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Topological Optimization of Multi - material Structures Based on Level Set Method

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Abstract. The level set topology optimization method solved by reaction diffusion equation not only can avoid the occurrence of numerical problems such as checkerboard, but also can control the structural complexity of the optimization result through adjusting the value of regularization parameter. In this paper, the level set topology optimization method was used to study the multi-material problem, and the topological optimization mathematical model of the multi-material structure was established. Two methods of introduced element stress are studied. The typical example solved by this model has realized the reasonable layout of multi-material.

1. Introduction

In general, the research of single material structure by topology optimization method is relatively common, but the optimization of different material layouts under multi-material structure optimization has higher practical engineering application significance. Based on the concept of material distribution, Thomsen [1] studied the topology optimization of multiphase materials firstly, and the article immediately attracted the attention of the majority of scholars. Bendsoe and Sigmund [2] extend the SIMP method to solve the material interpolation model with two-phase materials and voids, and proposed a method of sensitivity filtering to overcome the possible chessboard phenomenon in the optimization process. Besides, they applied this method successfully to the dual material topology optimization problem of compliant institutions. The interfacial properties of the material were improved and the effect of the interfacial properties on the structural optimization of the multiphase elastic and thermo elastic materials were studied by Allaire et al. [3]. In summary, the research on multi-material topology optimization has been paid more and more attention, but the topology optimization solved by reaction diffusion equation has little research on multi-material problem.

In this paper, the method of multi-material structure topology optimization based on the level set method is studied, which a mathematical model that the structural stiffness maximization was regard as the objective function, the element stress as the main constraint and the layout stability of material as the convergence condition is established. Finally, two numerical examples were given to demonstrate the effectiveness of the proposed method.

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2. The construction of Multi - material Structure Optimization Mathematical Model

2.1. The basic theory of level set topology optimization method

In the level set topology optimization method, the level set function is updated by solving the reaction diffusion equation that containing the topological derivative of the objective function. The changes of structural topology and new boundary generation are allowed in the optimization process. The level set function is shown in Figure 1.

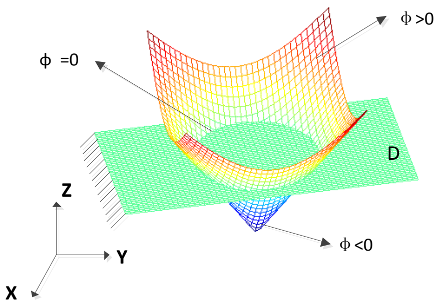

Figure 1 The illustration of level set function.

The boundary of the structure in the topology optimization problem is implicitly expressed by the value of the level set function. The specific expression is as shown in equation (1).

$$\begin{cases} 1 \geq \phi(x) > 0 & \forall x \in \Omega \setminus \partial\Omega \\ \phi(x) = 0 & \forall x \in \partial\Omega \\ 0 > \phi(x) \geq 1 & \forall x \in D \setminus \Omega \end{cases} \tag{1}$$

Where, $\phi(x) > 0$ is the material area, $\phi(x) = 0$ indicating the structural boundary, $\phi(x) < 0$ is the hole area, the level set function has the upper and lower boundaries.

The constrained optimization problem can be transformed into an unconstrained optimization problem by applying the Lagrangian multiplier method, and then the defined level set function was introduced into the optimization problem. Finally, the mathematical model of the final topology optimization was obtained [4].

$$\begin{cases} T\Phi(t+\Delta t) = Y \\ \phi = 0 \quad on\ \partial D \end{cases} \qquad \begin{aligned} T &= \bigcup_{e=1}^{N} \int_{V_e} \left(\frac{1}{\Delta t} N^T N + \nabla^T N \tau \nabla N \right) dV_e \\ Y &= \bigcup_{e=1}^{N} \int_{V_e} \left(C d_t \overline{F} + \frac{\phi(x,t)}{\Delta t} \right) N dV_e \end{aligned} \tag{2}$$

2.2. The construction of Multi - material Structure Optimization Model under Stress Constraint

According to the distribution of the element stress in design area, the stress range of each element material is determined, and the material with the appropriate properties is selected to be filled. From that materials of different performance can be added to the design area. According to the different focus of the optimization goal, two kinds of element stress introduction methods were adopted in this paper. One is that the stress of the element is calculated and the properties of the element material are distributed in each iteration. From that, the structural element layout of each step is affected by the previous step. The using of expensive material with a larger elasticity modulus is small and the cost of the material is reduced by this method.

