

Heat Transfer Analysis of Thermal Protection Structures for Hypersonic Vehicles

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Abstract. This research aims to develop an analytical approach to study the heat transfer problem of thermal protection systems (TPS) for hypersonic vehicles. Laplace transform and integral method are used to describe the temperature distribution through the TPS subject to aerodynamic heating during flight. Time-dependent incident heat flux is also taken into account. Two different cases with heat flux and radiation boundary conditions are studied and discussed. The results are compared with those obtained by finite element analyses and show a good agreement. Although temperature profiles of such problems can be readily accessed via numerical simulations, analytical solutions give a greater insight into the physical essence of the heat transfer problem. Furthermore, with the analytical approach, rapid thermal analyses and even thermal optimization can be achieved during the preliminary TPS design.

1. Introduction

Space vehicles encounter severe aerodynamic heating when traveling at high speeds in the atmosphere. Thermal protection systems (TPS) are required to maintain the underlying structural temperature within acceptable limits [1]. Since the inner surface temperature is usually taken as a design variable and the temperature distributions are needed to carry out the following linear static analysis as well as bulking analysis, it is of great need and importance to be able to predict the temperature profile through the TPS. Normally, numerical simulations can be adopted to achieve this purpose, however, analytical solutions, whether exact or approximate, are always useful in engineering analysis since they could offer a clear insight into the physical essence of the heat transfer problem [2].

De Monte [3] summarized a few analytical methods that could be used to address transient heat transfer problems, as well as their strengths for specific applications. Miller and Weaver [4, 5] developed analytical models based on techniques of integral transforms and separation of variables, and described the transient temperature distribution through an air-filled box structure and a multi-layered plate, respectively. Finite element (FE) models were also established for comparison purpose and showed a good fit. Ferraiuolo and Manca [6, 7] developed an analytical procedure and a semi-analytical procedure based on the integral method and the Green's function method for layered TPS and hot structures.

In the current work, analytical approaches are used to study the transient heat transfer problem of the TPS. Laplace transform and integral method are adopted to describe the temperature distribution through the TPS subjected to aerodynamic heating. Radiation to ambient environment and time



dependent heat flux are taken into account. Comparisons of the results from analytical approaches and one-dimensional (1-D) FE analyses are conducted for validation.

2. Model description

Excessive aerodynamic heating occurs when a space vehicle travels at hypersonic speeds through a planetary atmosphere. Since the temperature gradient along the TPS thickness is much larger than those along the other two directions, the heat transfer problem of a TPS is usually simplified to a 1-D issue [8], as shown in figure 1. In this study, thermal properties are assumed to be constant with respect to time and temperature.

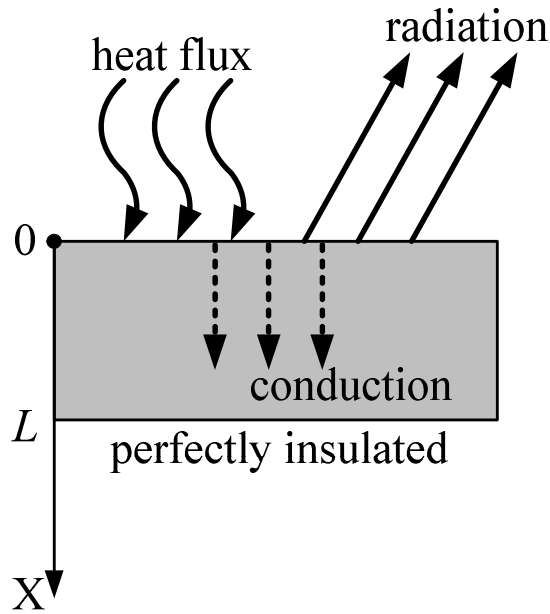


Figure 1. Illustration of boundary conditions.

As shown in figure 1, the TPS is subjected to a heat flux at $x=0$, while conservatively assumed to be adiabatic at $x=L$. Some heat is conducted through the TPS, and some heat is radiated away to the ambient environment.

The thermal conduction differential equation to be solved in this study is given as

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad (1)$$

where α is the thermal diffusivity. The boundary conditions to define this problem are

$$-k \left. \frac{\partial T}{\partial x} \right|_{x=0} = q_h - q_r = q_h(t) - \varepsilon \sigma (T_s^4 - T_a^4) \quad (2)$$

$$k \left. \frac{\partial T}{\partial x} \right|_{x=L} = 0 \quad (3)$$

and the initial condition is

$$T|_{t=0} = T_i \quad (4)$$

where k is the thermal conductivity, ε is the surface emissivity, σ is the Stefan-Boltzmann constant, q_h and q_r refer to the incident heat flux and radiation, T_s and T_a stand for the surface temperature and ambient temperature, respectively.

