

Theoretical calculation of the fragment initial velocity following aerial explosion of the cylindrical warhead with two terminals

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Abstract. In order to study the fragment initial velocity distribution of the cylindrical warhead with terminals and exploded in the air, this paper builds a theoretical calculation model of the side-wall and terminal velocity of the warhead with two terminals based on the energy conservation principle, and considering the explosive energy distribution and proportion. The calculation results obtained by the theoretical model are compared with the experimental results of previous scholars. In this paper, the influence of the warhead L/D and C/M on the side-wall and terminal initial velocity is examined. A good agreement between the theoretical calculation results and the experimental results of previous scholars is observed. This verifies feasibility of the calculation method proposed in this paper. When the warhead L/D is smaller than 2, along with increase of L/D, the side-wall fragment velocity increases and maintains at certain value. In terms of the terminal fragment initial velocity, when the warhead L/D is close to 1, the initial velocity is larger; but, when the L/D is larger than 1, the terminal fragment initial velocity decreases significantly.

1. Introduction

Scholars have come up with some methods to calculate the fragment speed after explosion of the warhead, and carried out some experiments to verify the feasibility of the calculation methods. Gurney [1] discovered in his early work that there is not a strong correlation between the initial velocities of fragments from bombs and shell. He then theoretically calculated the initial velocity distribution of the sphere, cylindrical shell and infinite slab after being driven by the explosive. GB/T Koch et al. [2] argued that Gurney energy worked out by Gurney equations does not take the detonation wave velocity, D . Thus, they used C-J equation to show that the total energy of the explosive serves as the initial energy, and then deduced the relation between the detonation velocity, D , of an explosive and its Gurney energy. Jean-Francois et al. [3] also raised the same issue, thinking that a series of problems pops up when the polytropic exponent is a constant, 3. They also pointed out the relation between the detonation velocity, D , and Gurney energy, and carried out experiments to confirm how to set the value of the polytropic exponent for different explosives. Hutchinson et al. [4] put forward the equivalent momentum distribution method, expounded on the reflex action of the blast wave during the explosion process of the warhead, and proposed the method to calculate the naked charge of the shelled warhead.



Kong et al. [5-6] tested the velocity of the side-wall fragment and the end-cover fragment when the terminal of the cylindrical warhead with two closed terminals is exploded. They also simulated the explosion process of the warhead using the Smoothed Particle Hydrodynamic (SPH) method, thus obtaining results of the fragment mass distribution. Their work also showed that the warhead with terminals cannot be worked simply using Gurney equations. Zhang et al. [7] studied the influence of thickness of the warhead shell on the location and velocity of fragments, and also examined the influence of fragment acceleration on the explosion shockwave. Li et al. [8] built the second-dimensional theoretical model to deduce the fragment velocity distribution of asymmetrically initiated warhead.

Based on the Gurney hypothesis, this paper proposes the method to calculate the fragment initial velocity after the cylindrical warhead with two terminals explodes in the air, and compares the calculation results with the results obtained by the above scholars to verify reliability of the calculation method put forward in this paper. Meanwhile, this paper examines how the length-to-diameter ratio (L/D) and the ratio of the explosive mass to the mass of the cylindrical shell and terminal (C/M) on the initial velocity of terminal and side-wall fragments.

2. Building of the theoretical calculation model

2.1. Model hypotheses

The calculation model in this paper is deduced based on the Gurney hypothesis [2]. Gurney equations are applicable to cylindrical warhead exploded at the center, the cylindrical shell exploded in the axial direction, and the infinite slab. Thus, if Gurney equations are applied to the warhead with two terminals, it might have the following shortages:

Calculating the relationship between Gurney energy and detonation velocity, D , without considering the influence of warhead explosion;

The actual warhead is not a cylindrical shell without terminals, nor is it a cylindrical charge which is initiated synchronously along the axial direction;

Failing to consider the energy consumed by the deformation failure of the combat shell.

