugs in PHP with nitrogen and hydrogen, a model for closed PHP is established. The following conclusions are obtained:

(1) Given a temperature difference, the long liquid slugs in PHP with hydrogen will oscillate periodically, whose period is smaller than 0.02 s.

(2) For PHP with hydrogen and nitrogen, the total heat transfer and sensible heat ratio will increase when heating temperature increases, and sensible heat ratio is usually greater than 90%.

(3) Compared to the PHP with water, those with hydrogen and nitrogen both have smaller temperature difference under the same heat transfer.

Although similarities and differences between PHP with water, nitrogen and hydrogen are shown, the present model only includes the movement and heat transfer of long liquid slugs, while the phenomenon of separation and combination of slugs in PHP is not considered. The nucleate vaporization and condensation are important factors of the driving forces of PHP, which have great impact on the PHP's performance. The model including nucleate vaporization is being performed, and the simulation results are in expectation.

5. References:

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Acknowledgments

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