

Parameter Estimation of the Precessing Cone-Shaped Warhead Based on the Micro-Doppler with Multi-aspect Observations

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Abstract. Micro-motion feature is one of the effective features used for radar target recognition in the middle section of the ballistic curve. The micro-Doppler expressions of the scattering center at the conical point and two sliding scattering centers in the conical bottom are given, firstly. Then by analyzing the micro-Doppler expressions, it is found that only the precession frequency can be estimated from the micro-Doppler expressions, and other parameters can't be estimated directly. But all precession and geometry parameters can be estimated by decreasing the number of parameters in the micro-Doppler expressions of two sliding scattering centers, which must use two groups of echoes illuminating the same warhead target from the cone top and the cone bottom. Finally, a new method of estimating all precession and geometry parameters is proposed, and it is validated by the simulation analysis.

1. Introduction

Due to long time and a target's inertial flight in mid-course phase, the mid-course is helpful for the missile defense system to achieve target identification of the ground based radar. In the mid-course flight, due to the separation of the missile and rocket and transverse interference during decoy release, the missile warhead will produce precession, which is closely associated with a target's shape and structure, quality distribution, kinetic characteristics and other physical quantities. Warhead precession causes micro-Doppler modulation in radar echo. Through micro-Doppler analysis and processing, the characteristic quantity of the warhead target can be extracted for identification.

Chen V C systematically studied the micro-motions and micro-Doppler phenomenon in radar observations, and analyzed the characteristics of a rigid target's various forms of micro-motions and the analysis and processing method of doppler effect [1,2]. For an ideal point target, its micro-Doppler expressions in the four basic micro-motions of vibration, rotation, rolling and coning were derived.

A model for the motion characteristics of the warhead target's scattering center was built, so as to obtain the micro-Doppler mathematical expression of a sliding scattering center, where a darkroom dynamic measurement experiment for a precession target was also designed to have verified this expression [3,4].

For parameter estimation of the precessing cone-shaped warhead, several feature extraction methods have been proposed. In [5], based on analysis of the scattering properties of the cone-shaped



target, the scattering centers' theoretical Instantaneous Frequency variations induced by precession are derived, according to the properties of IF variations of the top and bottom scattering centers under different radar aspects, the precession and geometry feature extraction method is proposed for estimating the target's parameters. In [6], using the linear sum of the radial distance of the strong scattering centers, got by taking sample from the echo signal in the wideband radar, the cycle of the micro-motion and the initial phase of the echo signal are estimated. Using the linear sum and difference of the radial distance of the separated sliding scattering centers and adopting the least square fitting and solving the multivariate nonlinear equation system, the target's micro-motion features and structure features are extracted. In [7], union equations of wideband and narrowband micro-Doppler information are analyzed, and the precession and geometry feature extraction method is proposed for estimating the target's parameters. In [8], the theoretical micro-Doppler cone target's scattering source is deduced, which is decomposed into synthesis of many order harmonic components by descending power formula of trigonometric function, then the target tracking technology is introduced to gain different scattering sources' micro-Doppler, and a method to gain the characteristics parameters is proposed by the relation between the harmonic component and the characteristics parameters. These methods don't estimate all precession and geometry parameters in the micro-Doppler mathematical expressions which are proposed in [3, 4].

In Section 2, the micro-Doppler mathematical expressions of three scattering centers are given. In Section 3, three micro-Doppler expressions are analyzed. Hough transform can only be used to extract precession frequency in the expression of the scattering center at the cone top. In contrast, Hough transform cannot estimate parameters directly in the expressions of the two sliding scattering centers until the parameter dimensions have been reduced. As a result, the conclusion that two groups of echoes illuminating the same warhead target from the cone top and the cone bottom must be both utilized in order to estimate all precession and structural parameters is obtained. In Section 4, with the formation of fore sight and back sight from a narrowband netted radar to a precession conical warhead simultaneously, a new parameter estimation method is proposed, the effectiveness of the parameter estimation method has been validated by the simulation analysis. Section 5 is the summary.

2. Micro-Doppler Models of the Scattering Centers of a Precessing Cone-Shaped Warhead Target

As shown in Fig. 1, z axis is defined as the axis of the cone. y axis is in the plane which is constituted by the incident direction of the electromagnetic wave and z axis. x axis meets right hand spiral law with y axis and z axis. The conical sphere center is o' with the radius r_b . The radius of its bottom circle is r_0 , and the height of the cone is h . In [9], it was pointed out that the positions of the three scattering centers of the conical warhead are a , c and d in Fig. 1, and the scattering coefficient expressions of the three scattering centers of the doom flat conical warhead were derived from geometric scattering theory.