In order to make up shortages of Gurney equations, this paper regards the warhead shell consists of two parts, namely the cylindrical shell and the top and bottom slabs, and builds the theoretical initial velocity calculation model of fragments after a cylindrical warhead with two terminals explodes based on the energy conservation principle. Meanwhile, the theoretical method to calculate the terminal and side-wall fragment initial velocity is also put forward, respectively. Before building the theoretical calculation model, this paper makes the following hypotheses:

Chemical energy of explosives is fully converted into the momentum of the explosive products (gas and shell), and the shell deformation energy;

The reflection action of the rarefaction wave and the shockwave in the reaction area on the shell is ignored. In other words, the shockwave triggered by the explosion maintains the spherical diffusion;

During the swelling process of the warhead, the side-wall axial length remains the same, and the major deformation is reflected as the radial extension and the bending deformation;

Before the warhead shell is fragmented, the density of explosive products is evenly distributed.

2.2. Energy calculation

Based on the shortages, hypotheses and simulated calculation results of Gurney equations described above, the fragment velocity of a cylindrical warhead exploded at the center is divided into two parts, namely the terminal velocity, v_t , and the side-wall velocity, v_s . According to the energy conservation principle and Hypothesis (1) and (3), the following equation can be obtained:

$$CE_e = \frac{1}{2} M v_0^2 + \frac{1}{2} \int_0^{r_f} 2\pi \rho(r) v^2(r) dr + \frac{M_t}{\rho_m} (\sigma_b \cdot \varepsilon_f) \quad (1)$$

Where, the left part represents the chemical energy generated by detonation of the explosive, while Item 1, Item 2 and Item 3 on the right stand for the shell momentum, the explosive product momentum and the shell deformation energy.

Therefore, the energy equation of the warhead explosion can be written as below:

$$CE_e = \frac{1}{2}M_s v_s^2 + \frac{1}{2}M_t v_t^2 + \frac{M_t}{\rho_m}(\sigma_b \bullet \varepsilon_f) + \frac{1}{2}\rho_a \int_0^{r_f} 2\pi x \left[\int_{-l/2}^{l/2} (v_x^2 + v_y^2) dy \right] dx \quad (2)$$

Where, C is the explosive mass; E_e is the explosive specific energy; M_s and M_t are the mass of the cylindrical shell mass and the terminal mass, respectively; v_s and v_t are the initial velocity of the warhead cylindrical shell and the terminal, respectively; ρ_a and ρ_m are the density of the explosive products of the explosive and the warhead shell, respectively; σ_b is the strength of the warhead shell materials; ε_f is the failure strain of the shell materials; v_x and v_y are the radial and axial velocity of explosive products at different positions, respectively; r_f is the radius of the side-wall explosive products when the warhead shell is fully detonated.

The previous research findings have already indicated the difference between the fragment velocity distribution of central explosion and the average fragment velocity distribution of the ideal explosion considered by Gurney equations. Therefore, the side-wall average fragment velocity is calculated. Along with increase of the length-to-diameter ratio (L/D), the side-wall average fragment velocity gradually increases, but the terminal fragment velocity decreases.

Here, the L/D coefficient is defined as k ($0 < k < 1$), which is used to indicate the explosive mass proportion used to drive the side wall. Therefore, the explosive proportion used to drive the terminal can be written as $(1-k)$. Besides, the explosive used to drive the terminal is cone-shaped. The lateral view of explosive distribution is shown in Fig. 1 below:

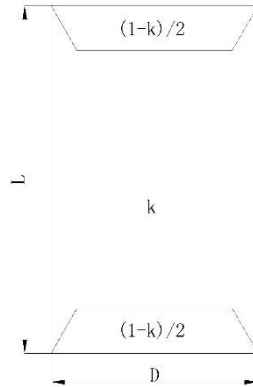


Fig 1. Schematic diagram of explosive distribution

Meanwhile, it is assumed that velocity distribution of explosive products driving the side wall features a linear distribution from the axis to the side wall; and the velocity of explosive products driving the terminal is also linearly distributed from the shell upper terminal to the bottom terminal.

