

# Single-tier city logistics model for single product

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**Abstract.** This research develops single-tier city logistics model which consists of suppliers, UCCs, and retailers. The problem that will be answered in this research is how to determine the location of UCCs, to allocate retailers to opened UCCs, to assign suppliers to opened UCCs, to control inventory in the three entities involved, and to determine the route of the vehicles from opened UCCs to retailers. This model has never been developed before. All the decisions will be simultaneously optimized. Characteristic of the demand is probabilistic following a normal distribution, and the number of product is single.

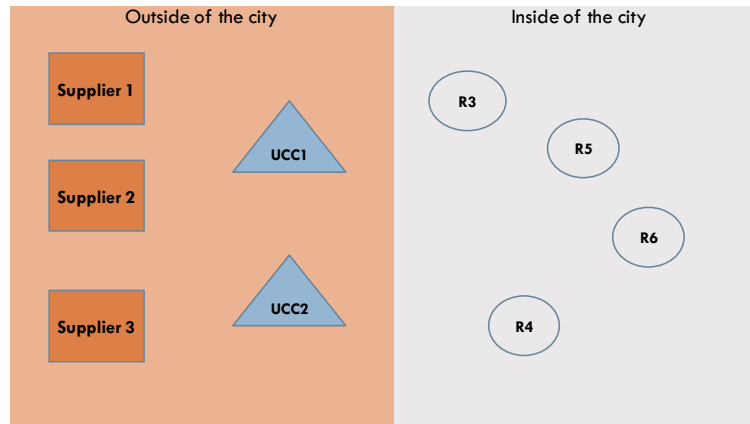
## 1. Introduction

This research continues the research of [1] to develop mathematical model for single-tier city logistics system. Single-tier city logistics system consists of three entities, that is, point of supplies, logistics facilities, and point of demands [2]. The location of point of supplies and logistics facilities is outside of the city and the location of point of demands is inside of the city. In this research, point of supplies are then called suppliers, logistics facilities are called UCCs (urban consolidation centers), and point of demands are called retailers. The problems that will be answered in this research are how to determine the location of UCCs, to allocate retailers to opened UCCs, to assign suppliers to opened UCCs, to control inventory in the three entities involved, and to determine the route of the vehicles from opened UCCs to retailers. All the decisions will be simultaneously optimized for city logistics is the process for totally optimizing the logistics and transport activities [3]. City logistics model that can answer the described problems simultaneously in single-tier city logistics system has never been developed before [4]. Characteristics of the demand is the probabilistic one, following a normal distribution, and the number of product is single.

## 2. Methodology

This research used [5] as reference model to develop the single-tier city logistics model. The entities involved in [5] were single supplier, multi depot, and multi retailers, while the entities in single-tier city logistics model that would be developed in this research were multi suppliers, multi UCCs, and multi retailers. The illustration of entities involved in this research can be seen at Figure 1.





**Figure 1.** Illustration of entities involved in single-tier city logistics system.

### 3. Mathematical model

Mathematical model for single-tier city logistics model for single product that will be developed in this research is described as follows.

#### 3.1. Index sets

|       |                                                               |
|-------|---------------------------------------------------------------|
| $K$   | set of retailers                                              |
| $J$   | set of potential UCCs                                         |
| $N_j$ | set of capacity levels available to UCC ( $j \in J$ )         |
| $I$   | set of suppliers                                              |
| $V$   | set of vehicles                                               |
| $M$   | merged set of retailers and potential UCCs, i.e. $(K \cup J)$ |

#### 3.2. Parameters and notations

At retailer

|              |                                                                            |
|--------------|----------------------------------------------------------------------------|
| $\mu_k$      | mean of demand at retailer $k$ (Unit/day) ( $\forall k \in K$ )            |
| $\sigma_k^2$ | variance of demand at retailer $k$ (Unit/day) ( $\forall k \in K$ )        |
| $h_k$        | inventory holding cost at retailer $k$ (Rp/unit/day) ( $\forall k \in K$ ) |
| $a_k$        | ordering cost at retailer $k$ (Rp/order) ( $\forall k \in K$ )             |
| $lt_k$       | lead time of retailer $k$ (Day) ( $\forall k \in K$ )                      |
| $s_k$        | shortage cost at retailer $k$ (Rp/unit) ( $\forall k \in K$ )              |
| $\alpha$     | fill rate                                                                  |
| $z_\alpha$   | left $\alpha$ -percentile of standard normal random variable $z$           |

