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Available pressure amplitude of linear compressor based on phasor triangle model

C X Duan[1], X Jiang[1], X Q Zhi[1], X K You[1] and L M Qiu[1]

[1]Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou 310027, China

limin.qiu@zju.edu.cn

Abstract. The linear compressor for cryocoolers possess the advantages of long-life operation, high efficiency, low vibration and compact structure. It is significant to study the match mechanisms between the compressor and the cold finger, which determines the working efficiency of the cryocooler. However, the output characteristics of linear compressor are complicated since it is affected by many interacting parameters. The existing matching methods are simplified and mainly focus on the compressor efficiency and output acoustic power, while neglecting the important output parameter of pressure amplitude. In this study, a phasor triangle model basing on analyzing the forces of the piston is proposed. It can be used to predict not only the output acoustic power, the efficiency, but also the pressure amplitude of the linear compressor. Calculated results agree well with the measurement results of the experiment. By this phasor triangle model, the theoretical maximum output pressure amplitude of the linear compressor can be calculated simply based on a known charging pressure and operating frequency. Compared with the mechanical and electrical model of the linear compressor, the new model can provide an intuitionistic understanding on the match mechanism with faster computational process. The model can also explain the experimental phenomenon of the proportional relationship between the output pressure amplitude and the piston displacement in experiments. By further model analysis, such phenomenon is confirmed as an expression of the unmatched design of the compressor. The phasor triangle model may provide an alternative method for the compressor design and matching with the cold finger.

1. Introduction

Linear compressors with the advantages of high efficiency, long life, low vibration and compact structure, are widely used in civil, spatial and military applications [1-2]. However, the output efficiency of the linear compressor is determined by the driven load besides its own structural parameters. The efficiency of poorly matched linear compressor may drop to less than half of that on the optimum status [3]. So, many researchers focus on how to achieve high efficiency and large acoustic power output through the match between the compressor and the load.

Previous researchers have proposed a series of linear compressor analysis models such as phasor analysis model, equivalent circuit model, comprehensive model of a miniature-linear fluid for electronics cooling and 1-D lumped parameter model [4-7]. Gan et al studied the acoustic impedance of the linear compressor through the RC load, and the influence of the acoustic impedance on the output acoustic power and the efficiency of the compressor was analysed [3]. Dai et al studied on the impedance matching between the linear compressor and the cold finger, and introduced that changing the piston diameter had a great influence on the impedance matching [8]. Dang et al proposed a design method about how to design cold finger or linear compressor, which needs iterative calculation based on the electrical circuit analogy model [9]. Actually, in order to make pulse tube cryocooler (PTC) work in the best condition, it needs not only the high efficient output of the compressor but also the high efficient working of the cold finger. However, it's difficult to achieve a good match between linear compressor and cold finger, since it requires both the compressor and cold finger working in the high efficient state simultaneously. Compressor output pressure amplitude is one of the most critical factors affecting the performance of the cold finger. Ray Radebaugh's analysis, the effect of He_3 and He_4 on the properties of the regenerator, indicates that increasing the pressure amplitude under the same input of the acoustic power can effectively reduce the mass flow in the regenerator, thereby reducing the regenerator loss [10].

In this paper, the phase triangle model is proposed through the analysis of compressor piston force and setting the piston speed as the reference. Under the known compressor operating and structure size parameters, compressor acoustic conversion efficiency and output acoustic power can be calculated by this phase triangle model. Furthermore, it can also calculate the output pressure amplitude in a given condition and the maximum value at silent power output state. Calculations match well with experimental results, and the deviation is less than 6%.