Based on the above assumption, the cone can be regarded as a cylinder of equal mass and with a diameter of D . Then, the energy equation can be written as below:

$$CE_e = \frac{1}{2}M_s v_s^2 + \frac{1}{2}M_t v_t^2 + \frac{1}{2}\rho_a \int_{L-h'}^L \pi r_f^2 \left(\frac{y-L+h'}{h'} v_t \right)^2 dy + \frac{1}{2}\rho_a \int_0^{r_f} 2\pi x (L-h') \left(\frac{x}{r_f} v_s \right)^2 dx + \frac{M_s}{\rho_m}(\sigma_b \bullet \varepsilon_f) \quad (3)$$

Where, h' is the height of the equivalent cylinder.

According to the above assumption about the velocity distribution, Eq. (3) can be simplified to obtain Eq. (4) below:

$$CE_e = \frac{1}{2}M_s v_s^2 + \frac{1}{2}M_t v_t^2 + \frac{1}{6}(1-k)Cv_t^2 + \frac{1}{4}kCv_s^2 + \frac{M_s}{\rho_m}(\sigma_b \cdot \varepsilon_f) \quad (4)$$

Set $E'_e = E_e - M_b(\sigma_b \cdot \varepsilon_f)/C\rho_a$, and then:

$$CE'_e = \frac{1}{2}M_s v_s^2 + \frac{1}{2}M_t v_t^2 + \frac{1}{6}(1-k)Cv_t^2 + \frac{1}{4}kCv_s^2 \quad (5)$$

Since the warhead shell thickness is far smaller than the warhead diameter and length, the mass ratio of the warhead cylindrical shell to the terminal can be written as:

$$\frac{M_t}{M_s} = \frac{2\rho\pi tR^2}{2\rho\pi tRL} = \frac{1}{2} \frac{D}{L} \quad (6)$$

After conversion, Eq. (6) can be written as Eq. (7) below:

$$CE'_e = \frac{1}{2}M_s v_s^2 + \frac{D}{4L}M_s v_t^2 + \frac{1}{6}(1-k)Cv_t^2 + \frac{1}{4}kCv_s^2 \quad (7)$$

According to the hypothesis that cylindrical momentum and deformation of the cylindrical shell are partially propelled by the explosive of kC , then:

$$kCE_e = \frac{1}{2}M_s v_s^2 + \frac{1}{4}kCv_s^2 + \frac{M_s}{\rho_m}(\sigma_b \cdot \varepsilon_f) \quad (8)$$

$$(1-k)CE_e = \frac{1}{2}M_t v_t^2 + \frac{1}{6}(1-k)Cv_t^2 \quad (9)$$

Based on the above hypotheses and conclusions, the value of k is analyzed in two sections, namely $0.5 < L/D < 2$ and $L/D > 2$, respectively.

When $L/D > 2$, the changing trend of the side-wall fragment velocity shows that the fragment velocity changes increasingly steadily. Meanwhile, the shell deformation energy is small, so it is temporarily ignored during calculation of the value of k . Then,

$$kCE_e = \frac{1}{2}M_s v_s^2 + \frac{1}{4}kCv_s^2$$

$$v_s = \sqrt{2E_e} \sqrt{\frac{kC}{M_s + 0.5kC}} = \sqrt{2E_e} \sqrt{\frac{k\beta}{\frac{2\varphi}{1+2\varphi} + 0.5k\beta}} \quad (10)$$

Where, $\varphi = L/D$, $\beta = C/M$.

The ratio of E_{v_s} to the cylindrical shell fragment velocity v_s in Gurney equations tends to be a constant. In other words, the value of v_s/v_s is free from the influence of the ratio of the explosive mass to the mass of the cylindrical shell and terminal (C/M) or β , then:

$$\frac{v_s}{v_c} = \frac{\sqrt{2E_e} \sqrt{\frac{kC}{M_s + 0.5kC}}}{\sqrt{2E_e} \sqrt{\frac{C}{M + 0.5C}}} = \frac{\sqrt{\frac{k\beta}{1+2\varphi + 0.5k\beta}}}{\sqrt{\frac{\beta}{1+0.5\beta}}} = \sqrt{\frac{k(1+0.5\beta)}{\frac{2\varphi}{1+2\varphi} + 0.5k\beta}} \quad (11)$$

It can be observed that, when $k = \frac{2\varphi}{1+2\varphi} = \frac{M_s}{M}$, $\frac{v_s}{v_c} = 1$, and that φ tends to be infinitely large, the value of k tends to be 1.