At UCC

|          |                                                                                                                        |
|----------|------------------------------------------------------------------------------------------------------------------------|
| $f_j^n$  | fixed cost for opening and operating UCC $j$ with capacity level $n$ (Rp/day) ( $\forall j \in J, \forall n \in N_j$ ) |
| $b_j^n$  | capacity with level $n$ for UCC $j$ (Unit/day) ( $\forall j \in J, \forall n \in N_j$ )                                |
| $d_{kl}$ | transportation cost between node $k$ and node $l$ (Rp/delivery) ( $\forall k, l \in M$ )                               |
| $h_j$    | inventory holding cost at UCC $j$ (Rp/unit/day) ( $\forall j \in J$ )                                                  |

$a_j$  ordering cost at UCC $j$  (Rp/order) ( $\forall j \in J$ )

$lt_j$  lead time of UCC $j$  (Day) ( $\forall j \in J$ )

$vc$  capacity of vehicle (Unit)

At supplier

$h_i$  inventory holding cost at supplier  $i$  (Rp/unit/day) ( $\forall i \in I$ )

$a_i$  ordering cost at supplier  $i$  (Rp/order) ( $\forall i \in I$ )

$lt_i$  lead time of supplier  $i$  (Day) ( $\forall i \in I$ )

$b_i$  capacity for supplier  $i$  (Unit/day) ( $\forall i \in I$ )

At system

$w$  transportation cost of truck (Rp/truck)

$pp$  capacity of truck (Unit)

$B$  number of retailers contained in set  $K$ , i.e.  $B = |K|$

### 3.3. Decision variables

At retailer

$NP_k$  order frequency of retailer  $k$  ( $\forall k \in K$ )

$E$  order frequency of every retailer

$Q_k$  lot size of retailer  $k$  (Unit) ( $\forall k \in K$ )

$MK_k$  number of shortage at retailer  $k$  (Unit) ( $\forall k \in K$ )

$RK_k$  reorder point at retailer  $k$  (Unit) ( $\forall k \in K$ )

$SS_k$  safety stock at retailer  $k$  (Unit) ( $\forall k \in K$ )

At UCC

$U_j^n$  1 if UCC $j$  is opened with capacity level  $n$ , 0 if otherwise ( $\forall j \in J, \forall n \in N_j$ )

$D_j$  demand of UCC  $j$  (Unit/day) ( $\forall j \in J$ )

$Y_{jk}$  1 if retailer  $k$  is assigned to UCC  $j$ , 0 if otherwise ( $\forall j \in J, \forall k \in K$ )

$NP_j$  order frequency of UCC $j$  ( $\forall j \in J$ )

$Z$  order frequency of every UCC

$Q_j$  lot size of UCC $j$  (Unit) ( $\forall j \in J$ )

$RK_j$  reorder point at UCC $j$  (Unit) ( $\forall j \in J$ )

$R_{klv}$  1 if  $k$  precedes  $l$  in route of vehicle  $v$ , 0 if otherwise ( $\forall k, l \in M, \forall v \in V$ )

$M_{kv}$  auxiliary variable defined for retailer  $k$  for subtour elimination in route of vehicle  $v$  ( $\forall k \in K, \forall v \in V$ )

$X_j$  number of truck at UCC $j$  (Truck) ( $\forall j \in J$ )

At supplier

- $D_i$  demand of supplier (Unit/day) ( $\forall i \in I$ )  
 $Q_i$  lot size of supplier  $i$  (Unit) ( $\forall i \in I$ )  
 $RK_i$  reorder point at supplier  $i$  (Unit) ( $\forall i \in I$ )  
 $X_i$  number of truck at supplier  $i$  (Truck) ( $\forall i \in I$ )  
 $G_{ij}$  1 if supplier  $i$  supplies UCC $j$ , 0 if otherwise ( $\forall i \in I, \forall j \in J$ )  
 $V_{ij}$  amount of demand of UCC $j$  supplied by supplier  $i$  ( $\forall i \in I, \forall j \in J$ )

At system

- $TC$  total cost (Rp/day)  
 $T$  single cycle time (Day)

### 3.4. The model

Mathematical model of single-tier city logistics model for single product is given below.

