IOP Conf. Series: Materials Science and Engineering **269** (2017) 012008 doi:10.1088/1757-899X/269/1/012008

Delamination Analysis of a Multilayered Two-Dimensional Functionally Graded Cantilever Beam

V Rizov[1]

[1]Professor, Department of Technical Mechanics, University of Architecture, Civil Engineering and Geodesy, 1046 - Sofia, Bulgaria

Abstract. Delamination fracture behaviour of a multilayered two-dimensional functionally graded cantilever beam is analyzed in terms of the strain energy release rate. The beam is made of an arbitrary number of layers. Perfect adhesion is assumed between layers. Each layer has individual thickness and material properties. Besides, the material is two-dimensional functionally graded in the cross-section of each layer. There is a delamination crack located arbitrary between layers. The beam is loaded by a bending moment applied at the free end of the lower crack arm. The upper crack arm is free of stresses. The solution to strain energy release rate derived is applied to investigate the influence of the crack location and the material gradient on the delamination fracture. The results obtained can be used to optimize the multilayered two-dimensional functionally graded beam structure with respect to the delamination fracture behaviour.

1. Introduction

The functionally graded materials are manufactured by mixing of two material constituents. These new un-homogeneous materials permit tailoring of their properties in one or more spatial directions during manufacturing. This fact is one of the basic advantages of the functionally graded materials over the traditional structural materials. Multilayered functionally graded structural members and components are made by bonding of layers of different functionally graded materials. Usually, the delamination fracture, i.e. the separation of layers, is the earliest failure mode of the multilayered structures [1, 2]. The delamination reduces the strength and stiffness, complicates the post-buckling behaviour of structural members loaded in compression and may lead even to catastrophic failure of the entire structure.

The main objective of the present paper is to develop an analysis of the delamination fracture behaviour of a multilayered two-dimensional functionally graded beam in terms of the strain energy release rate. The solution obtained is used to investigate the influence of crack location and material gradient on the delamination fracture.

2. Solution to the strain energy release rate

The present paper reports a delamination fracture analysis of the multilayered two-dimensional functionally graded cantilever beam shown in figure 1. The beam is made of an arbitrary number of horizontal layers. Perfect adhesion is assumed between layers. The material is two-dimensional functionally graded in the cross-section of each layer. Also, the layers may have different thickness and material properties. A delamination crack of length, a, is located arbitrary between layers. The external loading consists of a bending moment, M, applied at the free end of the lower crack arm. Obviously, the upper crack arm is free of stresses. The thicknesses of the lower and the upper crack arms are h_1 and h_2, respectively. The beam length is *l*. The beam has a rectangular cross-section of width, *b*, and height, 2*h*. The right-hand end of the beam is clamped.

The strain energy release rate, *G*, is expressed as [3].

$$G = \frac{dU}{dA} \tag{1}$$

where

$$dA = bda \tag{2}$$

is an elementary increase of the crack area, *A*. In (2) and (3), *U* is the beam strain energy, da is an elementary increase of the crack length.

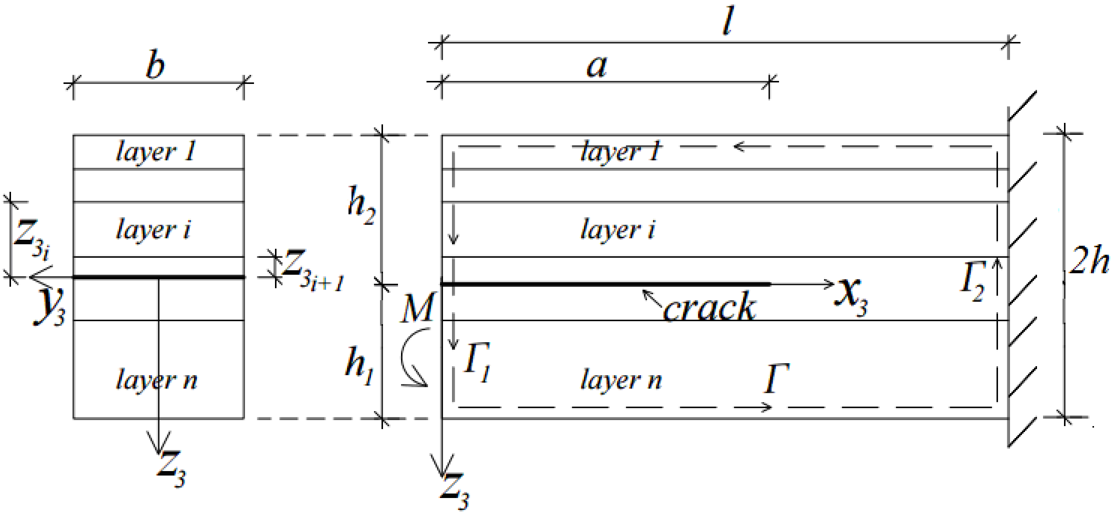

Figure 1. Geometry and loading of a multilayered two-dimensional functionally graded cantilever beam.