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Another way is to use only one material at the beginning of the structural optimization. The calculation of the element stress and the distribution of the element material properties are continued when the structural layout of the design area is stable. Then the iteration is continued until all the material distribution is stable. Only the structural shape of the latter stage will be affected in the way of stress introduction, and the optimized result is simple and easy to manufacture.

According to the finite element theory, the element stress calculation formula as shown in equation (3).

$$\sigma = BDu \tag{3}$$

Where, B represents the geometric matrix of the element, D is the elastic matrix of the element, the specific expression is shown in equation (4).

$$D_K = \frac{E_K}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1\ \ \mu}{2} \end{bmatrix} \tag{4}$$

From the equation (4), the stress of the element will also change when the elastic modulus of the element changed. Therefore, each element stress should calculate separately. Since the elastic matrix corresponding to each element is a 3×3 matrix, the elastic modulus matrix should be deformed in order to make the elastic modulus of each element corresponding to the elastic matrix.

$$\begin{bmatrix} E_k & E_k & E_k & & & & & & & \\ E_k & E_k & E_k & . & . & . & . & . & . & . \\ E_k & E_k & E_k & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \end{bmatrix} \qquad \frac{1}{1\ \ \mu^2}\times\begin{bmatrix} 1 & \mu & 0 & & & & & & & \\ \mu & 1 & 0 & . & . & . & . & . & . & . \\ 0 & 0 & \frac{1\ \ \mu}{2} & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \\ & . & & & & & & & & \end{bmatrix} \tag{5}$$

Where, D_k represents the elastic matrix of the Kth element, E_k represents the element in the elastic modulus matrix of the Kth element, and μ is the Poisson's ratio of the material. The elastic modulus matrix of each element is a 3×3 matrix and all the elements in the matrix are same, then the elastic modulus of the element can be calculated by the formula (6).

$$E_{\mathrm{ek}} = \frac{\sum_{\mathrm{i}=1}^{N} E_{K,i}}{N} \tag{6}$$

Where, E_{ek} represents the elasticity modulus of the Kth element and $E_{K,i}$ represents ith value in the elastic modulus matrix. Three kinds of materials are taken as an example.

$$\begin{cases} if \quad \sigma_K \in [\sigma_a, \sigma_b) \quad E_{e1} = \begin{bmatrix} E_1 & E_1 & E_1 \\ E_1 & E_1 & E_1 \\ E_1 & E_1 & E_1 \end{bmatrix} \\ if \quad \sigma_K \in [\sigma_b, \sigma_c) \quad E_{e2} = \begin{bmatrix} E_2 & E_2 & E_2 \\ E_2 & E_2 & E_2 \\ E_2 & E_2 & E_2 \end{bmatrix} \\ if \quad \sigma_K \in [\sigma_c, \sigma_d] \quad E_{e3} = \begin{bmatrix} E_3 & E_3 & E_3 \\ E_3 & E_3 & E_3 \\ E_3 & E_3 & E_3 \end{bmatrix} \end{cases} \tag{7}$$

In order to improve the efficiency of solving, only the element in the material design area is calculated for the unit stress. For the hole element, the element stress is directly set to a minimized value. The above treatment not only improves the efficiency of the calculation, but also facilitates the post-processing operation.