Objective function:

$$\begin{aligned} \min TC = & \sum_{j \in J} \sum_{n \in N_j} f_j^n U_j^n + \frac{E}{T} \sum_{v \in V} \sum_{k \in M} \sum_{l \in M} d_{kl} R_{klv} + \sum_{k \in K} \left[ \frac{a_k \mu_k}{Q_k} + h_k \left( \frac{Q_k}{2} + SS_k \right) + s_k MK_k \left( \frac{\mu_k}{Q_k} \right) \right] + \\ & \sum_{j \in J} \left[ \frac{a_j D_j}{Q_j} + h_j \left( \frac{Q_j}{2} + \sum_k (lt_k \mu_k + SS_k) Y_{jk} \right) + w X_j \frac{Z}{T} \right] + \\ & \sum_{i \in I} \left[ \frac{a_i D_i}{Q_i} + h_i \left( \frac{Q_i}{2} + \left( \sum_j \sum_k ((lt_j + lt_k) \mu_k + SS_k) Y_{jk} \right) G_{ij} \right) + w X_i \frac{1}{T} \right] \end{aligned} \quad (1)$$

Subject to:

$$\sum_{v \in V} \sum_{l \in M} R_{klv} = 1, \forall k \in K \quad (2)$$

$$\sum_{l \in K} \sum_{k \in M} Q_l R_{klv} \leq vc, \forall v \in V \quad (3)$$

$$M_{kv} - M_{lv} + (B \times R_{klv}) \leq B - 1, \forall k, l \in K, \forall v \in V \quad (4)$$

$$\sum_{l \in M} R_{klv} - \sum_{l \in M} R_{lkv} = 0, \forall k \in M, \forall v \in V \quad (5)$$

$$\sum_{j \in J} \sum_{k \in K} R_{jkv} \leq 1, \forall v \in V \quad (6)$$

$$\sum_{l \in M} R_{klv} + \sum_{l \in M} R_{jlv} - Y_{jk} \leq 1, \forall j \in J, \forall k \in K, \forall v \in V \quad (7)$$

$$\sum_{n \in N_j} U_j^n \leq 1, \forall j \in J \quad (8)$$

$$\sum_{k \in K} \mu_k Y_{jk} \leq \sum_{n \in N_j} b_j^n U_j^n, \forall j \in J \quad (9)$$

$$\sum_{k \in K} \mu_k Y_{jk} = \sum_{n \in N_j} D_j U_j^n, \forall j \in J \quad (10)$$

$$\sum_{j \in J} Y_{jk} = 1, \forall k \in K \quad (11)$$

$$\sum_{j \in J} G_{ij} \geq 1, \forall i \in I \quad (12)$$

$$\sum_{n \in N_j} D_j U_j^n = \sum_{i \in I} V_{ij} G_{ij}, \forall j \in J \quad (13)$$

$$\sum_{j \in J} V_{ij} = D_i, \forall i \in I \quad (14)$$

$$\sum_{j \in J} V_{ij} \leq b_i, \forall i \in I \quad (15)$$

$$SS_k = z_\alpha \sqrt{lt_k \sigma_k^2}, \forall k \in K \quad (16)$$

$$MK_k = \sqrt{lt_k \sigma_k^2} [f(z_\alpha) - z_\alpha \psi(z_\alpha)], \forall k \in K \quad (17)$$

$$RK_k = lt_k \mu_k + SS_k, \forall k \in K \quad (18)$$

$$RK_j = \left( \sum_k (lt_j + lt_k) \mu_k + SS_k \right) Y_{jk}, \forall j \in J \quad (19)$$

$$RK_i = \left( \sum_k lt_i \mu_k + \left( \sum_j \sum_k (lt_j + lt_k) \mu_k + SS_k \right) \right) Y_{jk}, \forall i \in I \quad (20)$$

$$T = \frac{Q_i}{D_i} = \frac{NP_j Q_j}{D_j} = \frac{NP_j NP_k Q_k}{\mu_k} \quad (21)$$