When $0.5 < L/D < 2$, the L/D of the warhead is relatively small, and the energy generated by the explosion is different from that of the warhead with a large L/D . Besides, according to the simulated calculation results^[7], it can be noticed that the terminal velocity decreases along with increase of L/D .

$$v_i = \sqrt{2E_e} \sqrt{\frac{(1-k)C}{M_i + \frac{2(1-k)C}{3}}} = \sqrt{2E_e} \sqrt{\frac{(1-k)\beta}{\frac{1}{1+2\varphi} + \frac{2(1-k)\beta}{3}}} = \sqrt{2E_e} \sqrt{\frac{1}{\frac{1}{(1+2\varphi)(1-k)\beta} + \frac{2}{3}}} \quad (12)$$

Along with increase of the explosive L/D or φ , the value of $(1+2\varphi)(1-k)\beta$ decreases. It can also be noticed that the value of k increases along with increase of φ . Since the derivative of Eq. (11) is smaller than 0, Eq. (13) below can be obtained:

$$k \geq 1 - \frac{c}{(1+2\varphi)} \quad (13)$$

Where, c is an integration constant; the maximum of k should satisfy $k \leq 0.8$ when $\varphi = 2$, and $k \geq 0$ when L/D is close to 0.

$$\begin{cases} 0.8 \geq 1 - \frac{c}{(1+2\varphi)} & (\varphi = 2) \\ 0 \leq 1 - \frac{c}{(1+2\varphi)} & (\varphi = 0) \end{cases}$$

Therefore, the value of c should be set to 1.

2.3. Result verification

In the experiment performed by Literature [6], the inner diameter of the cylindrical shell is 110mm; its height is 160mm; and its shell thickness is 6mm. The shell is charged with 1.9g TNT. Then, the warhead explodes in a fully-closed environment, and the actually measured fragment velocity is 1,389.1m/s.

In this paper, Gurney energy of TNT is set to be 2,370m/s, and the average initial speed of the side-wall fragment and the terminal fragment of the warhead can be given by Eq. (11) and Eq. (13), namely $v_s = 1487.3\text{m/s}$ and $v_t = 1331\text{m/s}$, respectively. Considering the side-wall mass is 2.59g and the terminal mass is 1.48g, the average velocity can be obtained via mass weighing, which is 1,430.3m/s. The calculation result shows good agreement with the experimental result, in that, the former only deviates from the latter by 2.96%.

3. Influence of the warhead L/D

In order to deepen the research, the theoretical calculation model of this paper is employed to analyze how the closed cylindrical warhead, side-wall fragment and terminal fragment velocity changes with different L/D . Under different research conditions, the charge mass of the warhead and the shell thickness are constant, being 150g and 2mm, respectively. The specific parameters of the cylindrical warhead are presented in Table 1, and the warhead fragment velocity is shown in Fig. 2.

The above calculation results show a good agreement between the side-wall fragment velocity changing trend with that described in Literature [3]. When the warhead L/D is small, the side-wall fragment initial velocity increases along with the increase of L/D . When the warhead L/D increases continuously, the side-wall fragment velocity maintains at a stable value. In terms of the terminal fragment initial velocity, when the L/D is smaller than 1, the velocity increases; but, when L/D is larger than 1, the terminal fragment initial velocity decreases sharply.

