

Inventory model optimization for supplier-manufacturer-retailer system with rework and waste disposal

A R Dwicahyani, E Kholisoh, W A Jauhari, C N Rosyidi and P W Laksono

Department of Industrial Engineering, Sebelas Market University, Surakarta, Indonesia

E-mail: anindyard94@gmail.com

Abstract. This study developed a model for a CLSC inventory system which consisted of a supplier, a manufacturer and a retailer. We applied single remanufacturing cycle and multiple manufacturing cycle, $(1,P)$, policy and performed a comparison to the previous study. We conducted an investigation of an imperfect manufacturing process whose defective items were being reworked. Other considerations were quality dependent return rate and waste disposal activity, for any returned items that did not exceed the acceptable quality level. Some parameters including demand, proportion of defect, waste and refurbished item were assumed deterministic and constant. We proposed a solution procedure and presented a numerical example to illustrate the application of the model. The results showed that $(1,P)$ policy allowed higher profit for the system than $(R,1)$ policy presented in the previous study.

Keywords: Inventory model, reverse logistics, closed loop supply chain, waste disposal, imperfect production, rework, quality dependent return rate

1. Introduction

Managing end-of-life (EoL) products returned from the market is a key to succeed a sustainable supply chain [1]. In addition, it also guarantees some benefits including reducing costs, maintaining customer loyalty, improving customer services and increasing market share [2, 3]. According to [4], activities related to EoL product management may include collection, inspection, cleaning, disassembly or product recovery that in the end will be resold to the market. Many manufacturing companies such as electronic products and automotive parts [5, 6], have broadly applied the concept of EoL product management. According to [7], since it has never been an easy way to manage EoL products from the market, some typical industries should do alliance between firms that later be called joint ventures collaboration. A supply chain that implements the concept of return item management is then classified as closed-loop-supply-chain (CLSC). By performing joint venture collaboration, some operations of the EoL product management are being performed by the help of certain firms in the joint venture [8]. Therefore, costs can be reduced and inventories within the supply chain can be managed more efficiently.

The model developed in this paper is an inventory model that aims to maximize joint total profit of all firms that collaborate in the supply chain. Formerly, some studies have initiated the development of inventory model in the system with product returns, including [9–11]. Some researchers have also extended the model by including some aspects such as collaborations, stochastic demand and returns,



dependent return rate, imperfect production and reworks [12–16]. Yuan and Gao [14] developed a cost minimization model of the CLSC inventory system and determined the optimal solutions for each party. The model was an extended study of [21], in which they investigated both $(R,1)$ and $(1,P)$ policies. The decision to choose the right number of manufacturing and remanufacturing cycles is surely a challenging effort in the system of CLSC, since each operation is performed sequentially due to the lack of production equipment and machinery. Teunter [15], [14] and [16] are some studies that investigated multiple cycle policy of remanufacturing and manufacturing operations in the system of reversed logistics.

However, most previous studies have not yet considered the collaboration in the supply chain and imperfect production process all at once. In this model, we aim to develop a model of CLSC inventory system that considers the imperfect manufacturing process, reworks, quality dependent return rate, as well as waste disposal activity. The model is intended to determine the optimal solution, including number of manufacturing cycle, number of shipment between parties, as well as retailer ordering cycle time, that maximizes joint total profit for all parties. Here, we also give a comparison of $(R,1)$ and $(1,P)$ policies to help the managers in choosing the most profitable and efficient policy of manufacturing and remanufacturing.

2. Notations

Some parameters used in the model development are denoted by these following notations.