$$NP_k = E, \forall k \in K \quad (22)$$

$$NP_j = Z, \forall j \in J \quad (23)$$

$$X_j = \left\lceil \frac{Q_j}{pp} \right\rceil, \forall j \in J \quad (24)$$

$$X_i = \left\lceil \frac{Q_i}{pp} \right\rceil, \forall i \in I \quad (25)$$

$$U_j^n \in \{0,1\}, \forall j \in J, \forall n \in N_j \quad (26)$$

$$Y_{jk} \in \{0,1\}, \forall j \in J, \forall k \in K \quad (27)$$

$$R_{klv} \in \{0,1\}, \forall k, l \in M, \forall v \in V \quad (28)$$

$$G_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \quad (29)$$

$$M_{kv} \geq 0, \forall k \in K, \forall v \in V \quad (30)$$

$$Q_i, Q_j, Q_k \geq 0, \forall i \in I, \forall j \in J, \forall k \in K \quad (31)$$

$$T > 0 \quad (32)$$

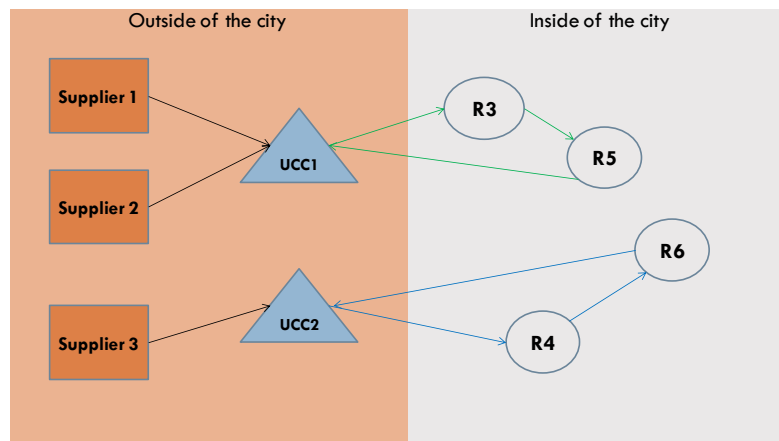
$$N, N_j, N_k \geq 1, N, N_j, N_k \in \text{int}, \forall j \in J, \forall k \in K \quad (33)$$

The difference between single-tier city logistics model that has been developed with the model of [5] is at constraints (12)–(15) where constraint (12) ensures that a supplier can supply more than one UCCs, constraint (13) ensures that demand at opened UCC must be fulfilled, constraint (14) ensures that demand at supplier is sum of the amount of demand of UCCs supplied by that supplier, and

constraint (15) ensures that amount of demand of UCCs supplied by a supplier does not exceed its capacity. The explanation of other constraints can be seen at [5].

#### 4. Numerical example

The model was then solved with Lingo 12.0 for some numerical examples. The data used were generated. Figure 2 gives the illustration for numerical example of 3 suppliers, 2 UCCs, and 4 retailers.



**Figure 2.** Illustration for numerical example of 3 suppliers, 2 UCCs, and 4 retailers.

From Figure 2 it was found that both UCC1 and UCC2 were opened. UCC1 was supplied by Supplier1 and supplier2 in accordance with UCC1's lot size. UCC2 was supplied by Supplier1 also in accordance with UCC2's lot size. UCC1 supplied R3 and R5 in accordance with their lot size and to serve R3 and R5 there was a route from UCC1, that is, UCC1-R3-R5-UCC1. UCC2 supplied R4 and R6 also in accordance with their lot size. To serve R3 and R6, there was also a route that started from UCC2, that is, UCC2-R4-R6-UCC2. All decisions were simultaneously optimized.

#### 5. Conclusion

This research has developed single-tier city logistics model for single product which has never been developed before. All the decisions are simultaneously optimized. The decisions are location-allocation of UCCs retailers, assignment of suppliers that supply opened UCCs, inventory control at all entities involved, and vehicle routes to serve retailers. This research also gives challenges for future studies to develop heuristic method to solve the model in large scale for the model itself belongs to NP-hard problems [6]. Considering multi product also can be a challenge for future studies.

#### 6. References

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