2. Phasor Triangle Model

Assume that the alternating motion of the piston and the working fluid are simple harmonic motion and set the piston speed direction as a reference. Displacement (S), velocity (v) of piston, acceleration (a) of piston, current (I), pressure (P) could be expressed as equations (1).

$$\begin{gathered} S = S_a\cos(\omega t - \frac{\pi}{2}) \\ v = S' = -\omega S_a\sin\left(\omega t - \frac{\pi}{2}\right) = \omega S_a\cos(\omega t) \\ a = v' = -\omega^2 S_a\sin(\omega t) = \omega^2 S_a\cos(\omega t + \frac{\pi}{2}) \\ I = I_a\cos(\omega t + \phi) \\ P = P_0 + P_a cos(\omega t + \theta) \end{gathered} \tag{1}$$

Where, ω is angular frequency, S_a is the amplitude of displacement, I_a is the amplitude of current, P_0 is the mean pressure, P_a is the amplitude of pressure, ϕ is the phase difference between current and velocity (called as current phase angle), and θ is the phase difference between pressure and velocity (called as pressure phase angle).

Figure 1 shows the force analysis of linear compressor piston, and right is the positive direction. The radial forces are all ignored, compared with axial forces. F_e is the electromagnetic force because of current in coil, F_k is the elastic force in spring, F_{bv} is the gas force in back volume, F_{cv} is the gas force in compression volume, F_f is the damping force of friction. Equation (2) is the force balance equation according to Newton's second law.

$$F_e + F_{bv} + F_k + F_{cv} + F_f = Ma \tag{2}$$

Forces in Figure 1 are listed in equation (3) based on their definitions [11].

$$\begin{gathered}F_e = \alpha I = \alpha I_a cos(\omega t + \phi) \\ F_k = Ks = -KS_a cos\left(\omega t + \frac{\pi}{2}\right) \\ F_{cv} = -P_{cv}A = -AP_0 - AP_{a,cv}\cos(\omega t + \theta_{cv}) \\ F_{bv} = P_{bv}A = AP_0 + AP_{a,bv}\cos(\omega t + \theta_{bv}) \\ F_f = -R_m v = -R_m \omega S_a \cos(\omega t)\end{gathered} \quad (3)$$

Phasor analysis method is a common transformation for replacing a harmonic equation in time domain with a phasor in frequency domain, seen in Figure 2. The bold character with a wave mark represents a phasor. The real part of a phasor is the harmonic equation, shown in equation (4).

$$\mathrm{Re}(\tilde{\boldsymbol{F}}) = F\cos(\omega t + \alpha) \quad (4)$$

Where, F is the force, α is the angle between F and its reference direction. Applying this phasor representation into equation (2), it comes to,

$$\tilde{\boldsymbol{F}}_{\boldsymbol{e}} + \tilde{\boldsymbol{F}}_{\boldsymbol{k}} + \tilde{\boldsymbol{F}}_{\boldsymbol{cv}} + \tilde{\boldsymbol{F}}_{\boldsymbol{bv}} + \tilde{\boldsymbol{F}}_{\boldsymbol{f}} = M\tilde{\boldsymbol{a}} \quad (5)$$

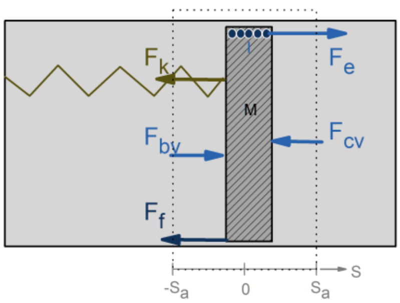

Figure 1. Force analysis of linear compressor piston (The range of motion is from -S_a to +S_a).

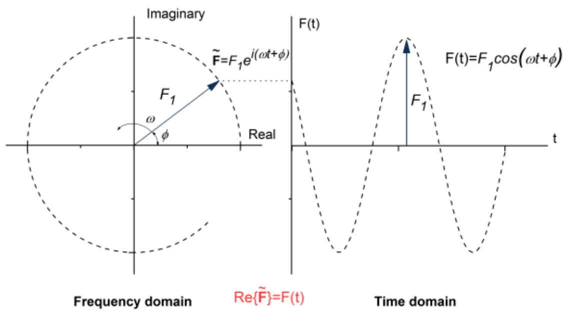

Figure 2. Transformation between a phasor in frequency domain and a harmonic equation in time domain.

