

Quadrotor trajectory tracking using PID cascade control

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Abstract. Quadrotors have been applied to collect information for traffic, weather monitoring, surveillance and aerial photography. In order to accomplish their mission, quadrotors have to follow specific trajectories. This paper presents proportional–integral–derivative (PID) cascade control of a quadrotor for path tracking problem when velocity and acceleration are small. It is based on near hover controller for small attitude angles. The integral of time-weighted absolute error (ITAE) criterion is used to determine the PID gains as a function of quadrotor modeling parameters. The controller is evaluated in three-dimensional environment in Simulink. Overall, the tracking performance is found to be excellent for small velocity condition.

1. Introduction

Recently, a growing interest in unmanned aerial vehicles (UAVs) has been shown among the research community. Quadcopters are flying vehicles that can be equipped with cameras and sensors to perform many complex tasks. Due to their high maneuverability and small size, quadrotors have been widely used. Their potential applications include search-and-rescue, homeland security, military surveillance, and earth sciences [1-3]. In general, quadrotors are naturally unstable, under-actuated, under damped, coupled and nonlinear systems that need to be controlled. Several control techniques can be utilized to control a quadrotor including proportional–integral–derivative (PID) [4, 5], linear quadratic regulator (LQR) [6], and backstepping and sliding mode control [7, 8]. Because of its simple structure and good stability, PID plays an important role in quadrotor control.

In this study, a PID controller is used for attitude and position control. Assuming small acceleration and small attitude angles, the corresponding control laws are derived. A Simulink model is developed where the performance of the controller is tested for different path-tracking cases.

2. Methodology

Figure 1 illustrates the quadrotor dynamic model with world (earth fixed) frame (x_w, y_w, z_w) , body-fixed frame (x_B, y_B, z_B) , rotor forces, F_i and rotor moments, M_i . The corresponding dynamic model is represented by Equation 1 to Equation 5 [4, 9], where r is the position vector of the center of mass of the quadrotor, F_i is the thrust force from rotor ‘ i ’, R is the transformation matrix from body frame to earth frame, p , q and r are the three components of angular velocity corresponding to roll, pitch and yaw motions, L is the distance from the center of mass to the axis of rotation of rotors, M_i is moment of rotor ‘ i ’ about center of mass, I is the inertia matrix, ω_i is rotor angular speed, ω_{des} is the desired rotor angular speed and k_F , k_M and k_m are constants.



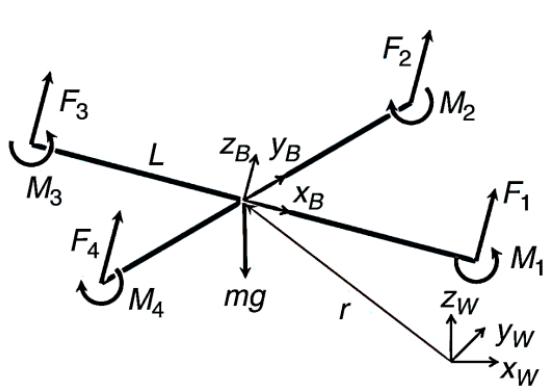


Figure 1: Quadrotor dynamic model representation

$$m \ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \sum F_i \end{bmatrix} \quad (1)$$

$$I \begin{bmatrix} \ddot{p} \\ \ddot{q} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (2)$$

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (3)$$

$$F_i = k_F \omega_i^2; \quad M_i = k_M \omega_i^2 \quad (4)$$

$$\dot{\omega}_i = k_m(\omega_i^{des} - \omega_i) \quad (5)$$

Figure 2 shows the cascade control loops [4, 9]. Position controller determines the required change in motor speed, $\Delta\omega_F$ to follow the hovering height, $r_{T,3}(t)$ and the required attitude angles: ϕ^{des} , θ^{des} and ψ^{des} based on position error, $r_T(t) - r(t)$. The attitude controller then determines the required change in the motor speeds: $\Delta\omega_\phi$, $\Delta\omega_\theta$ and $\Delta\omega_\psi$ in order to achieve the desired attitude angles. Thus the quadrotor will follow the specified trajectory, $r_T(t)$.

