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Dynamic Response of Two Rigid Foundations on Multi-Layered Poroelastic Medium

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Abstract. This paper presents dynamic response of two rigid foundations resting on a multi-layered poroelastic medium under time-harmonic vertical loading. The contact surface between the foundations and the layered medium is assumed to be smooth and fully permeable. This dynamic interaction problem is studied by employing a discretization technique and an exact stiffness matrix scheme. The accuracy of the present solution is verified and selected numerical results for impedances of two square foundations are also presented.

1. Introduction

The study of dynamic interaction between foundations and soils is of considerable importance in civil engineering. Several researchers investigated dynamic response of vertically loaded rigid foundations resting on an elastic half-space by employing a variety of analytical and numerical techniques. For example, Wong and Luco [1] presented dynamic response of rigid foundations of arbitrary shape resting on an elastic half-space by using a discretization technique. Subsequently, Wong and Luco [2] extended their technique to study interaction between two rigid square foundations. In addition, the case of two rigid rectangular foundations was also studied by Triantafyilidis and Prange [3]. Geomaterials are often two-phase materials consisting of an elastic solid skeleton with voids filled with water, commonly known as poroelastic materials. In the past, dynamic response of vertically loaded foundations involving poroelastic materials was investigated extensively by adopting Biot's poroelastodynamics theory [4]. For example, the case of a rigid rectangular foundation on a homogeneous poroelastic half-space was studied by Halpern and Christiano [5]. Vertical vibrations of a circular plate in a multi-layered poroelastic medium were also presented by employing the exact stiffness matrix method [6]. In practice, there exists a situation of a closely spaced foundation system where the interaction between adjacent foundations needs to be taken into consideration. A review of literature indicates that studies on dynamic interaction between two rectangular foundations and a multi-layered poroelastic medium have never been reported in the past.

In this paper, dynamic interaction between two rigid rectangular foundations and a multi-layered poroelastic medium subjected to time-harmonic vertical loading is presented. The geometry of foundations-layered system is shown in Figure 1. Each layer of the multi-layered half-space is governed by Biot’s poroelastodynamics theory [4]. The contact surface between the foundation and the layered medium is assumed to be smooth and fully permeable. To solve this interaction problem, the contact surface is discretized into a number of square elements. The unknown contract traction within each discretized element is then solved from the flexibility equations based on the influence functions, which are determined by employing the exact stiffness matrix method [6]. The accuracy of the proposed solution scheme has been verified, and selected numerical results for impedances of two identical square foundations resting on a multi-layered poroelastic half-space are also presented.