3. Numerical example and result analysis

3.1. The first Example of Multi - material Structures Topology Optimization

The validity of the method is verified by two typical examples. Assuming that there are three materials in the design area, the elasticity modulus are: $E^1 = 1.05\times10^{11}$pa ; $E^2 = 1.5\times10^{11}$pa ; $E^3 = 2.1\times10^{11}$pa ; $E_{\min} = 1\times10^{4}\ pa$. The example was calculated by using the method described in the previous section. A schematic diagram of the example is shown in Figure 2.

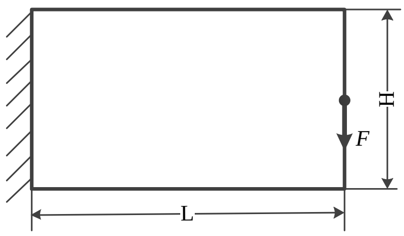
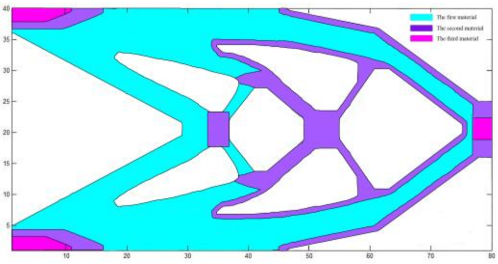


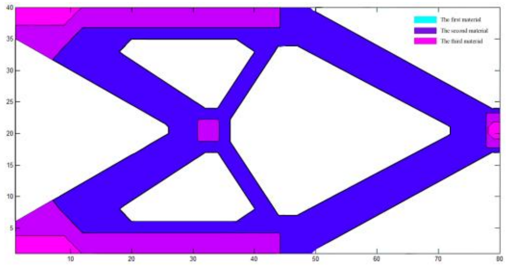
Figure 2 Schematic diagram of the example

Where, the length of structure is $L = 80\text{mm}$, width $H = 40\text{mm}$, thickness $t = 1\text{mm}$. The Poisson's ratio of the material is $\nu = 0.3$, the volume fraction of the material is 0.45. The material which the value of elasticity modulus is $E^1 = 1.05\times10^{11} pa$ was the initial structural material. The left end of the structure is constrained, and the vertical load $F = 4000\text{N}$ is concentrated in the middle position on the right side. The stress range of the element material is: $0\,pa < \sigma^1 \le 9\times10^8\ pa$; $9\times10^8\ pa < \sigma^2 \le 1.4\times10^9\ pa$; $1.4\times10^9\ pa < \sigma^2 \le 4\times10^9\ pa$.

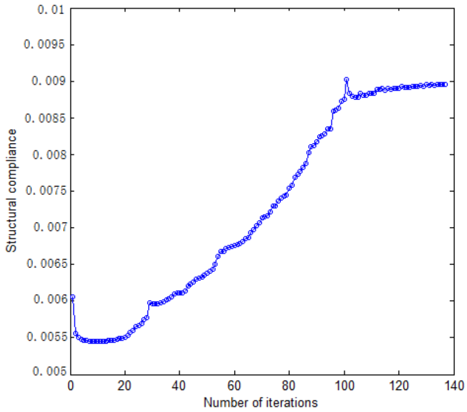
In this example, the stress in hole cell area is set to -1e-3. The 1st material is used to fill when the element stress is less than $9\times10^8\ pa$; the 2nd material is used to fill when the element stress is greater than $9\times10^8\ pa$ and less than $1.4\times10^9\ pa$; the 3rd material is used to fill when the element stress is greater than $1.4\times10^9\ pa$. In order to facilitate the subsequent processing and manufacturing, the obtained topological optimization results are properly processed. The final result graph and the structural compliance curve with the iteration are shown in Figure 3.