D : annual demand rate (units/year);	P_c : retail price (\$/unit);
A_r : ordering cost for retailer (\$/order);	P_r : wholesale price (\$/unit);
F_r : holding cost fraction for retailer's stock;	A_m : manufacturing set up cost (\$/batch);
A_R : remanufacturing set up cost (\$/batch);	N : total number of shipment per T ;
A_{mw} : raw material ordering cost (\$/order);	M : fixed cost to process all demand (\$);
F_M : holding cost fraction for serviceable stock;	r : remanufacturing rate ($r > D$, unit/year);
F_{mw} : holding cost fraction for raw material stock;	β : multiplying factor for p ($0 < \beta < 1$);
F_{rw} : holding cost fraction for recoverable stock;	γ : multiplying factor for r rate ($0 < \gamma < 1$);
P_{rm} : raw material purchasing price (\$/unit);	f : refurbishable item proportion ($0 < f < 1$);
P_u : used item purchasing price (\$/unit);	q : recoverable item proportion ($0 < q < 1$);
P_f : refurbished item selling price (\$/unit);	k : defective item proportion ($0 < k < 1$);
T : planning period of the model (year)	C_d : defective item holding cost (\$/unit);
T_R : remanufacturing cycle time (year);	C : annual return rate (unit/year);
T_P : manufacturing cycle time (year);	C_p : cost to manufacture (\$/unit);
T_w : setup time for reworking process (year);	C_R : cost to remanufacture (\$/unit);
T_{P2} : reworking process period (year);	C_f : cost to refurbish (\$/unit);
P : number of manufacturing cycle per T ;	C_w : cost of waste disposal (\$/unit);
C_{iscp} : cost to inspect and sort (manufactured items) (\$/unit);	C_{isc} : cost to inspect and sort (return items) (\$/unit);
A_s : supplier fixed cost for each order (\$/order);	P_s : supplier purchasing cost (\$/unit);
F_s : holding cost fraction ($0 < F_s < 1$);	L : fixed cost to process all orders (\$);

3. Problem description, mathematical model and solution procedure

The development of the model began by illustrating the relevant system. The proposed model concerns with a closed-loop-supply-chain (CLSC) inventory system considering three parties: a retailer, a manufacturer, and a supplier. This study develops the model belongs to [17] to a single-remanufacturing-multiple-manufacturing cycle policy $(1,P)$ model. The results from this model are then compared to the results from [17]. The CLSC inventory system is depicted in Figure 1.

In the system, manufacturer delivers products to retailer in n shipments during manufacturing process and m shipments during remanufacturing process. Manufacturer orders raw material from suppliers and receives the orders in l shipments per one cycle T_P . The problem here lies on how many shipments manufacturer and supplier should determine, hence ordering cost, holding cost, production cost, purchasing cost and other related costs could be minimized to gain the maximum joint total profit. Here, refurbishing refers to a process to increase the quality of a portion of recoverable used

items (f) that cannot be remanufactured, as its quality does not exceed the acceptable quality level. Refurbished items are then sold to the secondary market at a lower price, with $P_f < P_r$. Reworking process is needed, since manufacturing processes conducted by the manufacturer produce any defective items needed to be reworked immediately. The proportion of defective items (k) is assumed constant and known. In this study, we consider that amount of product returns (C) depends on quality level, q , where q is the portion of recoverable returned items. Hence, $(1-q)$ is the portion of items that disposed from the system. Return rate, C , is a portion of demand, D , returns to the system, then $0 < C(q)/D < 1$. The function $C(q) = Dbe^{-\varphi q}$ is adopted from [18], with $0 < b < 1$ and $\varphi > 1$.

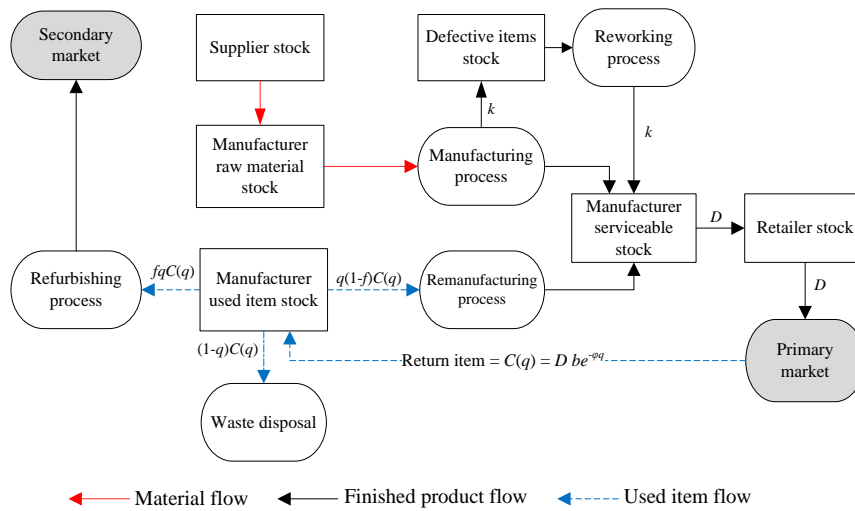


Figure 1. The proposed closed-loop supply chain inventory system.