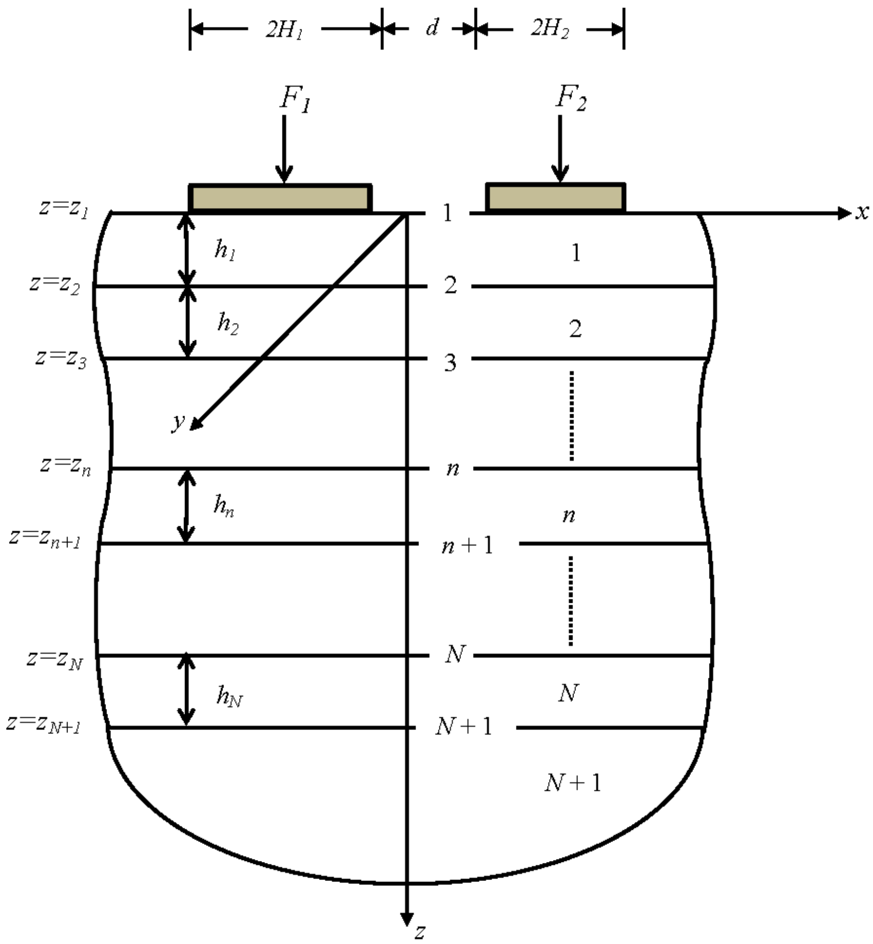

Figure 1. Geometry of rigid foundations on a multi-layered poroelastic medium.

2. Basic equations

Consider a poroelastic half-space with a Cartesian coordinate system (x, y, z) defined such that the z-axis is perpendicular to the free surface as shown in figure 1. Let $u_i(x, y, z, t)$ and $w_i(x, y, z, t)$ denote the average displacement of the solid matrix and the fluid displacement relative to the solid matrix in the i-direction ($i = x, y, z$) respectively. The constitutive relation can be expressed following Biot's theory for a poroelastic material by using the standard indicial notation as

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}e - \alpha\delta_{ij}p \quad , \qquad i, j = x, y, z \tag{1a}$$

$$p = -\alpha M \varepsilon_{kk} + M\zeta \tag{1b}$$

In equations (1a) and (1b), σ_{ij} is the total stress component of the bulk material; ε_{ij} is the strain component of the solid matrix; μ and λ are Lame' constants of the bulk material; p is the excess pore fluid pressure; and ζ is the variation of fluid content per unit reference volume. In addition, α and M are Biot's parameters accounting for compressibility of the two-phased material. In this study, the motion is assumed to be time-harmonic with the factor of $e^{i\omega t}$, where ω is the frequency of excitation.

Lu and Jeng [7] showed that the general solutions to Biot's equations can be obtained by using a Helmholtz representation and the Fourier integral transforms. The general solutions [7] can be expressed in the frequency-wave number domain in the following matrix form:

$$\mathbf{u}(k_x, k_y, z) = \mathbf{R}(k_x, k_y, z)\mathbf{c}(k_x, k_y) \tag{2a}$$

$$\boldsymbol{\sigma}(k_x, k_y, z) = \mathbf{S}(k_x, k_y, z)\mathbf{c}(k_x, k_y) \tag{2b}$$

where

$$\mathbf{u}(k_x, k_y, z) = \left[\mathrm{i}\bar{u}_x \ \mathrm{i}\bar{u}_y \ \bar{u}_z \ \bar{p}\right]^T \tag{3a}$$

$$\boldsymbol{\sigma}(k_x, k_y, z) = \left[\mathrm{i}\bar{\sigma}_{xz} \ \mathrm{i}\bar{\sigma}_{yz} \ \bar{\sigma}_{zz} \ \bar{w}_z\right]^T \tag{3b}$$

$$\mathbf{c}(k_x, k_y) = \left[A \ B \ C \ D \ E \ F \ G \ H\right]^T \tag{3c}$$

and the arbitrary functions $A(k_x, k_y)$ to $H(k_x, k_y)$ can be determined by employing appropriate boundary and continuity conditions.