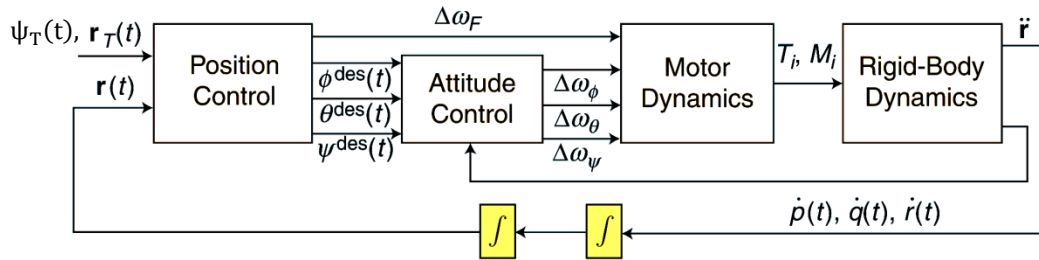


Figure 2: Nested control loops for position and attitude control

The vector of desired rotor speeds can be written as a linear combination of four terms as in Equation 6 [4, 9], where ω_h is the nominal rotor speed required to hover in steady state: $\omega_h = \sqrt{mg/(4k_F)}$ and the deviations from this nominal value are $\Delta\omega_F$, $\Delta\omega_\phi$, $\Delta\omega_\theta$ and $\Delta\omega_\psi$. $\Delta\omega_F$ results in a net force along the z-axis direction in the earth frame while $\Delta\omega_\phi$, $\Delta\omega_\theta$ and $\Delta\omega_\psi$ produce moments causing roll, pitch and yaw, respectively.

$$\begin{bmatrix} \omega_1^{des} \\ \omega_2^{des} \\ \omega_3^{des} \\ \omega_4^{des} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \omega_h + \Delta\omega_F \\ \Delta\omega_\phi \\ \Delta\omega_\theta \\ \Delta\omega_\psi \end{bmatrix} \quad (6)$$

Equation 7 is obtained by linearizing Equation 2 about the hovering operating point (assuming small attitude angles) [4, 9].

$$\ddot{p}^{des} = \frac{4k_F L \omega_h}{I_{xx}} \Delta\omega_\phi, \quad \ddot{q}^{des} = \frac{4k_F L \omega_h}{I_{yy}} \Delta\omega_\theta, \quad \ddot{r}^{des} = \frac{8k_M \omega_h}{I_{zz}} \Delta\omega_\psi \quad (7)$$

Near the hovering operating point, it is assumed that $\phi \approx p$, $\theta \approx q$ and $\psi \approx r$. Subsequently, the deviations from hovering required for attitude PD control are given by Equation 8. Since the position PID controller will generate angle references for the attitude controller, integral action is not necessary

for the latter. The integral term is needed to ensure steady state error, which is not required for the internal attitude loop.

$$\begin{aligned}\Delta\omega_\phi &= k_{p,\phi}(\phi^{des} - \phi) + k_{d,\phi}(p^{des} - p) \\ \Delta\omega_\theta &= k_{p,\theta}(\theta^{des} - \theta) + k_{d,\theta}(q^{des} - q) \\ \Delta\omega_\psi &= k_{p,\psi}(\psi^{des} - \psi) + k_{d,\psi}(r^{des} - r)\end{aligned}\quad (8)$$

Pitch and roll angles are used to control position in the horizontal plane, $\Delta\omega_\psi$ to control yaw angle and $\Delta\omega_F$ to control hover height. Assume a tracking trajectory with $r_T(t)$ and $\psi_T(t)$. PID controller is used to calculate the commanded acceleration (\ddot{r}_i^{des}) as in Equation 9 [4, 9].

$$(\ddot{r}_{i,T} - \ddot{r}_i^{des}) + k_{p,i}(r_{i,T} - r_i) + k_{i,i} \int (r_{i,T} - r_i)dt + k_{d,i}(\dot{r}_{i,T} - \dot{r}_i) = 0 \quad (9)$$

For near hover condition, $\ddot{r}_{i,T} \approx \dot{r}_{i,T} \approx 0$. Therefore, this algorithm is only successful in following a track with small velocity and acceleration. Linearizing Equation 1 about the hovering operating point leads to Equation 10 [4, 9].

$$\begin{aligned}\phi^{des} &= (\ddot{r}_1^{des} \sin \psi_T - \ddot{r}_2^{des} \cos \psi_T)/g \\ \theta^{des} &= (\ddot{r}_1^{des} \cos \psi_T + \ddot{r}_2^{des} \sin \psi_T)/g \\ \Delta\omega_F &= m \ddot{r}_3^{des} / (8 k_F \omega_h)\end{aligned}\quad (10)$$

By linearizing Equation 1 and Equation 2 around hovering operating point, plant transfer functions for position, roll, pitch, and yaw by $P(s) = \frac{K}{s^2}$ are approximated [4, 9]. In this case study, $K = 1/m$ for position, $K = \frac{4 k_F L \omega_h}{I_{xx}}$ for roll and pitch, and $K = \frac{8 k_M \omega_h}{I_{zz}}$ for yaw.