The assumption is that the back pressure chamber is adiabatic and has no power loss, thus θ_{bv} is 90°. Force phasor analysis of piston is shown in Figure 3 based on equation (5). It is almost the same with Figure 2 in the reference paper [11] except setting velocity phasor as the horizon axis. This difference makes it easier to express the power of forces in piston. The power of $\tilde{\boldsymbol{F}}_{\boldsymbol{cv}}$ as is shown in Figure 4 can be expressed as equation (6). The 1/2 on the right side of the equation is due to the use of amplitude value in the phasor-type.

$$< P\dot{V} >= \frac{1}{2}\left|\tilde{\boldsymbol{F}}_{\boldsymbol{cv}}\right|\left|\tilde{\boldsymbol{v}}\right| \cdot \cos\theta \quad (6)$$

Combined with phasor force, there is a right-angle triangle (called as phasor triangle) for pressure amplitude in Figure 3. Multiplying pressure amplitude with piston area is the length magnitude of the hypotenuse; the first term in the bracket under square root is the length magnitude of the vertical right-angle side (called as vertical side); and the second term in the bracket under square root is the length magnitude of the horizontal right-angle side (called as horizontal side). Base on the phasor triangle, the compressor output pressure amplitude can be defined as equation (7).

$$\mathrm{P}_a = \frac{\sqrt{[(\mathrm{M}\omega^2 - \mathrm{K} - \frac{\gamma \mathrm{A}^2 P_0}{V_{bv}})S_a - \alpha \mathrm{I}_a \sin\phi]^2 + (\alpha \mathrm{I}_a \cos\phi - R_m * \omega \mathrm{S}_a)^2}}{\mathrm{A}} \tag{7}$$

According to the energy balance, the output acoustic power is,

$$< P\dot{V} >=< W_e > -< W_f >= \frac{1}{2}(\alpha {\mathrm{I}_a}^2 * \omega \mathrm{S}_a * \cos\phi - R_m * {I_a}^2 * \omega \mathrm{S}_a * \omega \mathrm{S}_a) \tag{8}$$

Where, $< W_e >$ is the power of electromagnetic force, $< W_f >$ is the power of damping force. And the compressor acoustic power conversion efficiency can be expressed as equation (9). Where, $< \dot{Q} >$ is the Joule heat.

$$\eta = \frac{< P\dot{V} >}{< \dot{Q} > +< W_e >} = \frac{\alpha I_a * \cos\phi * \omega \mathrm{S}_a - R_m * \omega \mathrm{S}_a * \omega \mathrm{S}_a}{{I_a}^2 R_e + \alpha I_a * \omega \mathrm{S}_a * \cos\phi} \tag{9}$$

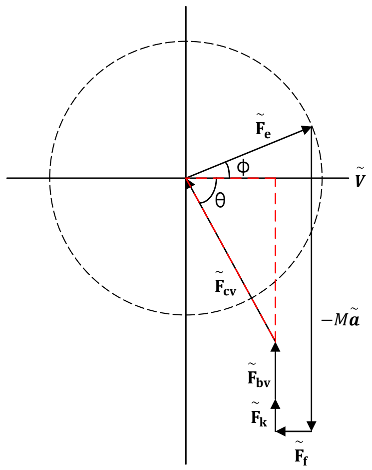

Figure 3. Force phasor analysis of linear compressor piston.

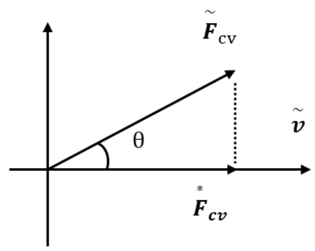

Figure 4. Power of force phasor.

3. Experimental analysis

A large capacity Stirling type PTC has been designed at 2.5 MPa charging pressure, 60 Hz to supply 650W@80K with 10 kW input electrical power. CFIC-2S297W, a dual-opposing pistons linear compressor, was chosen to offer 7.5 kW acoustic power for the cold finger. For simplifying calculation, the opposed dual piston model is converted to an equivalent single piston model, and the main design parameters are listed in Table 1.