This model aims to maximize the annual joint total profit (JTP) for manufacturer, retailer and supplier by determining the optimal retailer cycle time, T_r^* , manufacturer optimal number of shipment, m^* and n^* , supplier optimal number of shipment, l^* , and optimal number of manufacturing cycle, P^* . The inventory profile of the system is shown in Figure 2.

3.1 Retailer profit (TP_r)

Inventory cost to the retailer consists of ordering cost (OC_r), purchasing cost (PC_r) and holding cost (HC_r). Retailer profit (TP_r) is obtained by subtracting total revenue (TR_r) with total costs (TIC_r) as follows.

$$TP_r = TR_r - TIC_r = DP_c - \frac{A_r}{T_r} - DP_r - \frac{DF_r P_r T_r}{2} \quad (1)$$

3.2 Manufacturer profit (TP_m)

As shown in Figure 2, manufacturer has four types of stock, including raw material, serviceable items, defective items and recoverable items. Each kind of stock has different components of inventory cost. Inventory cost for raw material stock includes ordering cost (OC_{rm}), purchasing cost (PC_{rm}), and holding cost (HC_{rm}). Manufacturer serviceable inventory cost consists of production setup cost (SC_m), manufacturing cost (PC_m), reworking cost (rC_m), inspection cost (IC_m), remanufacturing cost (RC_m) and holding cost (HC_m). Manufacturer recoverable inventory cost (TIC_{rw}) consists of purchasing cost (PC_{rw}), inspection and sorting cost (ISC_{rw}), refurbishing cost (TC_f), waste disposal cost (WDC_{rw}) and holding cost (HC_{rw}). Manufacturer defective items inventory cost (TIC_d) is an incurred cost to hold stocks of defective items, the function of TIC_d is formulated below. Subsequently, we obtain the following equation for manufacturer total profit (TP_m).

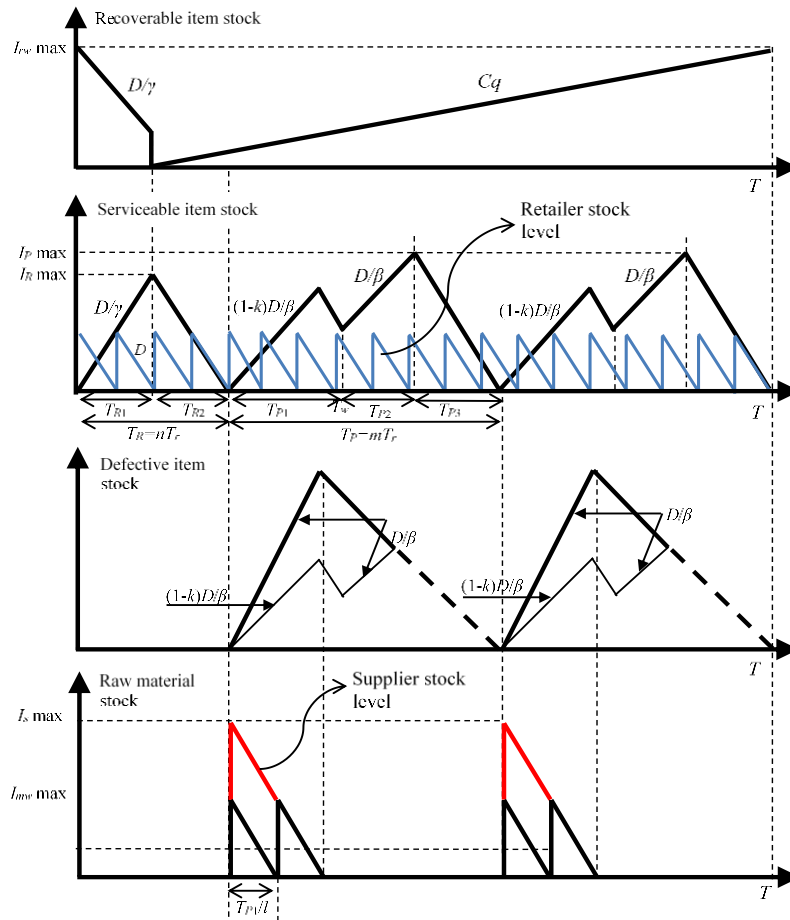


Figure 2. Retailer, manufacturer, and supplier stocks level ($l=2$, $P=2$, $n=4$, $m=7$, $N=18$).

