

Probabilistic Research of Long Composite Cylinders under Uniform External Pressure

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Abstract. This study focuses on the load and resistance factor design (LRFD) procedure for anisotropic long composite cylinders under external pressure. The theoretical analysis is performed, the performance function is presented, the target reliability levels are determined, the statistical characteristics of random variables are researched and the LRFD-based code is calibrated. The results show the LRFD-based design procedure provides explicit and consistent reliability for long composite cylinder, which could lead less conservative design.

1. Introduction

Fibre-reinforced composite materials are increasingly used in marine application over the past few decades due to their light weight and high resistance to salt water corrosion. These materials are now being applied to composite pressure hulls [1–4], autonomous underwater vehicle [5–7], and even offshore oil equipment [8–11], which is usually subjected to high pressure of seawater. For these structures, external hydrostatic pressure-induced buckling and crushing tend to dominate structural performance. This work aims to develop load and resistance factor design (LRFD)-based design procedures for anisotropic long composite cylinder subjected to external hydrostatic pressure. The performance function is presented, and the LRFD-based code is calibrated.

2. Theoretical Analysis

Closed-form buckling and crushing solutions of anisotropic long composite cylinders are important basic tools for validating various shell theories. The structural model used to represent the long composite cylinder under study is schematically shown in Fig. 1. The geometry of the cylinder is characterized by its inner radius r and wall thickness t . For “long” cylinder, the length l is not considered due to that the buckling of thin shells with a relatively high l/r ratio is more influenced by in-plane deformations rather than the remote end boundary conditions. For the sake of simplicity, the studied cylindrical wall is composed of 4 anisotropic plies with 90° winding angle and K -4 anisotropic plies with 0° winding angle of equal thickness. The stacking sequence can be denoted by $[90_4/0_{K-4}]_T$. The corresponding critical external pressure is p_{cr} .



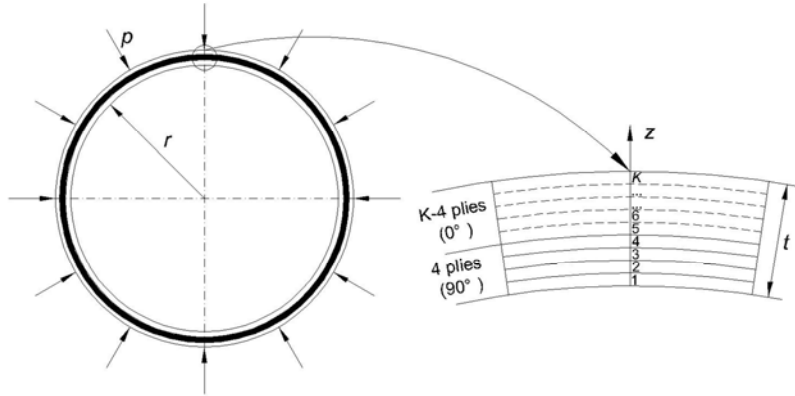


Figure 1. Structural model of the long composite cylinder under external pressure

The analytical buckling model is developed based on the classical laminate theory. The off-axis stiffness matrix of each composite layer can be expressed in term of the on-axis stiffness matrix as follows:

$$\begin{bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & m^4 + n^4 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2 - n^2)^2 \\ m^3n & -mn^3 & mn^3 - m^3n & 2(mn^3 - m^3n) \\ mn^3 & -m^3n & m^3n - mn^3 & 2(m^3n - mn^3) \end{bmatrix} \begin{bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{ss} \end{bmatrix} \quad (1)$$

Where $m = \cos \alpha$, $n = \sin \alpha$, α is the winding angle with respect to the cylinder axis, Q_{ij} ($ij=x, y, s$) are the on-axis stiffness coefficients, and Q_{ij} ($ij=1, 2, 6$) are the off-axis stiffness coefficients. The on-axis stiffness coefficients are defined in terms of material properties as follows:

$$Q_{xx} = E_L / (1 - \nu_{LT}\nu_{TL}) \quad (2)$$

$$Q_{yy} = E_T / (1 - \nu_{LT}\nu_{TL}) \quad (3)$$

$$Q_{xy} = E_L\nu_{TL} / (1 - \nu_{LT}\nu_{TL}) \quad (4)$$

$$Q_{ss} = G_{LT} \quad (5)$$

$$\nu_{LT} = E_L\nu_{TL} / E_T \quad (6)$$

Where E_L and E_T are the longitudinal and transversal elastic modulus, respectively; ν_{LT} and ν_{TL} are the longitudinal and transversal Poisson's ratios, respectively; and G_{LT} is the shear modulus.