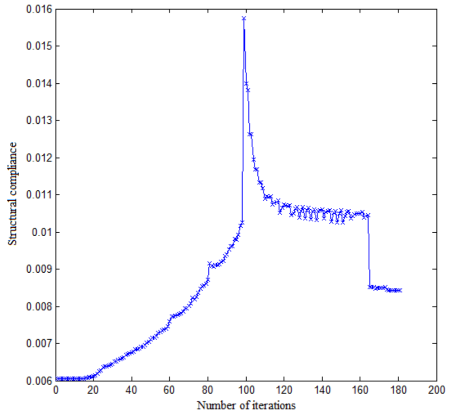
(a) Result of 1st stress introduced method



（b）Result of 2nd stress introduced method



(c) Curve of 1st stress introduced method



（d）Curve of 2nd stress introduced method

Figure 3 The topology optimization result of the first example

The total structural compliance of the 1st stress introduction method is: 8.9503e-3(N.m). The proportion of 1st material is 0.667; the proportion of 2nd material is 0.273; the ratio of 3rd material is: 0.060. The total compliance of the 2nd stress introduction method is that the proportion of the 1st material is 0.602, the proportion of the 2nd material is 0.353, and the ratio of the 3rd materials is 0.045.

3.2. The second Example of Multi - material Structures Topology Optimization

The schematic diagram of the second example is shown in figure 4

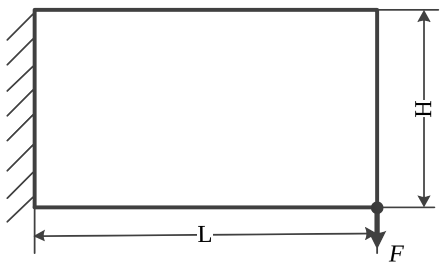
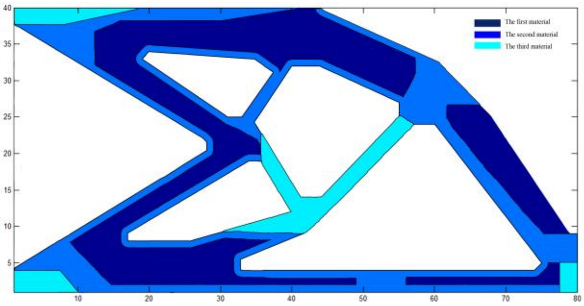
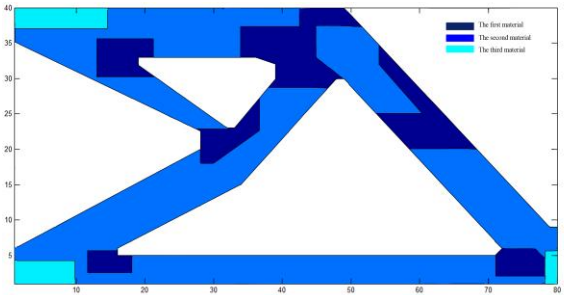


Figure 4 The schematic diagram of the second example

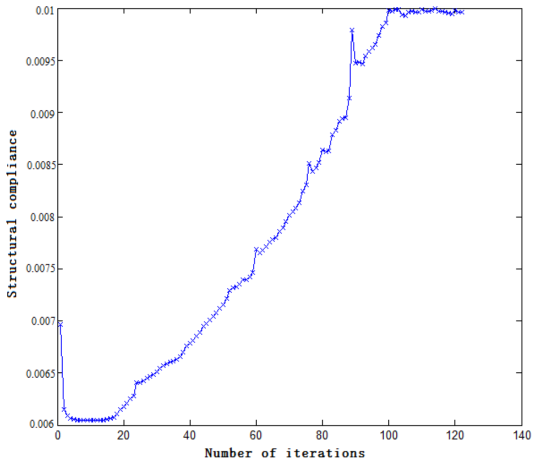
Where, the length of structure $L = 80\text{mm}$; width $H = 40\text{mm}$; the Poisson's ratio of the material is $\nu = 0.3$; the volume fraction of the material is 0.4; the value of vertical concentrated load is $F = 4000\text{N}$. The stress range and the elasticity modulus is same with the first example. In order to facilitate the subsequent processing and manufacturing, the obtained topological optimization results are properly processed. The final result graph and the structural compliance curve with the iteration are shown in Figure 5.