3. Analytical model

3.1. Integral method

The heat transfer problem becomes nonlinear once radiation is involved. Exact analytical solutions are always difficult or even impossible to obtain for nonlinear problems. Herein, integral method is adopted to gain approximate solutions. The differential equation of heat conduction is integrated over a phenomenological distance $\delta(t)$, i.e. thermal layer.

The integral method for a 1-D finite region is performed in two stages [2]. For the first stage where $\delta(t) < L$, according to the definition of thermal layer, boundary conditions at $x=\delta(t)$ can be described as

$$\left. \frac{\partial T}{\partial x} \right|_{x=\delta(t)} = 0 \quad (5)$$

$$T|_{x=\delta(t)} = T_i. \quad (6)$$

To simplify the mathematical derivation process, following expressions are used

$$Q_1(t) = \frac{q_h(t)}{k} \quad (7)$$

$$Q_2(T_s) = \frac{-\varepsilon\sigma(T_s^4 - T_a^4)}{k}. \quad (8)$$

Equation (1) is integrated from $x=0$ to $x=\delta(t)$. By introducing the boundary conditions of equations (2) and (5)-(6), together with expressions (7) and (8), the corresponding energy integral equation is obtained

$$Q_1(t) + Q_2(T_s) = \frac{1}{\alpha} \frac{d}{dt} \left[\int_0^{\delta(t)} T dx - T_i \delta(t) \right]. \quad (9)$$

A quadratic polynomial representation is chosen for the temperature profile

$$T_1(x, t) = a_0 + a_1 x + a_2 x^2. \quad (10)$$

By applying equations (2) and (5)-(6), the coefficients a_0 , a_1 and a_2 are determined, and the temperature profile can be further expressed as

$$T_1(x, t) = T_i + \frac{\delta(t)}{2} [Q_1(t) + Q_2(T_s)] \left(1 - \frac{x}{\delta(t)} \right)^2. \quad (11)$$

By setting $x=0$, the relation between the surface temperature T_s and the thermal layer $\delta(t)$ is obtained

$$T_s = T_1(0, t) = T_i + \frac{\delta(t)}{2} [Q_1(t) + Q_2(T_s)] \quad (12)$$

according to which the thermal layer $\delta(t)$ can be expressed as a function of T_s . Then the substitution of equation (11) and $\delta(t)$ into the energy integral equation (9) gives

$$\frac{3}{2} \alpha [Q_1(t) + Q_2(T_s)] = \frac{d}{dt} \left[\frac{(T_s - T_i)^2}{Q_1(t) + Q_2(T_s)} \right]. \quad (13)$$

The heat flux profile $q_h(t)$ is usually known as an input, and $\varepsilon\sigma T_a^4$ is omitted since it is small compared with the other terms in equation (2). By substituting $Q_1(t)$ and $Q_2(T_s)$, an ordinary differential equation (ODE) for the determination of T_s is obtained

$$2k^2(T_s - T_i) \left[q_h(t) + \varepsilon \sigma T_s^4 - 2\varepsilon \sigma T_i T_s^3 \right] \frac{dT_s}{dt} = \frac{3}{2} \alpha \left[q_h(t) - \varepsilon \sigma T_s^4 \right]^3 + k^2(T_s - T_i)^2 \frac{dq_h(t)}{dt}. \quad (14)$$

Equation (14) can then be integrated numerically with the initial condition (4). Specifically, Maple is employed here in this study to solve the ODE. Once the relation between T_s and t is established, temperature distribution for the range $0 < x < \delta(t)$ is therefore derived

$$T_1(x, t) = T_i + (T_s - T_i) \left[1 - \frac{Q_1(t) + Q_2(T_s)}{2(T_s - T_i)} x \right]^2 \quad (15)$$

while temperature for the range $\delta(t) < x < L$ stays at the initial value.