$$\begin{aligned}
 TP_m &= TR_m - TIC_{rm} - TIC_m - TIC_{rw} - TIC_d \\
 &= P_r D + P_f \left(\frac{n f D}{(1-f)(n+mP)} \right) - \left[\frac{A_{mw} P l}{T_r (n + Pm)} + \frac{m D P_{rm} P}{(n + Pm)} + F_{mw} P_{rm} \left(\frac{P D T_r \beta m^2}{2 l (n + Pm)} \right) \right] \\
 &\quad - \left[\frac{A_R + A_m P + A_w P + M(n+mP)}{T_r (n+mP)} + \frac{DC_p m P}{n+mP} + \frac{DC_p k m P}{n+mP} + \frac{DC_{iscp} m P}{n+mP} + \frac{DC_R n}{n+mP} \right. \\
 &\quad \left. + F_M \left(\frac{P_u n + m P_{rm} P}{n+mP} \right) \left(\frac{n(D T_r \gamma + \frac{C_q T_r (1-f)(1-\gamma)(n+mP)}{2})}{n+mP} + \frac{m D P (-2k T_w + 2 T_r \beta - T_r ((1+k+k^2)\beta - 1) n^2)}{2(n+mP)} - \frac{D T_r}{2} \right) \right] \\
 &\quad - \left[\frac{P_u n D}{(1-f)q(n+mP)} + \frac{C_{isc} n D}{(1-f)q(n+mP)} + \frac{C_f n f D}{(1-f)(n+mP)} + \frac{C_w n D (1-q)}{(1-f)q(n+mP)} + F_{rw} P_u \left(2 C_f q \gamma n + D T_r \gamma n^2 + C_q T_r ((\gamma-1)n - mP)^2 \right) \right] \\
 &\quad - \left[\frac{C_d k m P D (2 T_w + (1+k) T_r \beta m)}{2(n+mP)} \right] \tag{2}
 \end{aligned}$$

3.3 Supplier profit (TP_s)

Inventory cost to the supplier consists of delivery cost (DC_s), purchasing cost (PC_s) and holding cost (HC_s). Supplier profit (TP_s) is obtained by subtracting total revenue (TR_s) with total costs (TIC_s).

$$TP_s = TR_s - TIC_s = \frac{P_{rm}mPD}{(n+mP)} - \left(\frac{P(A_s+L)}{T_r(n+mP)} + \frac{P_s mPD}{(n+mP)} + \frac{F_s P_s D T_r \beta m^2 P(l-1)}{2 l (n+mP)} \right) \quad (3)$$

Finally, the joint total profit of the CLSC system (JTP) is formulated by equation (4) below.

$$JTP(T_r, n, m, P, l) = TP_r + TP_m + TP_s \quad (4)$$

3.4 Solution procedure

The optimal values of T_r , P , m , n and l that maximize $JTP(T_r, P, m, n, l)$ are determined by the following solution procedure. Here, we only guarantee local optimal solution. By substituting $(N=n+mP)$ to $(nDT_r=Cq(1-f)T_r(n+mP))$, we obtain $n = \frac{NCq(1-f)}{D}$ and $m = \frac{N(1-f)(1-\frac{Cq}{D})}{P}$. The function of n and m are then substituted into equation (4). Hence, the function of JTP in equation (4) becomes $JTP(T_r, P, N, l)$.

To obtain the optimal value of T_r , let the derivative of $JTP(T_r, P, N, l)$ with respect to (for short w.r.t.) T_r equal to zero, then we get $T_r^\#(P, N, l)$. The optimal value of N and l can be obtained by letting the derivative of equation (4) w.r.t. N and l equal to zero, then substituting the optimal value of $T_r^\#(P, N, l)$ to those equation respectively. Hence, we obtain the values of $N^\#(P, l)$ and $l^\#(P)$. The value of $T_r^\#$, $N^\#$ and $l^\#$ are used as approximations to find the optimal solution. The value of P^* , N^* , m^* , n^* , l^* and T_r^* that maximize $JTP(T_r, P, m, n, l)$ can be derived by the following procedure

Step 1 Given $C(q)=Dbe^{-\varphi q}$, calculate the value of $C(q)$

Step 2 Set $P^{(0)}=1$, then find the value of $l^\#, N^\#$ and $T_r^\#$

Step 3 Determine the optimal value of N which satisfies:

$$JTP(N^*-1, P^*, l^*, T_r^*) \leq JTP(N^*, P^*, l^*, T_r^*) \geq JTP(N^*+1, P^*, l^*, T_r^*) \quad (5)$$