By integrating the strain energy density in the lower crack arm (the upper crack arm is free of stresses) and the un-cracked beam portion $(a < x_3 < l)$, the beam strain energy is written as

$$U = a\sum_{i=1}^{i=n_L} \int_{z_{1i}}^{z_{1i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} u_{L_i} dy_1 \right) dz_1 + (l-a)\sum_{i=1}^{i=n} \int_{z_{2i}}^{z_{2i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} u_{U_i} dy_2 \right) dz_2 \tag{3}$$

where n_L and n are numbers of layers in the lower crack arm and the un-cracked beam portion, respectively. The strain energy densities in the *i*-th layer of the lower crack arm and the un-cracked beam portion are denoted by u_{L_i} and u_{U_i}, respectively. In (3), z_{1i} and z_{1i+1} are the coordinates, respectively, of the upper and lower surfaces of the *i*-th layer of the lower crack arm, y_1 and z_1, are the centroidal axes of the cross-section of the lower crack arm (z_1 is directed downwards). The centroidal axes of the cross-section of the un-cracked beam portion are denoted by y_2 and z_2.

By substituting of (2) and (3) in (1), one obtains

$$G = \frac{1}{b}\sum_{i=1}^{i=n_L} \int_{z_{1i}}^{z_{1i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} u_{L_i} dy_1 \right) dz_1 - \frac{1}{b}\sum_{i=1}^{i=n} \int_{z_{2i}}^{z_{2i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} u_{U_i} dy_2 \right) dz_2 \tag{4}$$

The strain energy density in the *i*-th layer of the lower crack arm is expressed as

$$u_{L_i} = \frac{1}{2} E_i \varepsilon^2 \tag{5}$$

where E_i is the distribution of the modulus of elasticity in the cross-section of the same layer, ε is the distribution of the longitudinal strain.

The modulus of elasticity is distributed in the cross-section of the *i*-th layer according to the following law:

$$E_i(y_1, z_1) = E_{B_i} + \frac{E_{T_i} - E_{B_i}}{b^2}\left(\frac{b}{2} + y_1\right)^2 + \frac{E_{S_i} - E_{B_i}}{z_{1i+1} - z_{1i}}(z_1 - z_{1i}) \tag{6}$$

where E_{B_i}, E_{T_i} and E_{S_i} are material properties (E_{T_i} and E_{S_i} govern the material gradient along the width and thickness of the layer, respectively).

The distribution of the longitudinal strain is analyzed by applying the Bernoulli's hypothesis for plane sections since the span to height ratio of the beam under consideration is large. It should also be mentioned that since the beam is loaded in pure bending (figure 1), the only non-zero strain is ε. Therefore, according to the small strain compatibility equations, ε is distributed linearly in the cross-section. Thus, the distribution of the longitudinal strain in the cross-section of the lower crack arm is written as

$$\varepsilon = \varepsilon_{C_1} + \kappa_{y_1} y_1 + \kappa_{z_1} z_1 \tag{7}$$

where ε_{C_1} is the strain in the lower crack arm cross-section centre, κ_{y_1} and κ_{z_1} are the curvatures of lower crack arm in the $x_1 y_1$ and $x_1 z_1$ planes, respectively.

By combining of (5), (6) and (7), one arrives at

$$u_{L_i} = \frac{1}{2}\left[E_{B_i} + \frac{E_{T_i} - E_{B_i}}{b^2}\left(\frac{b}{2} + y_1\right)^2 + \frac{E_{S_i} - E_{B_i}}{z_{1i+1} - z_{1i}}(z_1 - z_{1i}) \right]\left(\varepsilon_{C_1} + \kappa_{y_1} y_1 + \kappa_{z_1} z_1\right)^2 \tag{8}$$