In reference to Figure 3, with $C(s) = K_p + \frac{K_i}{s} + K_d s$, the closed loop transfer function is given by Equation 11.

$$\frac{y(s)}{r(s)} = \frac{K(k_d s^2 + k_p s + k_i)}{s^3 + K(k_d s^2 + k_p s + k_i)} \quad (11)$$

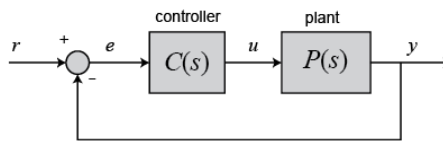


Figure 3: Transfer function for PID tuning

PID controller is designed using minimization of integral of time-weighted absolute error (ITAE) criterion [10]. The ω_n of the closed loop system is selected based on the desired settling time ($T_s = \frac{4}{\xi \omega_n}$ and $\xi \approx 0.8$). PID parameters are then determined by comparing the characteristic equation of the closed loop system with the optimum characteristic equation for ITAE as shown in Equation 12 [10]. The resulting PID controller parameters serve as the starting values for later tuning by trial and error.

$$s^3 + 1.75 \omega_n s^2 + 2.15 \omega_n^2 s + \omega_n^3 \quad (12)$$

3. Results and Discussions

The quadrotor parameters are tabulated in Table 1 and the controller gains are listed in Table 2, with $\omega_h = 109.68$ rev/s. The attitude controller (roll and pitch) is made ten times faster than the position controller by appropriate choice of settling time.

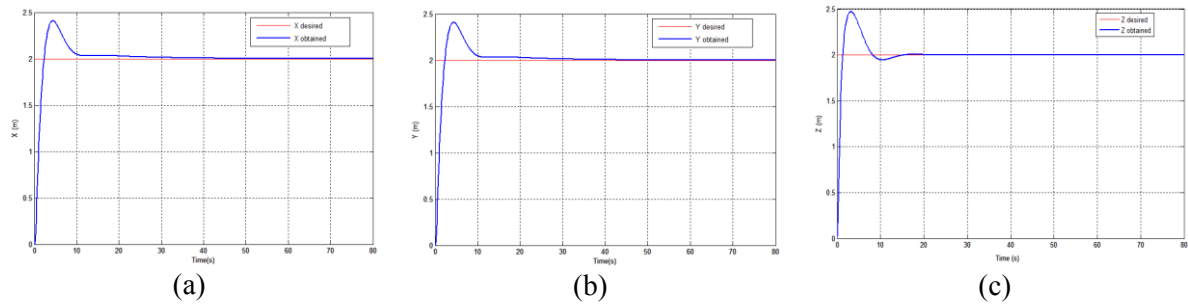
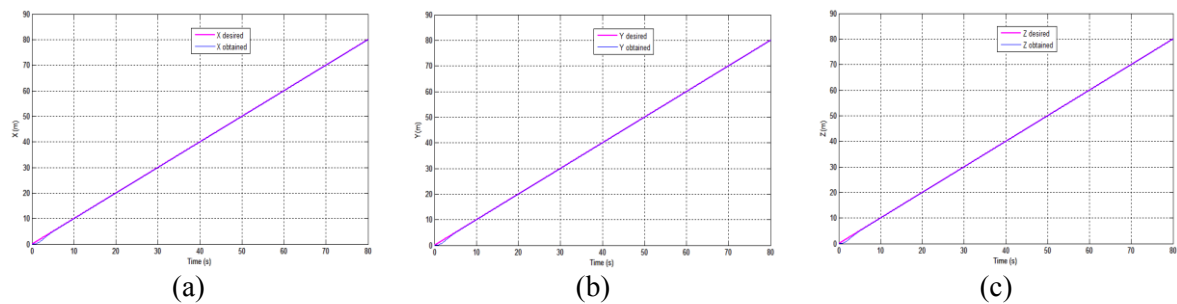
Table 1: Quadrotor parameters [4]

| Parameter | Value | Units |
|-----------|----------------------|----------------------------|
| m | 1.08 | Kg |
| L | 0.22 | m |
| I_{xx} | 0.1045 | Kg.m ² |
| I_{yy} | 0.1045 | Kg.m ² |
| I_{zz} | 0.2090 | Kg.m ² |
| k_F | 2.2×10^{-4} | N/(rev/s) ² |
| k_M | 5.4×10^{-6} | N m / (rev/s) ² |
| k_m | 20 | s ⁻¹ |

Table 2: Controller parameters

| Controller | Gain K | Settling time T_s | Frequency ω_n | k_p $2.15 \omega_n^2 / K$ | k_i ω_n^3 / K | k_d $1.75 \omega_n / K$ |
|----------------|-------------|------------------------|-------------------------|--------------------------------|---------------------------|------------------------------|
| Position (PID) | 0.952 | 6 s | 0.83 | 1.567 | 0.607 | 1.53 |
| Roll (PD) | 0.203 | 0.6 s | 8.3 | 734.9 | 0 | 71.81 |
| Pitch (PD) | 0.203 | 0.6 s | 8.3 | 734.9 | 0 | 71.81 |
| Yaw (PD) | 0.005 | 6 s | 0.83 | 291.55 | 0 | 298.37 |

Figure 4 shows the step response for step input of 2m in x , y or z . Desired track and obtained track are plotted, where an overshoot of 15% (x and y) and 25% (z). The rise time is 2s and the settling time is 15 s. A compromise is made for fast response (small rise time) at the expense of high overshoot. The tracking of the ramp trajectory in x , y , or z with a slope (velocity) equals to 1 is demonstrated in Figure 5. The Root Mean Square Error (RMSE) is about 0.0001.