The ending time of the first stage t_L is obtained by setting $\delta(t) = L$. For the second stage, i.e. $t > t_L$, the differential equation (1) is integrated from 0 to L

$$Q_1(t) + Q_2(T_s) = \frac{1}{\alpha} \frac{d}{dt} \int_0^L T dx. \quad (16)$$

It is assumed that the temperature has a quadratic polynomial distribution through the thickness

$$T_2(x, t) = a_0 + a_1 x + a_2 x^2. \quad (17)$$

Then the application of the boundary conditions (2) and (3) solves the coefficients and yields

$$T_2(x, t) = T_s - [Q_1(t) + Q_2(T_s)] x + \frac{Q_1(t) + Q_2(T_s)}{2L} x^2. \quad (18)$$

The substitution of this expression into the energy integral equation (16) gives the following relation

$$3\alpha [Q_1(t) + Q_2(T_s)] = \frac{d}{dt} [3T_s L - (Q_1(t) + Q_2(T_s)) L^2]. \quad (19)$$

Similarly, $Q_1(t)$ and $Q_2(T_s)$ are introduced into equation (19), and $\varepsilon \sigma T_a^4$ is neglected

$$(3kL + 4\varepsilon \sigma L^2 T_s^3) \frac{dT_s}{dt} = \frac{dq_h(t)}{dt} L^2 + 3\alpha q_h(t) - 3\alpha \varepsilon \sigma T_s^4. \quad (20)$$

By combining the initial condition of the second stage $T_s(t_L) = T_1(0, t_L)$, T_s can be solved numerically and then substituted into equation (18) to determine the temperature profile of the second stage $T_2(x, t)$.

3.2. Laplace transform and integral method

For cases where the incident heat flux is constant and the surface temperature is relatively low before the heat penetration approaches to the inner side of the TPS, radiation is neglected when its contribution is less than 5% of the aerodynamic heating

$$q_r = \varepsilon \sigma (T_s^4 - T_a^4) < 0.05 q_h. \quad (21)$$

In this simplified linear case, the Laplace transform method is used to obtain the temperature profile through the structure thickness, which is as follows

$$T_1(x, t) = T_i + \frac{q_h L}{k} \left[Fo + \frac{x^2}{2L^2} - \frac{x}{L} + \frac{1}{3} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos(n\pi \frac{x}{L}) e^{-(n\pi)^2 Fo} \right] \quad (22)$$

where $Fo = at/L^2$, is the Fourier number.

By setting $q_r=0.05q_h$, the ending time of the first stage t_r is obtained, after which the radiation must be taken into account as it is comparable to the aerodynamic heating. Therefore, integral method is adopted to gain approximate solutions for the second stage.

To simplify the mathematical derivation process, the following expression are defined

$$Q(T_s) = \frac{q_h - \varepsilon\sigma(T_s^4 - T_a^4)}{k}. \quad (23)$$

An energy integral equation similar to equation (16) is obtained by integrating equation (1) from 0 to L with the boundary conditions (2) and (3)

$$Q(T_s) = \frac{1}{\alpha} \frac{d}{dt} \int_0^L T dx. \quad (24)$$

A quadratic polynomial expression is employed to describe the temperature profile

$$T_2(x, t) = a_0 + a_1x + a_2x^2. \quad (25)$$

The coefficients are solved using (2) and (3). The resulting temperature profile becomes

$$T_2(x, t) = T_s - Q(T_s)x + \frac{Q(T_s)}{2L}x^2. \quad (26)$$

Afterwards, the temperature profile is substituted into the equation (24),

$$3\alpha Q(T_s) = \frac{d}{dt} [3T_sL - Q(T_s)L^2]. \quad (27)$$

Again, $\varepsilon\sigma T_a^4$ is ignored to simplify the derivation. The following ODE to determine the surface temperature T_s is obtained

$$(3kL + 4\varepsilon\sigma L^2 T_s^3) \frac{dT_s}{dt} = 3\alpha q_h - 3\alpha\varepsilon\sigma T_s^4. \quad (28)$$

T_s can be computed numerically with the initial condition $T_s(t_r)=T_1(0, t_r)$. Once T_s is obtained, it is substituted into equation (26) to calculate the temperature distribution.

4. Validation

4.1. Constant heat flux case

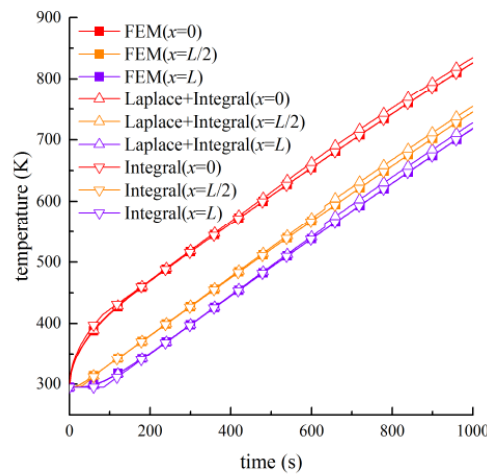
The leading-edge radius significantly influences the heat flux. Heat flux is inversely proportional to the square root of the radius. For hypersonic vehicles with sharp leading edges, the maximum temperature might exceed 2000°C, in which case the widely used C/C material in blunt leading edges is not feasible because of the severe oxidization issue. Ultra-high temperature ceramics (UHTCs) [9] have high melting temperature and good chemical and thermal stability. Unlike the working mechanism of traditional insulation materials, UHTCs normally have a high thermal conductivity and transfer heat through the material and reradiate it at cooler locations away from the stagnation region.