Table1. CFIC-2S297W prototype parameters in single piston model.

A (m^2)	M (kg)	k (N/m)	V (m^3)	α (N/A)	R_m (Ns/m)	R_e (Ω)	I_{max} (A)	S_{max} (mm)
0.0783	17.6	320000	0.028	45	140	0.23	84.85	13

Dynamic pressure signals in compression space and back space are measured by Endevco/8530B-500 sensors. Piston displacement is measured by Micro-Epsilon/VIP50GA5SA7I. As shown in the Figure 5, for the compressor operating at different frequencies, the theoretical calculated pressure amplitude is in good agreement with the experimental value, and the deviation is less than 6%. The maximum output pressure amplitude under the specific operating conditions can also be calculated by the phasor triangle model. Theoretical maximum pressure amplitude data is obtained by keeping compressor current and piston displacement constant, changing the current phase angle.

When without acoustic power output, compressor can be operated in a silent power output state. All of the input electric power is converted into Joule heat and damping loss, then the compression chamber pressure amplitude can reach the maximum. Phasor forces on compressor piston are shown in Figure 6. The maximum pressure amplitude can be calculate by equation (10).

$$\mathrm{P}_{a\_max} = \frac{\left(\mathrm{M}\omega^2 - K - \frac{\gamma \mathrm{A}^2 P_0}{V_{bv}}\right) * S_a + \sqrt{(\alpha I_a)^2 - (R_m * w S_a)^2}}{A} \tag{10}$$

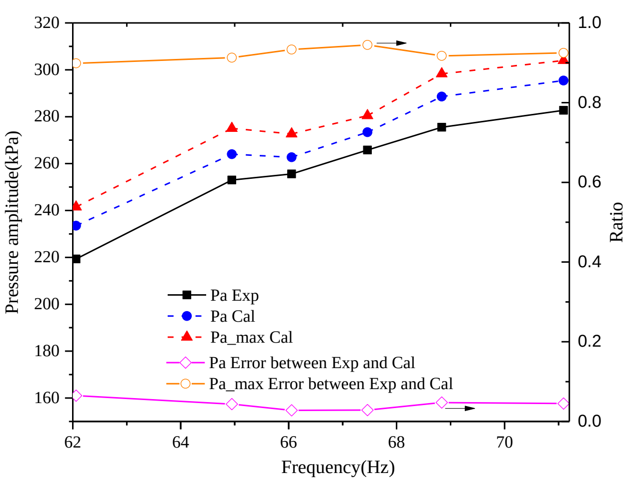

Figure 5. Pressure amplitudes in different frequency.

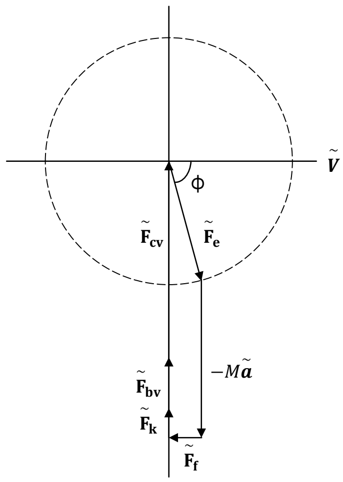

Figure 6. Phase diagram in silent power output state.

In Figure 5, the output pressure amplitude of the compressor used in the experiment is more than 90% of the theoretical maximum. This shows that the compressor output pressure amplitude at the experiment is close to its limit, and spending more effort to enhance the pressure amplitude is meaningless.

4. Proportion between pressure amplitude and displacement

As can be seen from Figure 5, the output pressure amplitude of CFIC-2S297W is close to the theoretical maximum output, but it is still far away from the design requirement of the cold finger, 326 kPa. After a large number of experiments in changing the size of transfer tube or inertance tube and the operating frequency, the output pressure amplitude still can not meet the design requirements. Meanwhile, the output pressure amplitude has a good linear relationship with the displacement of the piston in these experiments, as shown in Figure 7. In order to improve measurement accuracy of the sensor, the experiment has been done under the condition that the pressure amplitude is larger than 0.1 MPa and piston displacement is larger than 6 mm. It seems logical that more power input makes the piston displacement increase, and thus the pressure amplitude also increases. But such a good linear relationship is not good for the PTC.