3. Analysis of interaction problem

Consider a soil-structure interaction problem involving two rectangular foundations, namely Foundation 1 and Foundation 2, separated by a distance d and resting on a multi-layered poroelastic half-space as shown in figure 1. The contact areas under Foundations 1 and 2 are denoted by A_1 and A_2, respectively. The two foundations are assumed to be rigid, fully permeable and in smooth contact with the layered half-space. When Foundation 1 is subjected to time-harmonic vertical loading, Foundation 2 will also experience a part of radiated energy dissipated from Foundation 1. Consequently, Foundation 2 starts vibrating accordingly, and also generates some radiated energy that could affect Foundation 1. It is noted that the interaction between foundations would be significant if the two foundations are relatively close. To investigate this interaction problem, Foundation 1 is assumed to undergo time-harmonic vertical displacement of amplitude Δ_z^1 whereas the displacement of the second foundation is restrained to be zero. The following relationship can then be established:

$$\int G_{zz}^{11}(\mathbf{r},\hat{\mathbf{r}})T_z^1(\mathbf{r})dA_1 + \int G_{zz}^{12}(\mathbf{r},\hat{\mathbf{r}})T_z^2(\mathbf{r})dA_2 = \Delta_z^1 \tag{6a}$$

$$\int G_{zz}^{21}(\mathbf{r},\hat{\mathbf{r}})T_z^1(\mathbf{r})dA_1 + \int G_{zz}^{22}(\mathbf{r},\hat{\mathbf{r}})T_z^2(\mathbf{r})dA_2 = 0 \tag{6b}$$

In equations (6a) and (6b), $T_z^i(\hat{\mathbf{r}})$ (i = 1, 2) denote the normal contact traction generated at the contact surface of Foundation i; $G_{zz}^{ij}(\mathbf{r},\hat{\mathbf{r}})$ ($i = j$ = 1, 2) denotes the influence functions, which are the vertical displacement of Foundation i at any point with the position vector $\mathbf{r}$ due to a unit vertical point load applied at a point on Foundation j with the position vector $\hat{\mathbf{r}}$. Note that the influence functions $G_{zz}^{ij}(\mathbf{r},\hat{\mathbf{r}})$ are computed by employing an exact stiffness matrix scheme [6]. A number of $N_x^i \times N_y^i$ sub-elements are employed for Foundation i (i = 1, 2) where N_x^i and N_y^i denote the number of sub-elements in the x and y directions, respectively. In the numerical study, only the case of identical square foundations is investigated. The size and the number of sub-elements for both foundations are then identical. The computed impedance functions of dynamic interaction between both foundations denoted by K_{ij} ($i = j$ = 1, 2) can then be obtained as follows:

$$K_{11} = K_{22} = \int T_z^1(\hat{\mathbf{r}})dA_1 \tag{7a}$$

$$K_{12} = K_{21} = \int T_z^2(\hat{\mathbf{r}})dA_2 \tag{7a}$$

4. Numerical results and discussion

The accuracy of the present solution scheme is first verified. Wong and Luco [2] presented dynamic response of two identical rigid square foundations resting on a homogeneous elastic half-space with the Poisson's ratio of 0.333. The size of both square foundations is $2H \times 2H$ and the distance between the two foundations is denoted by d. In addition, $\delta = \omega H\sqrt{\rho\mu}$. The homogeneous elastic half-space is modelled in the present study by using 10 homogeneous layers with a thickness of $h/a = 0.2$ and an underlying half-space of identical material properties of $\nu = 0.333$ and negligibly small values of poroelastic parameters. In addition, each foundation is discretized into 64 or 8×8 sub-elements (see figure 2(a)). Figure 2(b) shows the comparison of non-dimensional impedance functions of two identical rigid square foundations between the present solutions and those given by Wong and Luco [2] for $d/H = 1$. Since the two foundations are identical, $K_{11} = K_{22}$ represents the total force under Foundation 1 and $K_{12} = K_{21}$ represents the total force under Foundation 2. It is evident that a good agreement between the two solutions is observed from the comparison shown in figure 2(b).