Considering a 120mm UHTC leading edge and the 1-D thermal model shown in figure 1 is used. ZrB₂ with SiC of 20% volume fraction is adopted and the material thermal properties are shown in table 1. The aerodynamic heating $q_h=200\text{kW/m}^2$ is applied at $x=0$. Radiation to the ambient atmosphere is considered with a constant emissivity of 0.86, which is typical for the TPS exterior surface treatments. Both ambient temperature and initial structural temperature are assumed to be 295K.

Table 1. Materials thermal physical properties.

Parameter	ρ (kg/m ³)	k (W/(m·K))	c (J/(kg·K))
ZrB ₂ /20vol%SiC	5570	99	625

The analytical results of the previously mentioned methods are compared with those obtained by FE analyses. The FE model is established using the 2-node link element (DC1D2) with a refined mesh density. Figure 2 shows the temperature variations with time for $x=0$, $L/2$ and L . It is seen that the combination of Laplace transform and integral method can predict the temperature accurately at the beginning while later give slightly higher results than the FE results. The prediction of the integral method has a small deviation at the early stage, after that it approximately coincides with the FE results.

**Figure 2.** Temperature variations with time at $x=0$, $L/2$ and L .

4.2. Variable heat flux case

The integral approximate method can also deal with variable heat flux cases. A transient heat flux shown in figure 3 is applied at $x=0$. The other parameters are the same as those used in the constant heat flux case in section 4.1.

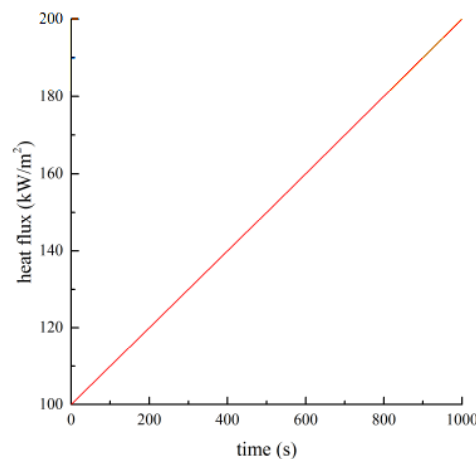
**Figure 3.** Heat flux profile.

Figure 4 shows the comparison between the results of integral method and FE analyses. Similarly, the maximum error occurs at the quite early stage of the whole heating process. Afterwards, the integral method shows a good fit with the FE results.

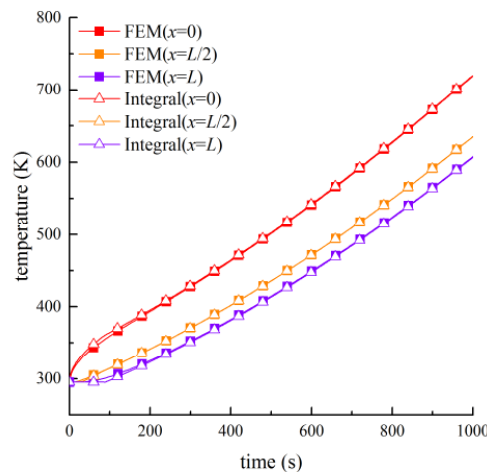


Figure 4. Temperature variations with time at $x=0$, $L/2$ and L .

5. Conclusion

The approaches of Laplace transform and integral method are adopted to predict the temperature distribution through the TPS thickness. Radiation boundary conditions are taken into account. Both constant heat flux and variable heat flux cases are studied. Comparisons with the FE results show that both approaches can predict the temperature distribution satisfactorily under constant heat flux and the integral method is also capable of solving variable heat flux cases with a relatively high accuracy.

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Acknowledgments

This work was supported by Funding of Jiangsu Innovation Program for Graduate Education [Grant No. CXLX13_163], the Fundamental Research Funds for the Central Universities [Grand No. NZ2016101] and a project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).