According to the classical laminate theory, the standard rigidity expression can be obtained:

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \\ k_x \\ k_\theta \\ k_{x\theta} \end{bmatrix} \quad (7)$$

Where, N_x , N_θ and $N_{x\theta}$ are the axial, hoop, and in-plane shearing forces, respectively; M_x and M_θ are the bending moments; and $M_{x\theta}$ is the twisting moment. A_{ij} , B_{ij} , D_{ij} are the classical laminate stiffness coefficients of membrane, coupling and bending, respectively. They can be obtained using Eqs. (8), (9) and (10).

$$A_{ij} = \sum_{k=1}^K Q_{ij} t_k \quad (8)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^K Q_{ij} (z_k^2 - z_{k-1}^2) \quad (9)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^K Q_{ij} (z_k^3 - z_{k-1}^3) \quad (10)$$

Where, K is the number of different layers in the stacking sequence, t_k is the thickness of k th layer, and z_k is the through thickness position of k th layer.

For the case of an anisotropic long composite cylinder, the out-of-plane displacements are restrained (that is: $\varepsilon_x = \gamma_{x\theta} = k_x = k_{x\theta} = 0$). Therefore, only the in-plane hoop strain (ε_θ) and circumferential curvature (k_θ) are taken into considerations. The rigidity expression can be written as

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x = 0 \\ \varepsilon_\theta \\ \gamma_{x\theta} = 0 \\ k_x = 0 \\ k_\theta \\ k_{x\theta} = 0 \end{bmatrix} \quad (11)$$

Accordingly, Eq. (11) can be simply reduced to

$$\begin{bmatrix} N_\theta \\ M_\theta \end{bmatrix} = \begin{bmatrix} A_{22} & B_{22} \\ B_{22} & D_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_\theta \\ k_\theta \end{bmatrix} = \begin{bmatrix} A_{ani} & B_{ani} \\ B_{ani} & D_{ani} \end{bmatrix} \begin{bmatrix} \varepsilon_\theta \\ k_\theta \end{bmatrix} \quad (12)$$

The final closed form solution for the critical buckling pressure is given by the following mathematical expression:

$$p_{cr} = \frac{3(A_{ani}D_{ani} - B_{ani}^2)}{A_{ani}(r+t/2)^3 + 2B_{ani}(r+t/2)^2 + D_{ani}(r+t/2)} \quad (13)$$

As described previously, the winding angle and stacking sequence are assumed deterministic ($[90_4/0_{K-4}]_T$) in this work. Therefore, in the critical buckling external pressure expression of Eq. (13), there are six variables including longitudinal modulus E_L , transversal modulus E_T , shear modulus G_{LT} , Poisson's ratio ν_{LT} , inside radius r and thickness t . Hence, the critical buckling external pressure can be rewritten as

$$p_{cr} = f(E_L, E_T, G_{LT}, \nu_{LT}, r, t) \quad (14)$$

3. Load and Resistance Factor Design

3.1. Performance Function

The LRFD method consists of the requirement that a factored strength of a structural component is larger than a linear combination a factored load effects as given by

$$\phi R \geq \sum_{i=1}^n \gamma_i L_i \quad (15)$$

Where R and L are the nominal strength (resistance) and load of the structure, respectively; ϕ and γ_i are the resistance and load factors, respectively, or partial safety factors.

To develop LRFD-based long composite cylinder design procedure, the performance function which corresponds to limit state for buckling mode should be defined first. In general, a mathematical expression of the performance function for a structure is given by

$$g = R - L \quad (16)$$

The resistance and load are replaced by critical buckling external pressure p_{cr} and design external pressure p . The performance function can be written as

$$g = p_{cr} - p \quad (17)$$

Considering the model uncertainty, and substituting Eq. (14) into Eq. (17), it can be derived as follows:

$$g = X_m f(E_L, E_T, G_{LT}, \nu_{LT}, r, t) - p \quad (18)$$

Where, X_m is the variable that accounts for uncertainty in the design by the use of classical laminate theory and closed-form buckling solutions of Eq. (13).

Various factors are applied in performance function corresponding to each variable. The design equation for the long composite cylinder can be written as

$$\gamma_{Xm} X_m f(\gamma_{EL} E_L, \gamma_{ET} E_T, \gamma_G G_{LT}, \gamma_\nu \nu_{LT}, \gamma_r r, t) \geq \gamma_p p \quad (19)$$

Where, γ_{EL} , γ_{ET} , γ_{GLT} , $\gamma_{\nu LT}$, γ_r , γ_p and γ_{Xm} are the partial safety factors of longitudinal modulus, transversal modulus, shear modulus, Poisson's ratio, inside radius, design external pressure, and model uncertainty, respectively.

3.2. Target Reliability Levels

In terms of the reliability-based LRFD method, the acceptable safety levels are expressed in terms of target reliability indices. The approach that has been adopted in developing most reliability-based

codes is to specify the target reliability index β . The target reliability index determines the probability of failure for structures. The approximation of the failure probability is given by:

$$P_f = \Phi(-\beta) \quad (20)$$

Where Φ is the cumulative density function of the standard normal distribution, P_f is the probability of failure, and β is the target reliability index or safety index.

3.3. Statistical Characteristics of Random Variables

As mentioned above, this work considers the uncertainties of the following parameters: longitudinal modulus E_L , transversal modulus E_T , shear modulus G_{LT} , Poisson's ratio ν_{LT} , inside radius r , design external pressure p , and model uncertainty X_m . These parameters are treated as probabilistic random variables, with each defined by its mean, coefficient of variation (COV) and distribution type.