The quantities, ε_{C_1}, κ_{y_1} and κ_{z_1}, in (8) are determined from the following equilibrium equations of the cross-section of the lower crack arm:

$$N_1 = \sum_{i=1}^{i=n_L} \int_{z_{1i}}^{z_{1i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} E_i \varepsilon dy_1 \right) dz_1, \; M_{y_1} = \sum_{i=1}^{i=n_L} \int_{z_{1i}}^{z_{1i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} E_i \varepsilon z_1 dy_1 \right) dz_1, \; M_{z_1} = \sum_{i=1}^{i=n_L} \int_{z_{1i}}^{z_{1i+1}} \left(\int_{-\frac{b}{2}}^{\frac{b}{2}} E_i \varepsilon y_1 dy_1 \right) dz_1, \tag{9}$$

where the axial force and the bending moments for y_1 and z_1 are denoted by N_1, M_{y_1} and M_{z_1}, respectively. Figure 1 indicates that $N_1 = 0$, $M_{y_1} = M$ and $M_{z_1} = 0$. After substituting of (6) and (7) in (9) the equations obtained should be solved with respect to ε_{C_1}, κ_{y_1} and κ_{z_1} by using the MatLab computer program.

Formula (8) can be applied also to obtain the strain energy density distribution, u_{U_i}, in the cross-section of the *i*-th layer in the un-cracked beam portion. For this purpose, y_1, z_1, z_{1i}, z_{1i+1}, ε_{C_1}, κ_{y_1} and κ_{z_1} should be replaced, respectively, with y_2, z_2, z_{2i}, z_{2i+1}, ε_{C_2}, κ_{y_2} and κ_{z_2}, where ε_{C_2}, κ_{y_2} and κ_{z_2} are the longitudinal strain in the centre of the cross-section of the un-cracked beam portion and the curvatures of the un-cracked beam portion in the x_2y_2 and x_2z_2 planes, respectively. Analogous replacements should be done in (9) to determine ε_{C_2}, κ_{y_2} and κ_{z_2}.

After substituting of u_{L_i} and u_{U_i} in (4), the integration should be performed by using the MatLab computer program.

The *J*-integral approach [4] is applied in order to verify the solution to the strain energy release rate derived in the present paper. The integration contour, Γ, which coincides with the beam contour, is used (figure 1). It can be observed in figure 1 that the *J*-integral is non-zero only in the segments, Γ_1 and Γ_2, of the integration contour (Γ_1 coincides with the free end of the lower crack arm, Γ_2 coincides with the clamping). Therefore, the *J*-integral value is obtained by summation, i.e. $J = J_{\Gamma_1} + J_{\Gamma_2}$, where the *J*-integral values in segments Γ_1 and Γ_2 are denoted by J_{Γ_1} and J_{Γ_2}, respectively. The MatLab computer program is used to perform the integration of the *J*-integral. The result obtained matches exactly the solution to the strain energy release rate, which is a verification of the fracture analysis of the two-dimensional functionally graded multilayered beam developed in the present paper.

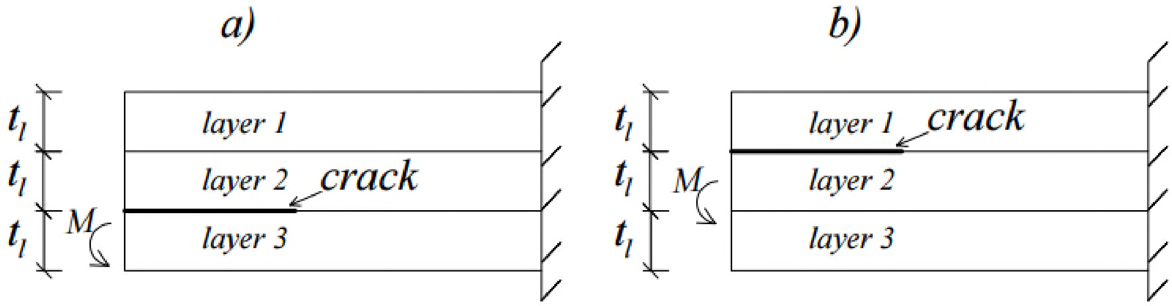

Figure 2. Two three-layered beam configurations.

The influence of material gradient and crack location on the delamination fracture behaviour is investigated.

For this purpose, calculations are carried-out by using the solution to the strain energy release rate (4). Two three-layered beam configurations are analyzed in order to evaluate the influence of the crack location (figure 2). Each layer has thickness of t_l (figure 2). The crack is located between layers 2 and 3 in the beam configuration shown in figure 2a. A beam with a crack between layers 1 and 2 is also analyzed (figure 2b). It is assumed that $b = 0.025$ m, $h = 0.0045$, $t_l = 0.003$ m and $M = 70$ Nm. The strain energy release rate is presented in non-dimensional form by using the formula $G_N = G/(E_{B_3}b)$. The material gradient in layer 3 is characterized by E_{S_3}/E_{B_3} ratio. The strain energy release rate is plotted against E_{S_3}/E_{B_3} ratio in figure 3 for the two three-layered beam configurations. The curves in figure 3 indicate that the strain energy release rate decreases with increasing of E_{S_3}/E_{B_3} ratio (this is due to the increase of the beam stiffness). It can also be observed in figure 3 that the strain energy release rate decreases when the crack location is changed from this shown in figure 2a to that in figure 2b. This finding is attributed to the increase of the lower crack arm thickness.