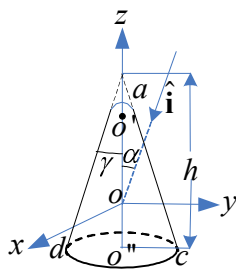


Figure 1. Scattering centers.

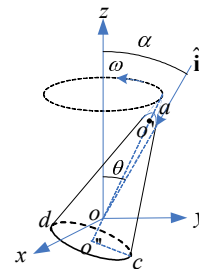


Figure 2. Precession model.

As shown in Fig. 2, z axis is defined as the precession axis of the conical warhead. o is the center of mass, and the warhead target precesses with the precession angle θ and the precession frequency

ω . ω is positive in the direction shown in Fig. 2, and is negative in opposite direction. The incident direction of radar wave is that $\mathbf{i} = -\sin \alpha \mathbf{y} - \cos \alpha \mathbf{z}$, of which α is the average visual angle of the radar, and the scattering direction is that $\mathbf{s} = -\mathbf{i}$.

The micro-Doppler expression of scattering center a [3] is

$$f_{micro-a} = 2 |oo'| \frac{f_0}{c} \omega \sin \alpha \sin \theta \cos(\omega t + \phi_0) \quad (1)$$

In (1), f_0 is radar frequency, c is the speed of light.

The micro-Doppler expression of scattering center c and d [3] is

$$f_{micro-cd} = \frac{2f_0}{c} \omega \sin \alpha \sin \theta \cos(\omega t + \phi_0) [-|oo'| + k \sin \alpha \sin \theta \sin(\omega t + \phi_0) + k \cos \alpha \cos \theta] \quad (2)$$

$$k = \pm r_0 / \sqrt{1 - [\sin \alpha \sin \theta \sin(\omega t + \phi_0) + \cos \alpha \cos \theta]^2} \quad (3)$$

“+” represents scattering center d and “-” represents scattering center c .

When illuminating the warhead target from the cone top, $\theta < \alpha < 90^\circ - \theta$, scattering center a and c are continuously visible, d is invisible. When illuminating the warhead target from the cone bottom, $90^\circ + \theta < \alpha < 180^\circ$, scattering center c and d are continuously visible, a is invisible.

3. Analysis of Micro-Doppler Expressions

3.1. Micro-Doppler Expression Analysis of the Scattering Centers

Equation (1) is a sine function, so Hough transform can be used to estimate ω , the initial angle ϕ_0

and the amplitude $I_m = 2 |oo'| \frac{f_0}{c} \omega \sin \alpha \sin \theta$.

Equation (2) is a non-sine function, and it includes six parameters that are $|oo'|$, ω , α , θ , ϕ_0 and r_0 . The focus is the estimation of four parameters: $|oo'|$, α , θ and r_0 . The basis for Hough transforms for parameter estimation is the difference of the curves under different parameter combinations. If a large difference in a parameter combination corresponds to a small difference in the curve, it indicates that the curve is not appropriate to use Hough transform for parameter estimation. In order to study the feasibility of using Hough transform to estimate these parameters, a numerical simulation traversal method is used for the curve's difference analysis under different parameter combinations.

First, the parameter space is subdivided. The range of $|oo'|$ is $[0.1m, 2m]$, with the interval of $0.1m$; the ranges of α and θ are $[1^\circ, 90^\circ]$ and $[1^\circ, 20^\circ]$ respectively, both with the interval of 1° ; the range of r_0 is $[0.1m, 1m]$, with the interval of $0.1m$. Next, supposing $\omega = 2\pi rad/s$, $\phi_0 = 0^\circ$, sampling rate $f_s = 1000Hz$, and sampling time is $1sec$. Then the sum of squares of the differences between the curves corresponding to each group of $(|oo'|, r_0, \alpha, \theta)$ and curves corresponding to all groups of $(|oo'|, r_0, \alpha, \theta)$ traversed is calculated and saved. The sum of squares of the differences between two curves are named ΔD . Finally, only two examples are listed here due to limited space. Table 1 is the result of finding the eight minimum ΔD and their parameter combinations for $f_{micro-c}$ when $(|oo'|, r_0, \alpha, \theta) = (0.5m, 0.4m, 30^\circ, 5^\circ)$. Table 2 is the similar result for $f_{micro-d}$. It can be found from the examples that there is a large difference between parameter combinations and the truth value, but little difference in their curves, which has proved that the parameter combinations cannot be estimated with Hough transform.

Table 1. ΔD for $f_{micro-c}$.