Step 4 Repeat step 2 to 5 for $P^{(a+1)}=P^{(a)}+1$ to find the optimal value of P which satisfies:

$$JTP(N^*, P^*-1, l^*, T_r^*) \leq JTP(N^*, P^*, l^*, T_r^*) \geq JTP(N^*, P^*+1, l^*, T_r^*) \quad (6)$$

Step 5 Derive the optimal value of l which satisfies:

$$JTP(N^*, P^*, l^*-1, T_r^*) \leq JTP(N^*, P^*, l^*, T_r^*) \geq JTP(N^*, P^*, l^*+1, T_r^*) \quad (7)$$

Step 6 Find the combination value of P^* , N^* , T_r^* and l^* that gives the maximum value of JTP

Step 7 Given a known N^* and P^* , calculate the value of $n^* = \frac{NCq(1-f)}{D}$ and $m^* = \frac{N(1-f)(1-\frac{Cq}{D})}{P}$

4. Numerical example and analysis

We present a numerical example to illustrate the application of the proposed P(1,P) model. The following parameters are adopted from [14]: $D=2000$ unit/year, $A_r=\$100$ /order, $A_{rw}=\$350$ /order, $A_{mw}=\$350$ /collection, $A_s=\$200$ /order, $A_m=\$2000$ /batch, $A_R=\$2500$ /batch, $P_c=\$180$ /unit, $P_r=\$160$ /unit, $P_{rm}=\$100$ /unit, $P_u=\$60$ /unit, $P_s=\$80$ /unit, $P_f=\$120$ /unit, $F_r=0.3$ /unit/year, $F_M=0.5$ /unit/year, $F_{mw}=0.4$ /unit/year, $F_{rw}=0.25$ /unit/year, $F_s=0.2$ /unit/year, $M=\$350$ /batch, $L=\$150$ /delivery, $C_j=\$20$ /unit, $C_d=\$30$ /unit, $C_w=\$5$ /unit, $C_{isc}=\$1$ /unit, $C_{iscp}=\$1$ /unit, $C_p=\$23$ /unit, $C_r=\$15$ /unit, $\gamma=0.4$, $\beta=0.6$, $f=0.3$, $k=0.1$, $q=0.83$, $b=0.95$ and $\omega=1.5$. Using the proposed solution procedure explained before, we obtain the optimal results as shown in Table 1.

Table 1. Optimization results of the proposed P(1,P) model.

P (cycles)	l (shipments)	N (shipments)	T_r (year)	TP_r (\$/year)	TP_s (\$/year)	TP_m (\$/year)	JTP (\$/year)
1	2	16	0.065	\$35,349	\$27,351	\$59,080	\$121,780
		17	0.062	\$35,417	\$27,321	\$58,799	\$121,537
		18	0.059	\$35,471	\$27,292	\$58,516	\$121,279
		19	0.057	\$35,514	\$27,264	\$58,232	\$121,010
2^*	3^*	16	0.095	\$34,374	\$27,692	\$60,556	\$122,622
		17*	0.091*	\$34,536*	\$27,682*	\$60,411*	\$122,628*
		18	0.087	\$34,676	\$27,671	\$60,260	\$122,607
		19	0.083	\$34,799	\$27,659	\$60,105	\$122,563
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
10	5	16	0.219	\$29,025	\$28,071	\$51,249	\$108,345
		17	0.208	\$29,492	\$28,096	\$51,353	\$108,941
		18	0.199	\$29,911	\$28,118	\$51,434	\$109,463
		19	0.191	\$30,291	\$28,139	\$51,495	\$109,925

The number of returned items calculated using equation (1) is 548 units. Table 1 shows the optimal solutions of the case. The comparison of the proposed P(1,P) model with its prior P(R,1) model belongs to [17] is shown in Table 2.

Table 2. Comparison of the optimization results of P(R,1) and P(1,P) policy.

Policy	R^* (cycles)	P^* (cycles)	l^* (shipments)	m^* (shipments)	n^* (shipments)	T_r^*	JTP^*
P(R,1)	1	1	2	5.4	4.6	32 days	\$121,979
P(1,P)	1	2	3	4.6	7.8	33 days	\$122,628

We conclude that (1,P) policy gives higher profit to the system rather than (R,1) policy. Hence, the system should choose only one remanufacturing cycle and at least one manufacturing cycle to gain maximum profit. However, the decision may be different if the values for each parameter are changed.