Table 1. Parameters of the cylindrical warhead with different L/D

Case	L/D (φ)	β	k
1	0.500	0.81	0.53
2	0.667	0.84	0.60
3	0.750	0.84	0.63
4	1.000	0.85	0.70
5	1.333	0.84	0.76
6	1.500	0.84	0.79
7	2.000	0.81	0.84

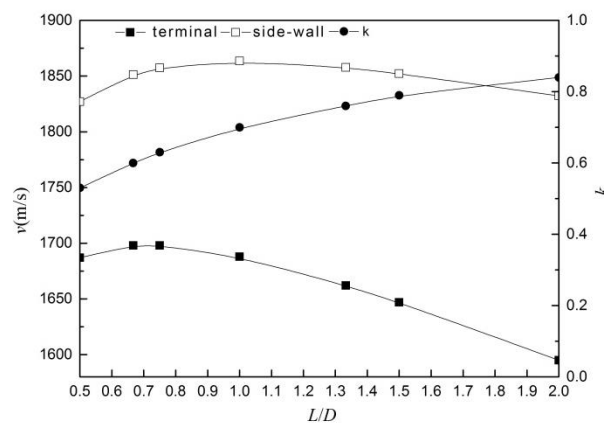


Fig 2. Fragment velocity curve of the cylindrical warhead with different L/D

4. Influence of the warhead C/M

In this section, the L/D remains constant, and the influence of different C/M on the side-wall fragment initial velocity and the terminal fragment initial velocity is examined. In different research cases, the charge mass of the warhead and the L/D of the warhead are the same, being 150g and 1, respectively. The warhead C/M is changed by adjustment of the shell thickness. The warhead parameters are presented in Table 2. The warhead fragment initial velocity is shown in Fig. 3.

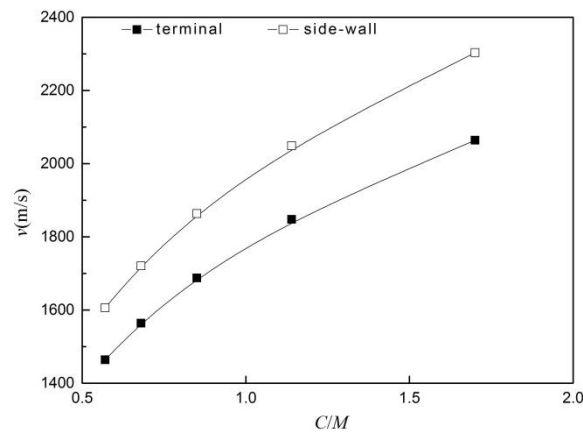


Fig. 3 Fragment initial velocity curve of the cylindrical warhead with different C/M

Table 2. Parameters of the cylindrical warhead with different C/M

Case	Shell thickness t (mm)	β	k
1	1	1.70	0.70
2	1.5	1.14	0.70
3	2	0.85	0.70
4	2.5	0.68	0.70
5	3	0.57	0.70

As one notices in Fig. 3 and Table 2, as the warhead C/M increases, the fragment initial velocity slows down its increase. This is because the increasing momentum proportion absorbed by the explosive products along with decrease of the shell thickness. Meanwhile, according to Eq. (11) and Eq. (13), the fragment initial velocity of the warhead is limited.

5. Conclusion

Based on the Gurney hypothesis, this paper studies how to calculate the fragment initial velocity of a cylindrical warhead with two terminals and exploded in the air, and how the fragment initial velocity changes with different C/M and L/D , respectively. The fragment initial velocity calculation method proposed in this paper is used to analyze the influence of the warhead L/D on the fragment initial velocity. Below is a summary of conclusions of this research:

Based on the hypothesis about the energy distribution model and proportion of the explosive in driving the side wall and the terminal, a fragment initial velocity calculation model formed after explosion of the warhead with two terminals and exploded in the air. The model can be used to calculate the terminal and side-wall fragment initial velocity. The calculation results are compared with the experimental results of the literature. The good agreement between the two verifies reasonability and accuracy of the theoretical model put forward in this paper;

When the warhead L/D is smaller than 2, along with increase of L/D , the side-wall fragment velocity increases and maintains at certain value. In terms of the terminal fragment initial velocity, when the warhead L/D is close to 1, the initial velocity is larger; but, when the L/D is larger than 1, the terminal fragment initial velocity decreases significantly;

Along with increase of the C/M , the fragment initial velocity increase gradually weakens, and the fragment initial velocity is limited.

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