	$ oo'' $	r_0	α	θ	ΔD
1	0.5	0.4	30	5	0
2	0.3	0.3	34	6	7.3946e-7
3	0.8	0.6	30	4	1.0306e-6
4	1.5	0.6	16	3	1.1898e-6
5	1.5	0.6	3	16	1.1898e-6
6	0.3	0.4	44	6	1.4157e-6
7	0.8	0.8	39	4	1.819e-6
8	0.2	0.2	31	7	2.2785e-6
9	0.7	0.9	46	4	2.9512e-6

Table 2. ΔD for $f_{micro-d}$.

	$ oo'' $	r_0	α	θ	ΔD
1	0.5	0.4	30	15	0
2	0.7	0.5	24	12	1.6304e-5
3	1	0.7	19	9	3.5134e-5
4	1	0.7	9	19	3.5134e-5
5	1.1	1	16	7	3.8213e-5
6	1.1	1	7	16	3.8213e-5
7	1.2	0.8	17	8	7.446e-5
8	1.2	0.8	8	17	7.446e-5
9	0.4	0.3	36	19	7.7438e-5

3.2. Dimension Reduction Analysis of Micro-Doppler Expressions of the Scattering Centers

It can be found from the above analysis that $|oo''|$, α , θ and r_0 cannot be estimated simultaneously with Hough transform. But when $90^\circ + \theta < \alpha < 180^\circ$, merely c and d are visible. Using the relationship of $f_{micro-c}$ and $f_{micro-d}$, two new expressions f_α and f_β are derived.

$$f_\alpha = -\frac{2f_0}{c} |oo''| \omega \sin \alpha \sin \theta \cos(\omega t + \phi_0) \quad (4)$$

$$f_\beta = \pm \frac{2f_0}{c} \omega \sin \alpha \sin \theta \cos(\omega t + \phi_0) \frac{r_0 [\sin \alpha \sin \theta \sin(\omega t + \phi_0) + \cos \alpha \cos \theta]}{\sqrt{1 - [\sin \alpha \sin \theta \sin(\omega t + \phi_0) + \cos \alpha \cos \theta]^2}} \quad (5)$$

Comparing (5) with (2), it can be found that f_β don't contain the parameter $|oo''|$, namely, one dimension has been reduced in parameters.

ω and ϕ_0 can be obtained from f_α , so for f_β , r_0 is a magnification coefficient, and the rest parameters are α and θ . Likewise, a numerical simulation traversal method is used to make the difference analysis of a curve under different parameter combinations.

First, the parameter space is subdivided. The ranges of α and θ are $[90^\circ, 180^\circ]$ and $[1^\circ, 20^\circ]$, with the interval both for 0.2° . $r_0 = 0.5m$, $\omega = 2\pi rad/s$, $\phi_0 = 0^\circ$, sampling rate $f_s = 1000Hz$, and the sampling time is 1 sec. Then, the sum of squares of differences between parameter groups for f_β is calculated and saved. Finally, as the parameter space is two-dimensional, Fig. 3 and Fig. 4 show the contour maps of two typical parameter groups corresponding to sums of squares of differences. It can be seen from the figures that the least values are all found near the truth value, so Hough transform can be used for accurate estimation of (α, θ) .

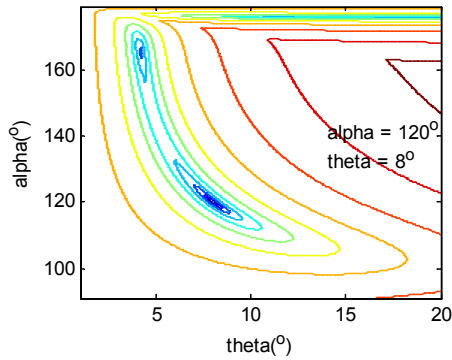


Figure 3. Contour map of curve distance when $(\alpha, \theta) = (120^\circ, 8^\circ)$.

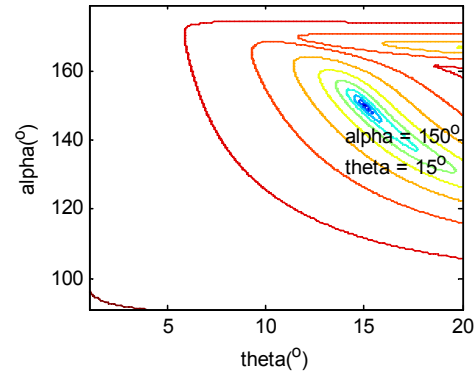


Figure 4. Contour map of curve distance when $(\alpha, \theta) = (150^\circ, 15^\circ)$.