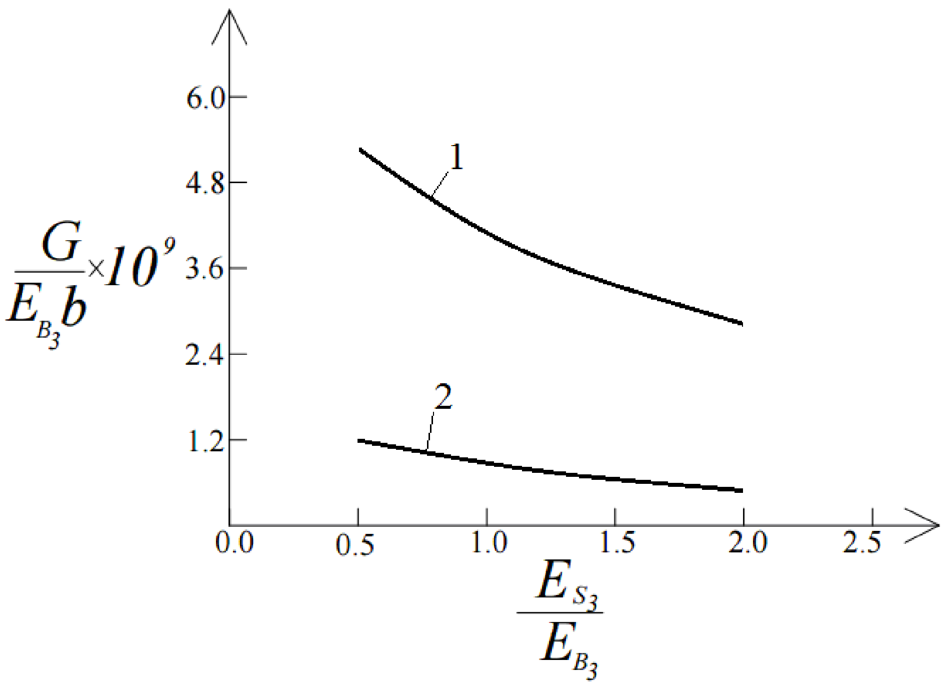

Figure 3. The strain energy release rate in non-dimensional form presented as a function of E_{S_3} / E_{B_3} ratio (curve 1 – for the beam configuration shown in figure 2a, curve 2 – for the beam configuration shown in figure 2b).

3. Conclusions

An analysis of the delamination in a multilayered two-dimensional functionally graded beam is performed. A solution to the strain energy release rate is derived. The solution is applicable for a beam made of an arbitrary number of horizontal layers. Perfect adhesion is assumed between layers. Each layer may have individual thickness and material properties. The material is two-dimensional functionally graded in the cross-section of each layer (the modulus of elasticity varies continuously along the thickness as well as along the width of each layer). The delamination is located arbitrary between layers. The solution obtained is verified by analyzing the delamination fracture with the help of the *J*-integral. The influence of the material gradient and the crack location on the strain energy release rate is investigated. It is found that the strain energy release rate decreases with increasing the thickness of the lower crack arm (the upper crack arm is free of stresses). The increase of E_{S_3} / E_{B_3} ratio leads also to decrease of the strain energy release rate. The results obtained can be applied for optimization of multilayered two-dimensional functionally graded beams with respect to the delamination fracture behaviour.

References

[1] Dahan I, Admon U, Sarei J, Yahav B, Amar M, Frage N, Dariel MP 1999 *Mater. Sci. Forum* **308-311** 923-9.
[2] Hsueh C H, Tuan W H, Wei W C J 2009 *Script. Mater.* **60** 721-4.
[3] Alexandrov A 1990 *Fundamentals of theory of elasticity and plasticity* (M: Vishaya shkola).
[4] Broek D 1986 *Elementary engineering fracture mechanics* (Springer).

Acknowledgments

The present study was supported financially by the Research and Design Centre (CNIP) of the University of Architecture, Civil Engineering and Geodesy (UACEG), Sofia (Contract BN – 198/2017).