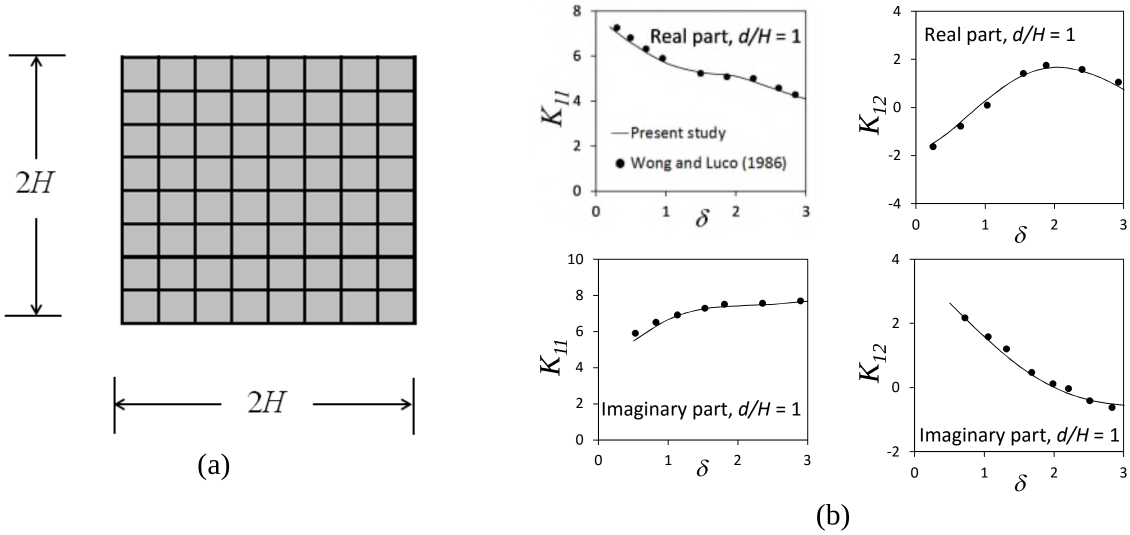

Figure 2. (a) Discretization of contact area; (b) Comparison of impedance functions K_{11} and K_{12} for two identical rigid square foundations with $d/H = 1$ resting on a homogeneous elastic half-space.

The dynamic interaction between two rigid foundations and a multi-layered poroelastic half-space is presented next. Figure 3 shows impedance functions K_{11} and K_{12} of two identical square foundations of size $2H \times 2H$ with various distances between the foundation, i.e., $d/H = 0.5$, 1 and 2, resting on a multi-layered poroelastic half-space. The layered half-space under consideration consists of two poroelastic layers with the same thickness H overlying a poroelastic half-space. The material properties of the two layers and the half-space are given in table 1. Both real and imaginary parts of the impedance functions K_{11} and K_{12} are shown in figure 3. Both foundations are fully permeable and the discretization shown in figure 2(a) is employed. It can be seen from figure 3 that the impedance functions of the two foundations depend significantly on the distance d. Both real and imaginary parts of K_{11} varies smoothly over the frequency range $0 < \delta < 3$ for all values of d/H. As expected, the maximum response is found in the case of the shortest distance between two foundations, i.e., $d/H = 0.5$ followed by the $d/H = 1$ and 2 respectively. The case of single foundation is also shown in figure 3 for comparison, and it is found that the impedance of two foundations converges to that of single foundation when $d/H > 2$ implying that K_{11} of two-foundation system can be obtained from the analysis of single-foundation system when the distance between them is greater than their width. It is also found that impedance function K_{12} shows complicated variations with frequency and its values do not diminish as the distance d/H increases. This indicates that an unloaded foundation would experience a radiated energy dissipated from a loaded foundation even though the distance between them is greater than their width. Additional parametric studies are currently underway and the results will be discussed in a future publication.