When $90^\circ + \theta < \alpha < 180^\circ$, the curves of $f_{\text{micro-c}}$ and $f_{\text{micro-d}}$ can be extracted from Time-frequency diagram of narrowband RCS sequences, so ω and ϕ_0 can be estimated from f_α , r_0 , α and θ can be estimated from f_β .

4. Parameter Estimation Method of Narrowband Netted Radar

It can be seen from Section 3 that only the precession frequency ω can be estimated when $\theta < \alpha < 90^\circ - \theta$. When $90^\circ + \theta < \alpha < 180^\circ$, through dimension reduction, the correct values can be estimated for parameters $(\omega, \alpha, \theta, \phi_0, r_0, |oo''|)$. However, $|oo'|$ cannot be obtained because scattering center a is invisible. Therefore, it can be concluded that all precession and structure parameters cannot be solved when $\theta < \alpha < 90^\circ - \theta$ or $90^\circ + \theta < \alpha < 180^\circ$. To combine the echoes illuminating at the cone top and the cone base, the information of scattering center a and c and the information of scattering center c and d can be utilized at the same time. The average visual angle is α_b and the initial angle is ϕ_b when $90^\circ + \theta < \alpha < 180^\circ$. The average visual angle is α_f and the initial angle is ϕ_f when $\theta < \alpha < 90^\circ - \theta$. All precession and structure parameters can be estimated without the help of other information.

4.1. New Parameter Estimation Method

A new parameter estimation method is as follows:

1. First, we get Time-frequency diagram of narrowband RCS sequences when $90^\circ + \theta < \alpha_b < 180^\circ$. Next, the micro-Doppler curves of scattering center c and d can be extracted from Time-frequency diagram by Hough transform, but the curves' parameters are wrong. Then the sum of the numerical value of two micro-Doppler curves is corresponding with f_α , so ω and ϕ_b can be estimated. The difference of the numerical value of two micro-Doppler curves is corresponding with f_β , so r_0 , α_b and θ can be estimated. Finally, $|oo''|$ is calculated.

2. First, we get Time-frequency diagram of narrowband RCS sequences when $\theta < \alpha_f < 90^\circ - \theta$. Then the micro-Doppler curves of scattering a and c can be extracted from Time-frequency diagram by Hough transform. Absolutely, $f_{\text{micro-a}}$ is a sine function, and $f_{\text{micro-c}}$ is a non-sine function, so their micro-Doppler curves can be distinguished. α_f and ϕ_f can be estimated from $f_{\text{micro-c}}$ because $(\omega, \theta, r_0, |oo''|)$ are known. Finally, $|oo'|$ is calculated.

In this method, in a certain period of time, two monostatic radars illuminate the same warhead target from the cone top and the cone bottom respectively. After their echo data is transmitted to the data fusion unit, all precession and structural parameters can be estimated.

4.2. Simulation

The parameter estimation method is validated by every simulation. An example is given here due to limited space. Simulation parameters: radar frequency $f_0 = 10\text{GHz}$, $|oo'| = 1.8175\text{m}$, $|oo''| = 1\text{m}$, $\omega = 2\pi\text{rad/s}$, $\theta = 10^\circ$, $\alpha_f = 45^\circ$, $\alpha_b = 135^\circ$, the warhead's initial angle in xoy plane for $\phi_f = \phi_b = 0^\circ$, $r_0 = 0.5\text{m}$, $r_b = 0.03\text{m}$, half conical angle $\gamma = 9.46^\circ$, $f_s = 400\text{Hz}$, sampling time is 1sec. Table 3 shows the comparison of the estimated value and real value. All parameters are close to the truth value.

Table 3. The Comparison of estimated values and truth values.

Parameter	truth value	estimated value
ω	$2\pi\text{rad/s}$	$2\pi\text{rad/s}$
ϕ_b	0°	-1°
α_b	135°	138°
θ	10°	9.4°
r_0	0.5m	0.5m
$ oo'' $	1m	1.11m
α_f	45°	50.1°
ϕ_f	0°	-0.9°
$ oo' $	1.8175m	1.6379m

5. Conclusion

In this paper, micro-Doppler expressions of the three scattering centers for a precession conical warhead are given firstly. Next, from the analysis of the three expressions, it can be found that only precession frequency can be estimated through Hough transform of time-frequency diagram of RCS sequences, and other parameters can be estimated after dimension reduction for the micro-Doppler expressions of sliding scattering centers. Then the method of estimating all precession and geometry parameters is proposed based on two groups of echoes illuminating at the cone top and the cone base. Finally, the proposed method is validated by the simulation analysis.

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