Figure 4: Step input response in (a) x , (b) y and (c) z Figure 5: Tracking ramp trajectory in (a) x , (b) y and (c) z

The quadrotor is also commanded to move in a circle (radius = 6m and velocity = 2m/s) in x - y plane, x - z plane or y - z plane as shown in Figure 6. It has been shown that the tracking performance is excellent with RMSE of 0.0002.

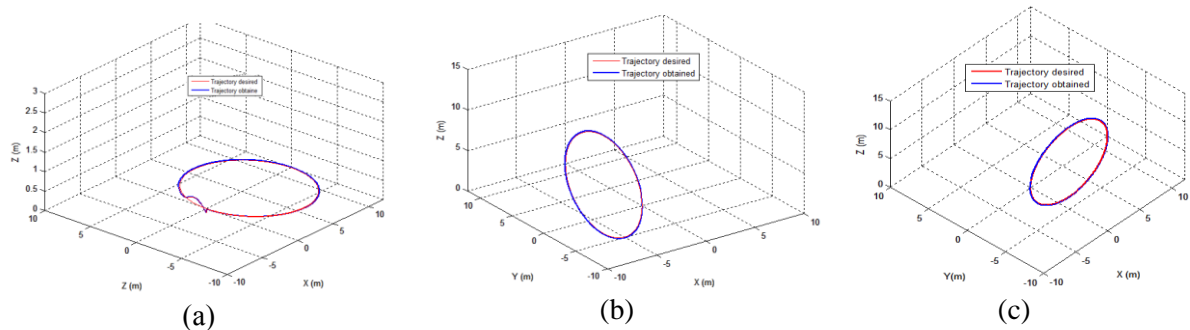


Figure 6: Tracking of a circular trajectory in (a) x - y plane, (b) x - z plane and (c) y - z plane

The quadrotor has been commanded to move in a sinusoidal path in one direction (x , y or z) with 1m amplitude and different time periods (80s, 20s and 10s) as shown in Figure 7. The RMSE increases as the period decreases (velocity increases) from 0.0002 for period = 80s to 0.045 for period = 10s. This demonstrates the limitation of the current algorithm to the small velocity trajectories as it is an assumption in the development of this control method.

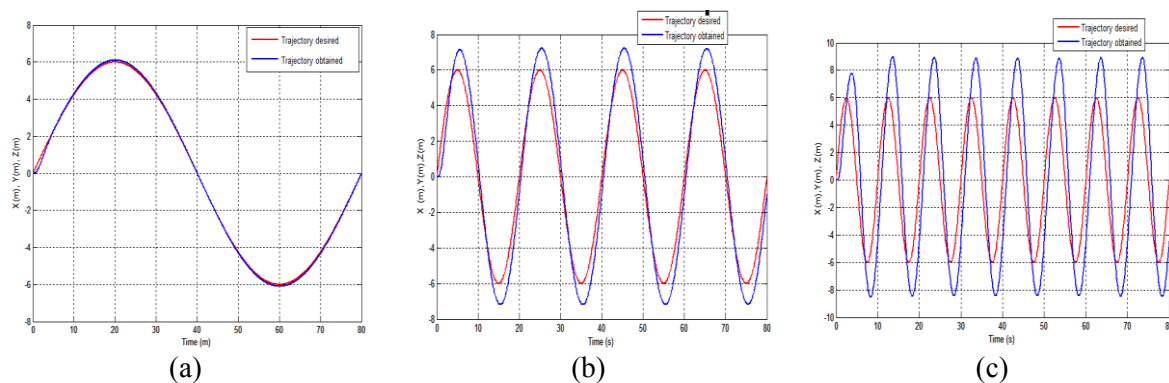


Figure 7: Tracking a sinusoidal trajectory with different periods (a) 80s, (b) 20s and (c) 10s

4. Conclusions

The PID cascade control method is implemented for quadrotor tracking of a specified trajectory. The controllers are based on near hover condition. The PID gains are obtained as the function of quadrotor modeling parameters using ITAE criterion. Tracking performance has been tested for ramp, circular and sinusoidal trajectories. The performance is found to be acceptable as long as velocity is small. For the future work, tracking of large angle and large velocity trajectories will be considered, which entail the modification of control algorithm.

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