The uncertainties of material properties (E_L , E_T , G_{LT} , ν_{LT}) arise from the manufacturing defect. The mechanical properties and their statistical characteristics of the composite material used in the subsequent experiment are acquired through a great deal of laboratory tests, as shown in Table 1.

The uncertainties of dimension (r) arise from the geometric imperfections during filament winding process. Its statistical characteristics are also acquired through a great deal of measurements of long composite cylinders. For a design external pressure (p), an allowable deviation is acceptable. The uncertainties are fitted with a normal with a mean of 1.5 and a COV value of 0.05.

Besides the randomness of material properties, geometric imperfections and load eccentricity, the model uncertainty is also introduced, which arises from assumptions and simplification used in the derivation. In general, the statistical characteristics of model uncertainty are achieved based on the extensive data collection and data analysis. However, in the absence of sufficient and good quality data, professional expertise and engineering judgment have to be employed. In this work, the mean value of model uncertainty is assumed to be 1.09 with a COV of 0.07.

Table 1. Statistical characteristics of random variables

Variables	Unit	Means, μ	COV, C	Distribution types
E_L	GPa	135	0.10	Normal
E_T	GPa	10	0.06	Normal
G_{LT}	GPa	5	0.08	Normal
ν_{LT}	—	0.3	0.08	Normal
r	mm	21.5	0.05	Normal
p	MPa	1.5	0.05	Normal
X_m	—	1.09	0.07	Normal

3.4. Calculation of partial safety factors

The partial safety factors can be calculated by applying advanced first-order second-moment method (AFOSM), which is a convenient tool to assess the reliability of composite structure. For the given reliability index β and statistical characteristics of random variables given above, the thickness of the long composite cylinder and the partial safety factors corresponding to each target reliability level can be calculated using the AFOSM and Eq. (21).

$$\gamma_{x_i} = \frac{x_i^*}{\mu_{x_i}} \quad (21)$$

Where x_i^* ($E_L, E_T, G_{LT}, \nu_{LT}, r, p, X_m$) is the value of the variables at the design point; μ_{xi} is the mean value of the variables; and γ_{xi} is the partial safety factor.

4. Results and Sensitivity Analysis

Due to that $p=p_{cr}$ and $p=p_a$ when $\beta=0.2$ and 5.2 respectively, when the external hydrostatic pressure is 1.5MPa, the thickness of 1.36mm ($K=17$) corresponds to the critical buckling thickness, and the thickness of 1.84mm ($K=23$) corresponds to the minimum allowable thickness for engineering application.

5. Conclusion

A LRFD-based design procedure for long composite cylinder subjected to external hydrostatic pressure is presented. The results show that the LRFD-based design procedure provides explicit and consistent reliability for long composite cylinder, which could lead less conservative design.

Acknowledgments

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References

- [1] D. Graham, Composite pressure hulls for deep ocean submersibles, *Compos. Struct.* 32 (1995) 331–343.
- [2] D. Graham, Buckling of thick-section composite pressure hulls, *Compos. Struct.* 35 (1996) 5–20.
- [3] K.A. Corona-Bittick, E. Baker, G. Leon, J. Hall. Filament winding of the navy composite storage module, *SAMPE J.* 37 (2001) 52–56.
- [4] D. Jackson, M. Dixon, B. Shephard, E. Keadze, J. Lummus, M. Crews, et al., Ultra-deepwater carbon fibre composite pressure vessel development, dual element buoyancy unit (DEBU), *SAMPE J.* 43 (2007) 61–70.
- [5] C.T.F. Ross, A conceptual design of an underwater vehicle, *Ocean Eng.* 33 (2006) 2087–2104.
- [6] T.J. Ossce, T.J. Lee, Composite pressure hulls for autonomous underwater vehicles. In: *IEEE Oceans Conference Record*, Vancouver, BC, Canada, 29 September–4 October 2007. No. 4449124.
- [7] V. Carvelli, N. Panzeri, C. Poggi, Buckling strength of GFRP under-water vehicles, *Compos Part B* 32 (2001) 89–101.
- [8] B. Cai, Y. Liu, Q. Fan, Y. Zhang, Z. Liu, S. Yu, R. Ji, Multi-source information fusion based fault diagnosis of ground-source heat pump using Bayesian network, *Applied Energy*, 114 (2014) 1–9.
- [9] O.O. Ochoa, M.M. Salama, Offshore composites: Transition barriers to an enabling technology, *Compos. Sci. Technol.* 65 (2005) 2588–2596.
- [10] B. Cai, Y. B. Zhao, H. L. Liu, M. Xie, A data-driven fault diagnosis methodology in three-phase inverters for PMSM drive systems, *IEEE Transactions on Power Electronics*, 32(2017): 5590–5600.
- [11] C. Alexander, O.O. Ochoa, Extending onshore pipeline repair to offshore steel risers with carbon–fiber reinforced composites, *Compos. Struct.* 92 (2010) 499–507.