Table 1. Material properties of the layered system considered in figure 3.

Location	$\mu^{\dagger}$	$\lambda^{\dagger}$	$M^{\dagger}$	$\rho^{\dagger\dagger}$	$\rho_f^{\dagger\dagger}$	$m^{\dagger\dagger}$	α	$b^{\dagger\dagger\dagger}$
First layer	2.5	5	25	2	1	3	0.95	1.5
Second layer	1.25	1.88	18.8	1.6	1	1.8	0.98	0.75
Half-space	10	10	20	2.4	1	4.8	0.9	4.5

$^{\dagger} \times 10^8$ N/m^2; $^{\dagger\dagger} \times 10^3$ N/m^2; $^{\dagger\dagger\dagger} \times 10^6$ N/m^2.

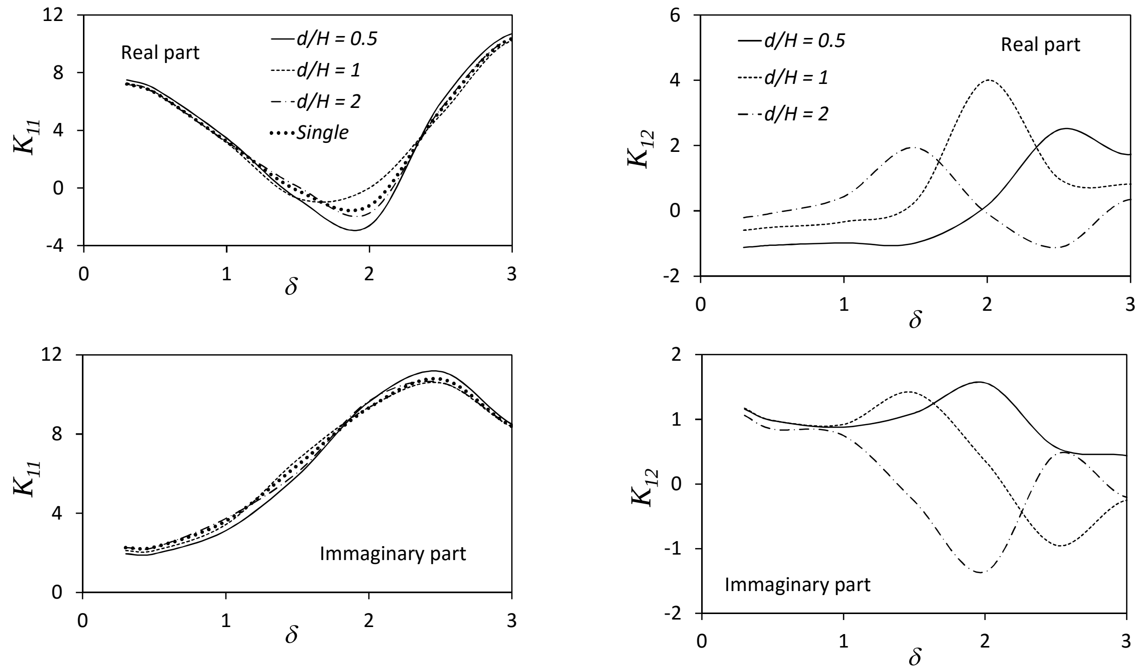

Figure 3. Impedance functions K_{11} and K_{12} of two identical rigid square foundations.

5. Conclusions

Dynamic response of two rigid foundations resting on a multi-layered poroelastic half-space is successfully studied in this paper by employing the discretization technique and the exact stiffness matrix scheme. Selected numerical results for impedance functions K_{11} and K_{12} for two identical rigid square foundations are presented, and it is found that the impedance functions are clearly influenced by the frequency of excitation and the distance between the two foundations. Extensive parametric studies on foundations of other shapes will be presented in a future publication.

6. References

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