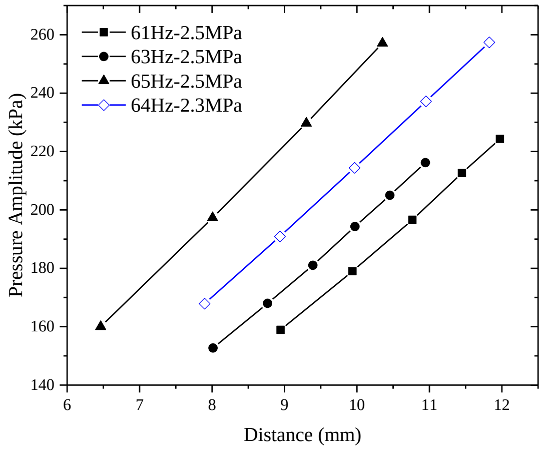

Figure 7. Pressure amplitude versus displacement in experiments.

A portion of the experimental data used in Figure 7 is shown in Table 2.

Table 2. Experimental data used in Figure 7.

f(Hz)	P_0(MPa)	I(A)	S_a(mm)	Pa(MPa)	θ(°)
63.98	2.33	54.13	11.83	0.2574	-87.6
61.09	2.53	47.76	11.97	0.2243	-87.5
63.07	2.51	58.01	10.95	0.2162	-83.7
64.95	2.47	60.00	10.35	0.2571	-84.0

It can be seen from the experimental data that θ is less than -80°. So when calculating the length of the hypotenuse in the phasor triangle, the length of the horizontal side can be neglected compared to the vertical side. Then equation (7) can be simplified as equation (11).

$$\mathrm{P}_a \mathrm{A} \approx (\mathrm{M}\omega^2 - K_s - \frac{\gamma \mathrm{A}^2 P_0}{V_{bv}}) S_a - \alpha \mathrm{I}_a * \sin\phi \tag{11}$$

While in the case of large displacement, S_a>7 mm, the first term on the right side of the equation (11) is much larger than the second term, as shown in equation (12).

$$\left(\mathrm{M}\omega^2 - K_s - \frac{\gamma \mathrm{A}^2 P_0}{V_{bv}}\right) S_a \gg \alpha \mathrm{I}_a * \sin\phi \tag{12}$$

Then, equation (11) can be further simplified as equation (13).

$$\mathrm{P}_a = \frac{(\mathrm{M}\omega^2 - K_s - \frac{\gamma \mathrm{A}^2 P_0}{V_{bv}}) S_a}{\mathrm{A}} \tag{13}$$

By equation (13), the phenomenon that the pressure amplitude is proportional to the displacement of the piston can be explained.

A well-designed compressor should directly meet the needs of the regenerator hot end phase angle 0 ~ -30°. At this time, the piston angle is about -30° ~ -60°, and the length of horizontal side is roughly the same as vertical side in the phasor triangle. In the experiment, the phase angle of the compressor piston is less than -80°. So the vertical side is far longer than the horizontal side in the phasor triangle. The compressor working in this condition needs more volume of transfer tube, thus the refrigerator system will lose the advantage of compact structure.

5. Conclusion

The phase triangle method is used to transform the force equation into a triangular model by phasor analysis method. This model can also be used to calculate the pressure amplitude extremum under the given frequency and charging pressure. It has significant practical meaning to select and use the compressor, avoiding a lot of experiments. The experimental data shows that the deviation of output pressure amplitude between calculated and the measured is less than 6%.

The model also explains the experimental phenomenon that the pressure amplitude is proportional to piston displacement in the CFIC-2S297W. It points out that compressors having the proportional phenomenon need larger volume of transfer tube and make the whole refrigerator system less compact.

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