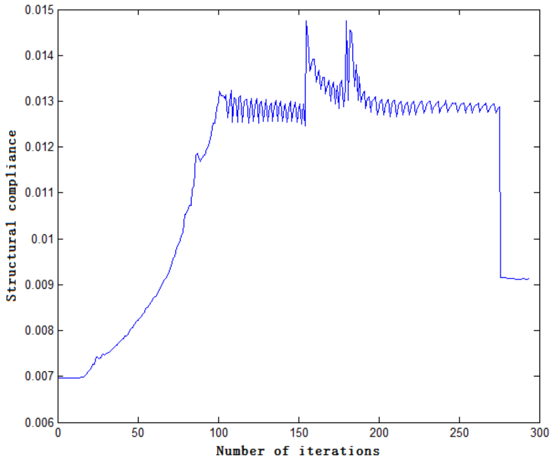
(a) Result of 1st stress introduced method



(b) Result of 2nd stress introduced method



(c) Curve of 1st stress introduced method



(c) Curve of 2nd stress introduced method

Figure 5 The topology optimization result of the second example

The total structural compliance of the 1st stress introduction method is: :$1.0071e-002(N.m)$. The proportion of 1st material is 0.440; the proportion of 2nd material is 0.447; the ratio of 3rd material is: 0.113. The total compliance of the 2nd stress introduction method is $9.0433e-003(N.m)$ that the proportion of the 1st material is 0.237, the proportion of the 2nd material is 0.697, and the ratio of the 3rd materials is 0.066.

It can be concluded that although the volume constraints are the same in the two examples, but the topological results are also significant differences when different element stress introduction method was used. The topology optimization structure is more complicated, but the 1st material is used in a large amount, and the other two materials are used less when using the 1st stress introduction method, which reduces the cost of using the material. The topology optimization structure is simple and easy to manufacture when the 2nd stress introduction methods are adopted.

Compared with the result solved by two methods, the structural compliance obtained by the 1st element stress introduction method is gentle with the iteration, and no large fluctuation is observed. The curve of the structural compliance obtained by the 2nd stress introduction method change drastically since the original support structure in the structure is deleted. Finally, the iterative curve of two kinds of introduced stress tends to be stable and convergence stably.

4. Conclusion

In this paper, the level set topology optimization method solved by reaction diffusion equation is used to study the multi - material problem. The corresponding mathematical model is established. A distribution method of material properties based on element stress is introduced, and the defined method of elastic modulus is established. In the process of topology optimization, the performance of the material is fully utilized, the cost of the material is reduced, the stable topology optimization results are obtained, and the following conclusions are obtained:

(1) In this paper, a multi-material topology optimization model with element stress as the main constraint and element material layout stability as the convergence condition is established. A stable structure is obtained according to the method of element stress distribution.

(2) Based on the calculation of the element stress in the process of topology optimization, the calculation method of the elastic modulus is established, and the dynamic change of the material properties in the optimization process is realized.

(3) In this paper, two methods of introduced element stress are studied. Through the calculation and analysis of the engineering examples, the differences of the topological optimization results obtained by the two methods are analyzed, and the stable topology optimization results are obtained.

The research of this paper can provide reference for the scholars in this field, and apply the level set topology optimization solved by reaction diffusion equation to the practical basis.

References

[1] Thomsen J. Topology optimization of structures composed of one or two materials [J]. Structural optimization, 1992, 5(1-2):108-115.

[2] Sigmund O. Design of multi-physics actuators using topology optimization-part II: Two-material structures [J]. Computer methods in applied mechanics and engineering, 2001, 190(49):6605-6627.

[3] Allaire G, Dapogny C, Michailidis G D G. Multi-phase structural optimization via a, level set method. Esaim Control Optimization & Calculus of Variations, 2014, 20(2):576-611P.

[4] M Otomori, T Yamada, K Izui, S Nishiwaki. Matlab code for a level set-based topology optimization method using a reaction diffusion equation [J]. Structural and Multidisciplinary Optimization, 2015,51(5): 1-14.