5. Conclusion and further research

This study has developed an inventory model of a CLSC system that implements (1,P) policy and considers inspection, sorting, waste disposal, imperfect manufacturing and reworks. From the proposed model and solution procedure, the system might maximize its annual joint total profit among three parties, i.e. retailer, manufacturer, and supplier, by determining the optimal value of number of shipments for each party, number of manufacturing cycles and retailer cycle time. From the comparison results to the model of [17], the (1,P) policy is better than (R,1) policy since (1,P) policy gives higher value of JTP than (R,1) policy. For further research, this model may be extended by adding some more considerations such as leadtime, shortage, stochastic demand and returns and environmental effects.

References

- [1] Kleindorfer P R, Singhal K and Van Wassenhove L N 2005 Sustainable operations management *Prod. Oper. Manag.* **14** pp 482–492
- [2] Chaves G d L D and Martins R S 2005 Diagnóstico da logística reversa na cadeia de suprimentos de alimentos processados no oeste paranaense *VIII Simpósio de Administração da Produção, Logística e Operações Internacionais (SIMPOI)* (São Paulo)
- [3] Andrade R, Vieira J M, Lucato W and Vanalle R 2014 Review of the relationship between reverse logistics and competitiveness *Adv. Mat. Res.* **845** pp 614–617

- [4] de Brito M P and Dekker R 2004 A framework for reverse logistics *Reverse Logistics: Quantitative Models for Closed-Loop Supply Chains* eds Dekker R, Fleischmann M, Interfurth K and Van Wassenhove L N (Berlin: Springer) pp 3–27
- [5] Xia W-H, Jia D-Y and He Y-Y 2011 The remanufacturing reverse logistics management based on closed-loop supply chain management processes *Procedia Environmental Science* **11** pp 351–354
- [6] Akçalı E and Çetinkaya S 2011 Quantitative models for inventory and production planning in closed-loop supply chains *Int. J. Prod. Res.* **49** pp 2373–2407
- [7] Dwicahyani A R, Jauhari W A and Jonrinaldi 2017 A regular production-remanufacturing inventory model for a two-echelon system with price-dependent return rate and environmental effects investigation *J. Phys. Conf. Ser.* **855**
- [8] Kasper A C, Berselli G B T, Freitas B D, Tenório J A S, Bernardes A M and Veit H M 2011 Printed wiring boards for mobile phones: Characterization and recycling of copper *Waste Management* **31** pp 2536–45
- [9] Shcrady D A 1967 A deterministic inventory model for repairable items *Nav. Res. Logist. Q.* **14** pp 57–71
- [10] Nahmias N and Rivera H 1979 A deterministic model for repairable item inventory system with a finite repair rate *Int. J. Prod. Res.* **17** pp 215–221
- [11] Teunter, R H 2004 Lot-sizing for inventory systems with product recovery *Comput. Ind. Eng.* **46** pp 431–441
- [12] Haji A, Haji R and Sajadifar S M 2008 Lot sizing with non-zero setup times for rework *J. Syst. Sci. Syst. Eng.* **17** pp 230–240
- [13] Chung S L, Wee H M and Yang P C 2008 Optimal policy for a closed-loop supply chain inventory system with remanufacturing *Math. Comput. Model.* **48** pp 867–881
- [14] Yuan K F and Gao Y 2010 Inventory decision-making models for a closed-loop supply chain system *Int. J. Prod. Res.* **48** pp 6155–87
- [15] Bazan E, Jaber M Y and Zanoni S 2017 Carbon emissions and energy effects on a two-level manufacturer-retailer closed-loop supply chain model with remanufacturing subject to different coordination mechanisms *Int. J. Prod. Econ.* **183** pp 394–408
- [16] Jauhari W A, Dwicahyani A R and Kurdhi N A 2017 Lot sizing decisions in a closed-loop supply chain system with remanufacturing *Int. J. Proc. Manag.* **10** pp 381–409
- [17] Kholisoh E, Jauhari W A and Rosyidi C N 2016 A closed-loop supply chain model for supplier-manufacturer-retailer system with rework and waste disposal *Proc. 2nd. Int. Conf. Industrial, Mechanical, Electrical, Chemical Engineering* (Yogyakarta: Universitas Sebelas Maret)
- [18] El Saadany A M A and Jaber M Y 2010 A production/remanufacturing inventory model with price and quality dependent return rate *Comput. Ind. Eng.* **58** pp 352–362